Optimal Control
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Lecture - 24
Interior Point Method for Solving Optimization Problems (Contd.)

So, last class we were discussing about the interior point method, that means if you have a linear programming problem or the convex optimization problem, we can solve by using interior point method. What is this method is that? First you have to guess one point inside the feasible region, and then we start the process, iterative process.

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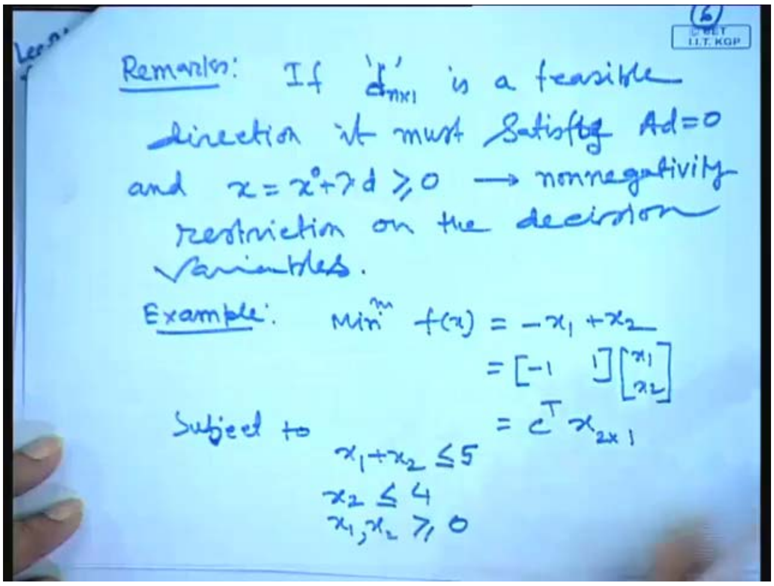

So, if you see this we have considered a simple example, minimise this function, that function is linear subject function, subject to equality and inequality constants. In this case all are inequality constants. Then, if you represent that problem in graphically, we will see this is the feasible region. If you see, this one is the feasible region. And we have taken initial, if you take the initial point a here whose coordinates is 1.5 and 1.5, that means these are the equidistance from the coordinate axis.

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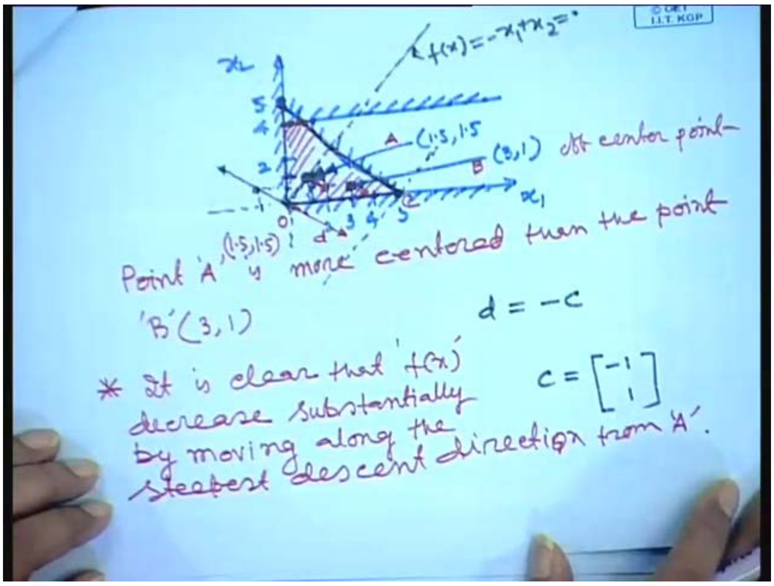

Another point have shown it take, this is the point b whose coordinates at 3 comma 1, this is half of centre point. If you start the process with b point, half centre point then you will see we are moving towards the, one side of the wall. So, we are in turn, we are on the outside on the border of these interior point, what is the called feasible region. It is not an interior point in the, on the border of the feasible point. To avoid that one, what we did it.

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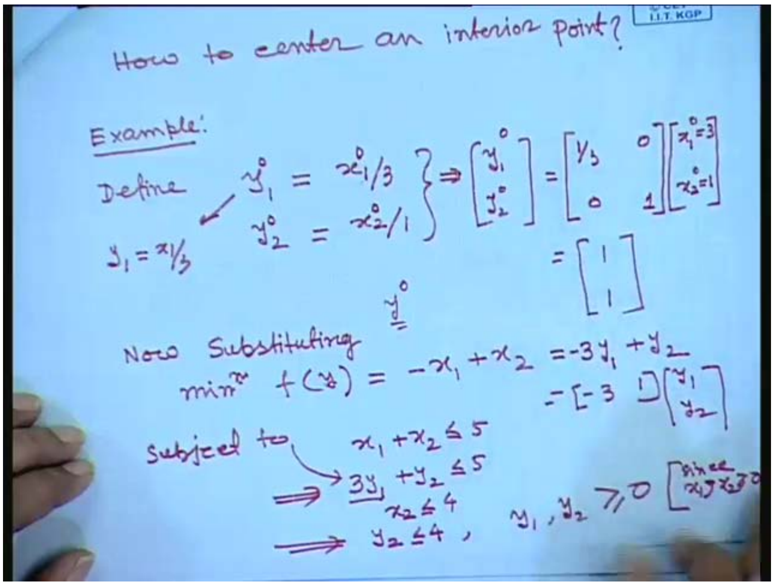

If you see this one, we have made a one simple transformation, that transformation is called affine transformations, means linear transformation so that, in transform coordinate system the coordinates are in the equidistance from the coordinate axis. So, that we have shown it how to select the, that is transform system matrix, how to selected it. We have considered that transfer function coordinate y 1 0, y 2 0.

Then, what is the initial and original coordinate axis what is the x 1 0, that we are dividing with the, that element. If you divide by these elements similarly here, then in transform system that value will be 1 that is what we have shown it. That means in transform systems, in transform coordinates the our initial interior point is equidistant from the two coordinate axis that we have seen.

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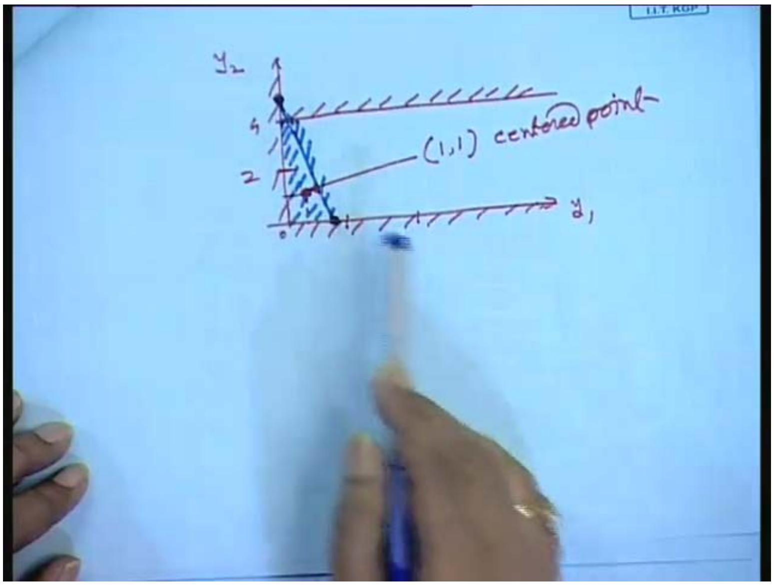

Now will see more general, if you see the transform system. Now you see our point is here, b point now transformed into a new coordinate axis 1 1 point, that is equidistance from y 1 1, y 1 axis and y 2 axis, they are in equidistance 1 1 point. In general, now we can, will show it how to proceed that one.

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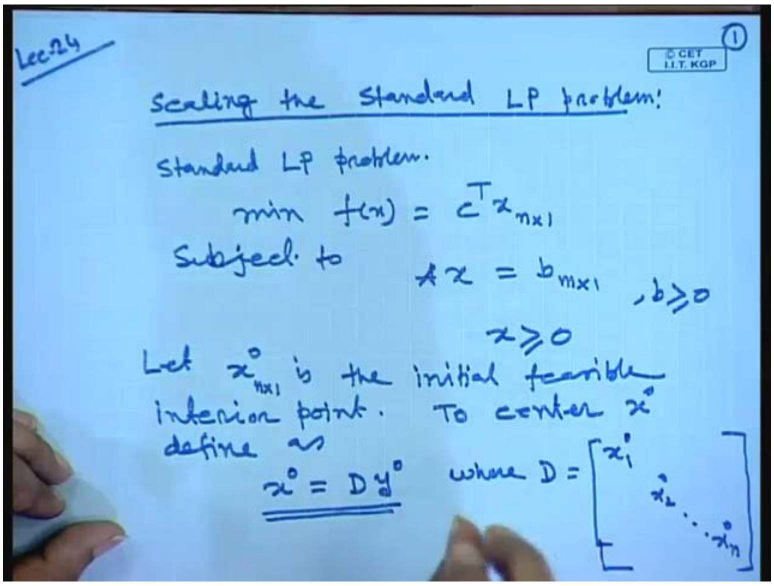

So, first we consider the scaling of a what is called standard LP problem, how to do, then after doing scaling that will, in other words how to select that transformation matrix in new coordinate axis.

So, the scaling, the standard LP problem so standard LP problem if u recollect, whatever is the LP problem is given, we have to convert into standard LP problem first. So, our standard LP problem is minimise f of x, which is nothing but c transpose x, whose dimension if you consider n cross 1. Then subject to, subject to A into x is equal to b, b is the number of equality constant. Even in original problem if there is an inequality constant is there, then we know how to convert into a equality constant by introducing slag variable, surplus variable and artificial variable. So, we know these things.

So, where b is, in the standard LP problem is b is greater than equal to 0 and x is greater than equal to each element of x is greater than equal to 0. So, this is the our standard LP problem. Let, x superscript 0 whose dimension n cross 1 is the initial, is the initial feasible interior point, feasible interior point. To centre this point and this point we are considering it is half centred, that means the distance from the coordinate axis are not equal each. So, we consider how to centre at 0 defined as, to centre 0 we defined as x 0 is equal to D y of 0, y of 0 means that coordinates systems. So, this transformation we have used.

Next question is, what should be the choice of D? that D will select d like this way x of 0, that if you consider the, How many elements of x of 0 are there? any elements so each element of 0 2 0 dot dot x n 0. So, this is the choice of our transformation matrix, which will convert the coordinate x that initial what is called interior point in the feasible region, will convert into a what is call transform system equivalent, what is called the free interior point.

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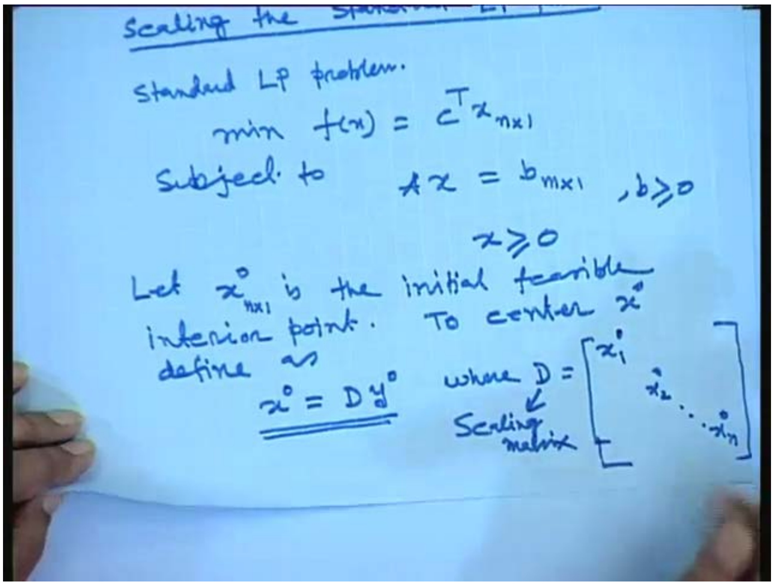

So, if you see this one and this is called the scaling matrix or transformation matrix. So, if you look at this one now, what is y 0, then you take the transpose of that one, agree? y 0, y 0 of this one is nothing but a D inverse, you have to take inverse of D, D inverse x of 0 and what is D inverse? You see D inverse is nothing but a 1 by x 1 0, because it is a diagonal matrix and reciprocal inverse of this one, reciprocal of each diagonal element. So, this then 1 by x 2 of superscript 0, and this way 1 by x n superscript 0. So, this is our D multiplied by x 0, x 0 is x 1 superscript x 2 superscript 0 and dot dot x n superscript 0.

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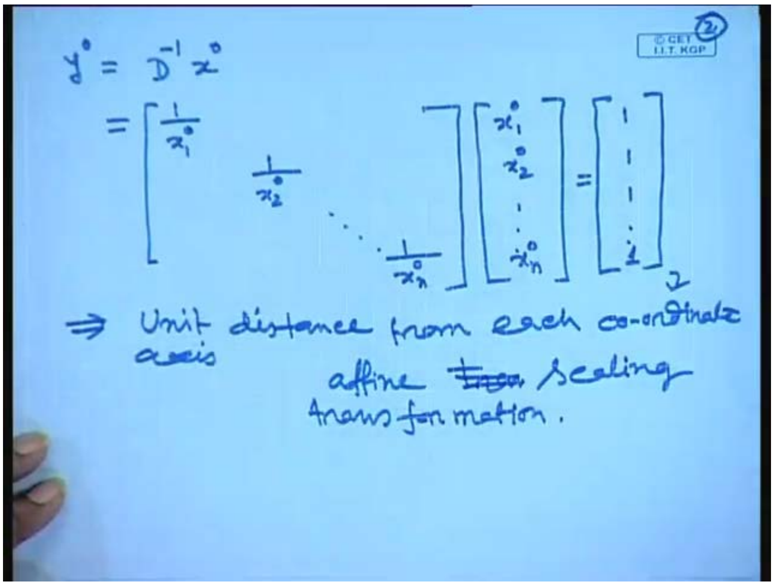

So, if you multiplied by these thing, you see in coordinate system the points will be 1 1 1 dot dot last element is 1. This indicates that in coordinate axis, that all the point, new point in coordinate axis are equidistance from each axis and they are in one distance y 1 coordinates is, where the distance is 1 from this one. So, that point in coordinate axis point is equidistant from each axis. So, it is now centered point in that transform, what is call axis. So, you can write it this implies that unit distance from each coordinate axis from each coordinate axis.

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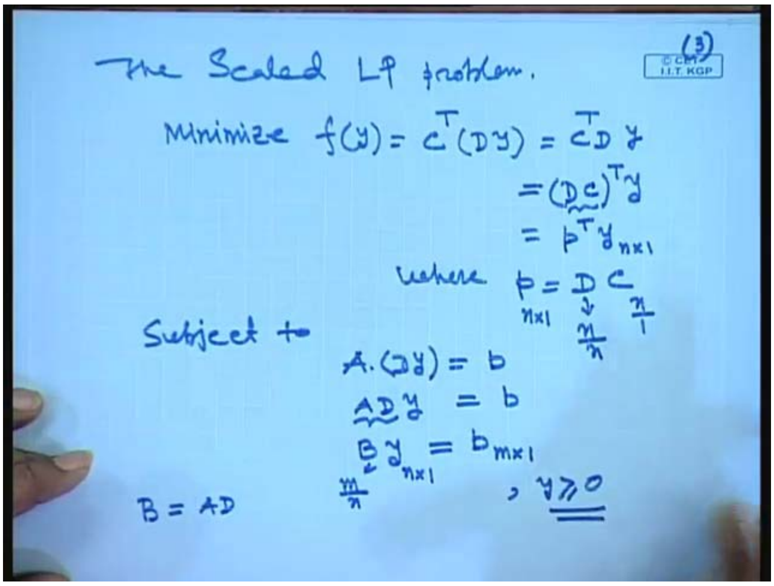

So, this transformation is called linear transformation because they are related with the linear expressions that sometimes, it is called affine transformations, affine transformations or affine scaling transformation. Now, what is the scaled LP problem, original problem is was that one, now we have made transformation.

Now we will say what is the our scaled LP problem, the scaled LP problem, now we are considering the scale LP problem. Now, the scale LP problem is minimise f of x now x, I will express in terms of transformed coordinates f of y is equal to c transpose original objective function was c transpose x. But x is now you see is related to y, new coordinate systems is D y, in place of x I have written D y. So, it is nothing but a C transpose D y, which I can write it if you see it is nothing but a D C whole transpose into y. That means, we know the properties of product of matrices and whole transpose we can write it in reverse ordered each matrices transposes.

Since D is diagonal matrix so we do not need it for to write here D transpose. So, this so let us call this D C I denoted by small p and this if you see it is a vector and D C transpose is a row vector and D C is a column vector. So, that D C, this into y so where p is equal to D C and this dimension you can say that, this dimension is n row 1 column and this dimension is n by n. So, this three dimensional will be n cross 1.

So, this is the now transform coordinate system, our objective function is a equivalently I can write for original system objective function, equivalently it transformed to p transpose y. y is the transformed coordinates and p is what, D into C that means scaling matrix multiplied by C objective function, that coefficient matrix of objective function. So, that subject to what? Subject to our A x is equal to b, that is standard LP problem. Now A x, I convert into a transform domain that is our D into y is equal to our b.

Now we can write it A into D y is equal to b then I denoted by A into D is equal to b, B into y is equal to b into capital B into y is b and that dimension m cross 1, y dimension, if you see that y dimension is n cross 1 here also, I written n cross 1. So, and D dimension naturally this dimension is m cross n. So, our b is what? Our b is equal to A into D and p is equal to D into C, so keeping this thing in mind, not only these our y is all greater than equal to 0. Since the x is greater than equal to 0, in this expression if you see that, how they are related x is greater than equal to 0 and the scaling matrix. That each element is positive quantity, then this if you take inverse, this will be a greater than equal to 0, that y is.

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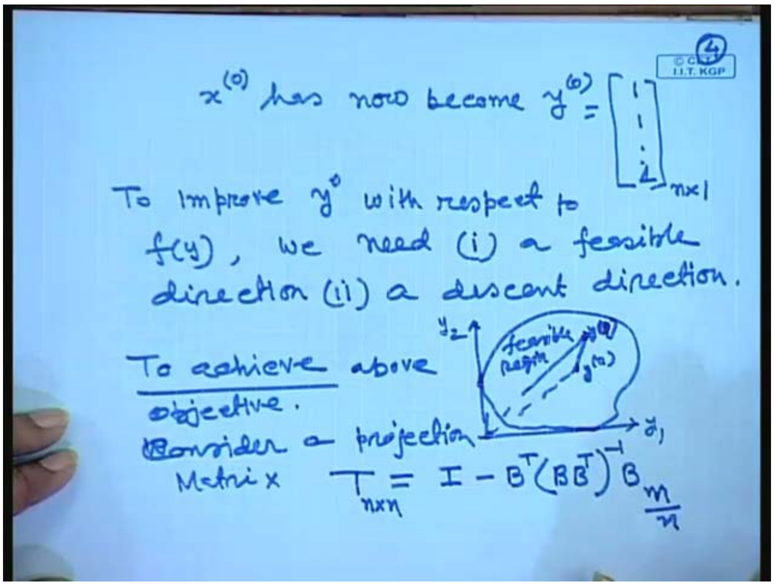

So, with this one our original feasible interior point, our original feasible interior point x has now become, has now become y of 0 and these coordinates are if you see we have seen 1 1 dot dot and last is 1. And this dimension is n cross 1, this indicate we have made in transform coordinate axis. Now, our interior point at the centre means, it distance from each coordinate axis are same in this case 1 1 1.

So, this is what we have seen, Now let us seem now our job is in transform coordinate system from y 0, we have to move in such a direction so that, we can achieve two objectives. One objective is to, we have to move in such a direction, one object is it should be in the interior point, that after the movement is made, it then that point, view point must be in the interior point, in the feasible region. Next is, that point must be, that point should be is such that, that objective function below should reduce compared to the earlier values. So, two objectives were to meet. So, to improve y 0 with respect to f of y, we need one a feasible solution or feasible direction one first objective, second is we need a descent direction.

So, this object we have to prove. So, if you see for two dimensional case if you say, if this is the our feasible region, let us call this our feasible. Now y 0 is this one, we have move from here in such a direction, then our vector coordinates are of this one is here, this y of 0, this is y of 0, this is y of 1 and it should be in the feasible region. This is the feasible region and not only this, the function below at y is equal to y 0 on the transformed coordinate axis, what is the function value and that function value must decrease from this value.

So, two objectives we have to achieve and for that one, we have to see which direction will move it, such that both the, I mean objectives are satisfied. To achieve this one, what we are doing it here, first we will to achieve above objectives, what we consider a, what is called position matrix. Consider a projection matrix, with the knowledge of our equality constraints, equality constraints in transform domain. In equality constant is B y is equal to small b, with the knowledge of b matrix we will form a what is called projection matrix. And that projection matrix, this projection t is defined as I minus B transpose then B, B transpose whole inverse into B. And this dimension, if you see this one that dimension of B is n cross n and if you proceed this one, this dimension is t dimension n cross 1.

So, with the knowledge of B matrix in the equality constraints of our transform coordinate system say, them then we formed a what is called projection matrix. This projection matrix indicates that, if you multiplied by these by B, the result is 0. And this positive projection matrix in addition to of this positive, what is called the projection matrix, this projection matrix as some properties, you one can easily verify that properties.

So, one property is there that the T transpose is equal to T, now look at this one point, if you take the transpose of this one, then it will be nothing but a same matrix you will get it. It is easy to prove from the structure of that T. Next is that, whatever just put the value of T, take the transpose of this one, this is one property. And second property is T square is nothing but a T, how look this one, proof of second one is easy to check, you multiply T into T and T is what I minus B transpose B of B transpose whole inverse into B.

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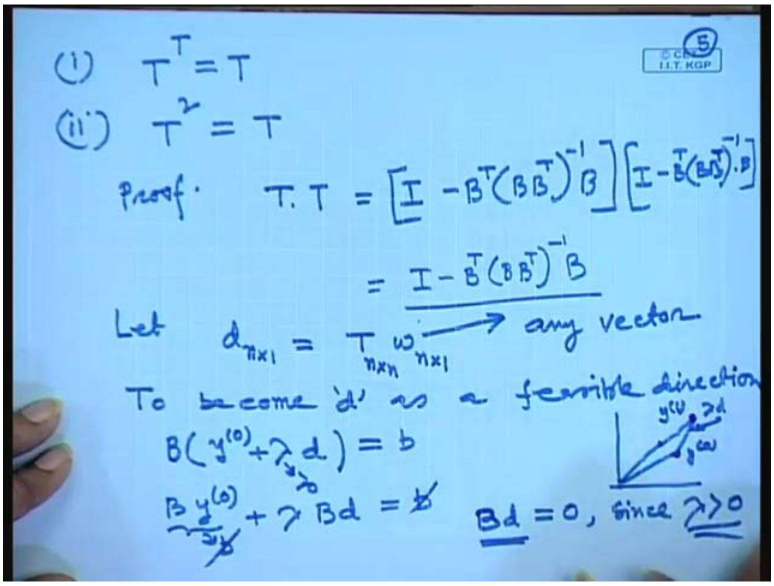

So, that you multiplied by I minus B transpose B of B transpose whole inverse, whole inverse, then multiplied by B. So, if you expand this one, ultimately you will get it I minus B, B of B transpose whole inverse B. So, kindly expand this one and prove yourself this equal to this. So, there are two properties are there for projection matrix.

Now, I told you I have to find out the direction in such a way so that, the our view point from y 0 to y 1, let us consider viewpoint that y 1 point must be in the interior of the feasible region, interior point. Second, the direction what will go that is the new point, that new point function value, objective function value must be less than the previous objective function value at y is equal to y 0 of y superscript 0. So, this two objective are fulfilled so our choice of this one, let the our direction vector choice n cross n is equal to T into w, T dimension n cross n and w is any vector of dimension n cross 1, it is any vector dimension and so when is any vector then D is any directions.

So, now see this one what will be it became, to become D, to become D as a feasible direction. In other words, to be a interior point of the feasible region D, then see this one D has to satisfy this condition, that B into let us call our new point, old plus lambda, some scalar which value is greater than 0 multiplied by D. That is our, that means if you think of it in two dimensional case, we are here now y 0, we moved some other D, D is in directions and that quantity is lambda into D and our new coordinates, when new point is that one, which is your y of this.

And this must satisfy our equality constraints, that is our b, now let us call B y 0 is equal to lambda is a scalar quantity, I can take it anywhere in the second term. This equal to b and this since y 0 was the interior point, that it must satisfy the, our constant. So, this value is our b so this b, this b cancelled so our things is coming that ultimately, this boils down to beta is equal to B into d is equal to 0.

Since, lambda is greater than 0, any scalar quantity, this positive quantity that means, form that the direction is there, in this direction you just multiply. So, this condition if it is satisfy B into d, if it is 0, B into d is 0, it means that our new point belongs to the, in the feasible region, many interior point. So, our d choice is here, once you select the our projection matrix w can be any matrix because now see why it is any matrix.

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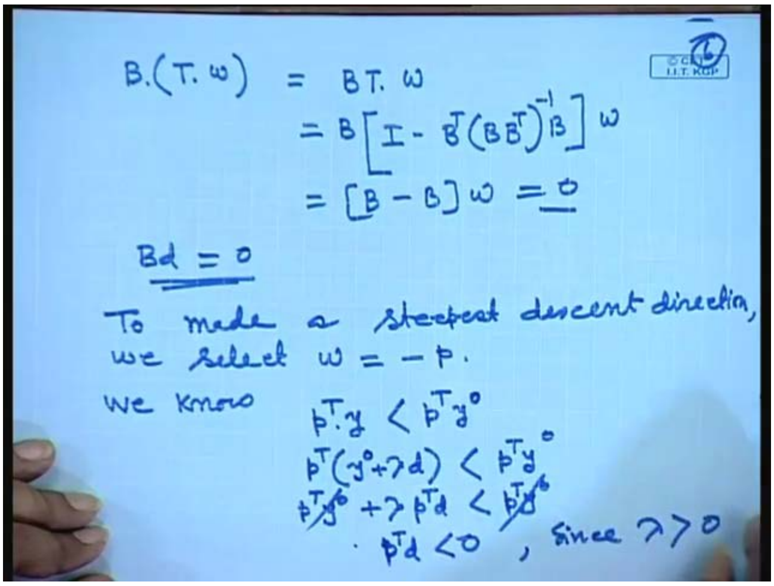

Now you see B into d, B these what we have selected T into w and w is a vector any direction that are these vector. So, let us call it is nothing but a B into T into w and what is B? this and what is T? T am writing I minus B T of B, B T transpose whole inverse into B into w.

So, if you expand that one, it will B minus B multiplied by 0, for any value of vector this result is 0. Only the condition is there, if you see our that B matrix what A and d, that must be a full rank, that is the only conditions, then this inversion is exist. So, this is the so our condition is B into d, B is formed if it is 0 indicates, the direction d should be in such a direction so that, B into d is equal to 0. Then that means we moved from initial point to another point, which is also in the interior point of this our transform coordinates, interior point in the transform coordinates.

So, this is the our direction we got it, now what is a guarantee that you have moved in that form y 0 to y 1. When the direction of d, that is no doubt it is a this equal to 0, B into d is 0, we proved that it is a interior point, but there is no guarantee that what is called objective function value, you will get what is called, what is the objective function value will get less than the, previous objective function value at y is equal to y 0.

So, in order to ensure that one, what we did you see this one. Now, next is your see to make second achievement, to make a steepest descent direction, to make a steepest value comma we select w vector, previously we took w is any vector. But it will satisfy that our, if remove any indication it will be inside the, our interior region. But now question I,s how to get that value of new value, new point which will give you the objective function value, less than the previous value of our previous point, that below objective function value. So, if you select w is equal to minus p, let us see what will be the our cost function values.

We know, our cost function, objective function what is called coordinate system objective function value is what, p transpose what is this y. And that value, if it is descend is that value must be less than equal to p transpose y of 0. And let us call p transpose y is moved from y 0 to y 1 so it is a y 0 plus lambda into d and that objective function value must be less than p transpose y 0. Now, you expand this one y p transpose y 0 plus lambda p transpose d is less than p transpose y 0. So, this this cancel so this implies that p transpose d is less than must be less than 0.

So, if this is the descent direction, means objective function value will decrease if you move from y 0 to new point y 1, that objective function value will decrease, provided this condition is satisfied. Now, since lambda is greater than 0 so this condition is satisfy, when if you choose w is equal to minus p.

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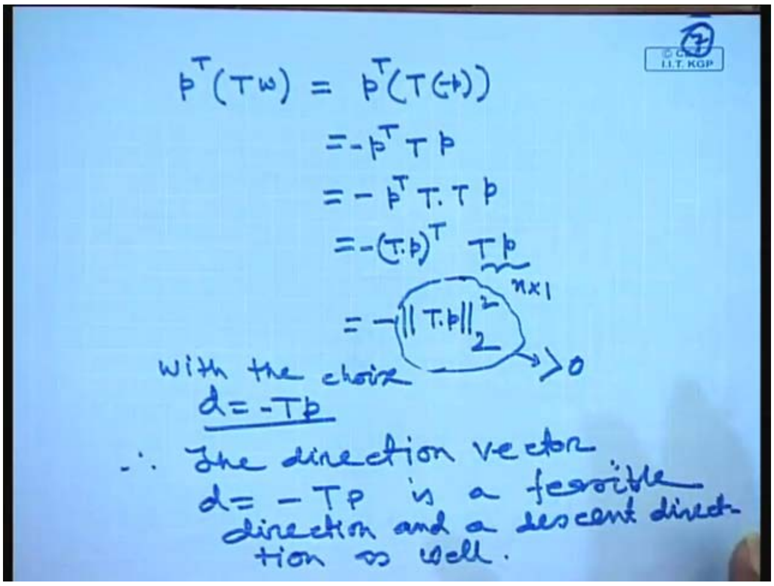

Then it is obvious that, p transpose d will be less than 0 p and d is if you choose d is what, if you see T w, d absent in place of d, we consider. Now I am telling the, if you select the w instead of any direction, now I am w is equal to minus, w is I am selecting minus of p w, I am selecting minus of p, you see here, minus p here, if you select it, these that minus p this one. So, it will be coming now p transpose T w minus p minus is called I have to get minus here, then p. Just now excluding the stretched, what this call properties of T, T I can write T square because T square is equal to T, T into T into p. So, I am writing now, I am writing this one T into p, there also T into p whole transpose minus.

So, this and this is what if you see this one, this is our vector of dimension n row 1 column, this will be a, whole thing will be n row 1 column. So, it is nothing but a clear norm of the vector p into T square and this quantity, this quantity is always greater than equal to 0, when this is preferably minus so this indicate with the choice of p is equal to what is call w is equal to minus p, that we are getting with the choice of, with the, with the choice of d is equal to T minus p, d is equal to T minus p. We are achieving the two objectives simultaneously, one is feasible direction we are moving, another is the objective function new point, the objective function value at the new point, value is less than the objective function value at y is equal to the, that earlier point that means, in the y of 0, is less that ensures.

So therefore, the direction vector d is equal to minus T p is a feasible direction and a descent direction as well. So, what is our coming in short that means, if the original problem is given, if it is half centred is they are you transformed this into a new coordinate axis. By selecting the, what is call the scaling matrix after you rest to the what is called coordinate, what is the new coordinate system then you select from your starting point of y 0.

You move in such a direction and that direction d is equal to minus T p and p is the nothing but a coefficient of transform coordinate system, objective function coefficient matrix or coefficient vector, this one p. And T is formed with the knowledge of B matrix, which is called projection matrix, if you select this one, d equal to minus t p will simultaneously achieve both the objectives.

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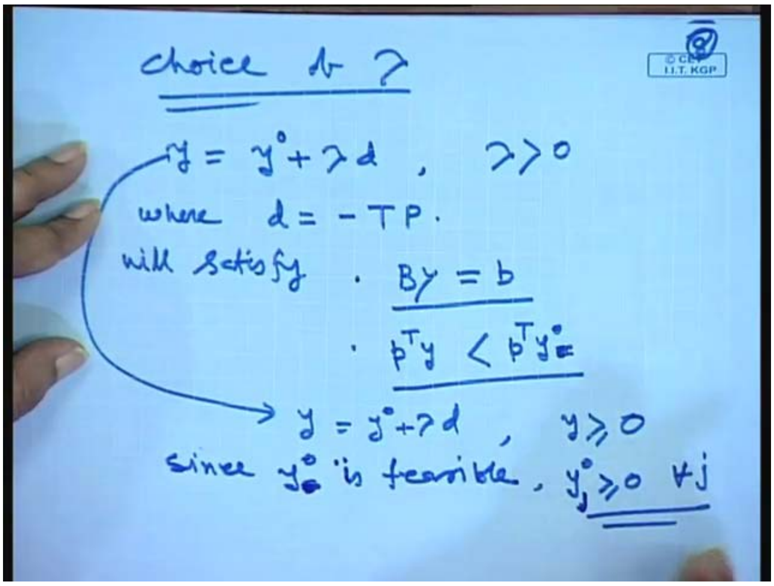

So, this is now, let us take one simple and see, now before that will see, what should choice of our lambda? What should be our choice of choice of that means, in d direction how far will move form y of 0 to what is, what length in the direction of d will move it. So, the lambda will decide what length. So, how to choice of d now, look at this expression y is new point, is we will get into the knowledge of initial point plus lambda into d. Where, lambda is greater than equal to 0 and our d where our choice of d is equal to minus T into p and this choice, this choice will satisfy both the objectives, will satisfy, what will satisfy? Equality constraints in transform coordinate axis B y is equal to small b, this will satisfy one.

Another condition is what will satisfy, p transpose y will be less than equal to p transpose y 0, sorry superscript 0. This both the condition will satisfy this one, now let us see what is mean, this one, if you see this one y is equal to y 0 of lambda d and in our what is called transform coordinate system, our LP problems that means, scaled LP problem you will see, the y values will be greater than equal to 0, y value is greater than equal to 0.

Since y 0 is greater than equal to 0, lambda is positive quantity. So, we have a two choice on l, if d, d element of, elements of d, if it is a positive than what is called the ith element of d is positive. Then it ensure the ith element of new point also positive. Now question is, if the ith element of d is negative then we may get the difference when though y 0 is positive, lambda is positive. But ith element of d is negative, if it is negative these two, the difference of these two we can get negative.

But that depends on the choice of lambda and we can make the choice of lambda so that, in that situation the ith element of y must be positive greater than equal to 0. So, let us see that since you can write it, since y 0, sorry y of 0 is feasible that means y j, jth component of y j is greater than equal to 0, for all j. This is, this implies feasible in this implies.

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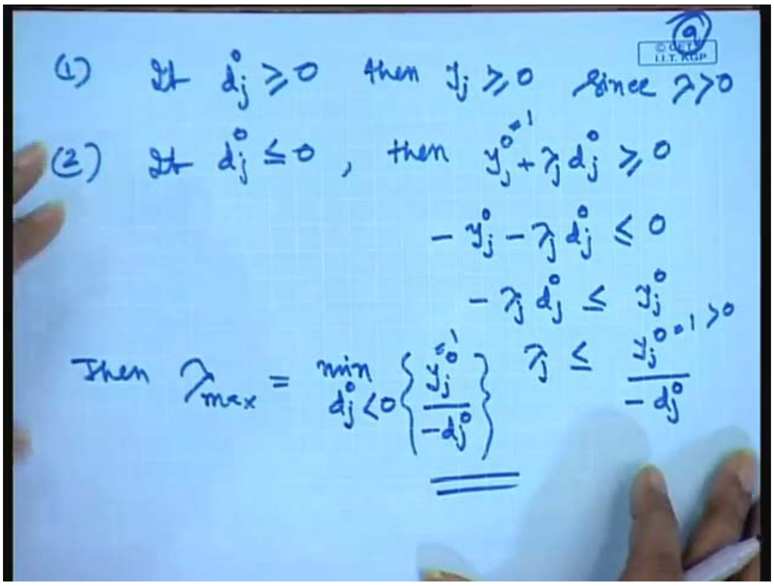

Now, there are two situation I have considered, first situation is if d j of 0 is all are greater than equal to 0, this then y j will be greater than equal to 0. Since, lambda is greater than 0, this is straight forward, there are two. Second situation if d j 0 is less than equal to 0, then y j since lambda is positive, y j is jth element of y j can be negative or positive, depending upon the choice of lambda. So, you are now keeping the choice of what is called, lambda should be make so that, we can get that y jth component of y is positive. Then you can write it y j, y j 0 plus lambda j, d j 0 what we want, greater than 0. If d is less than 0, this we want greater than 0.

Now, both side you multiplied by minus that you know y j 0 value is what, positive and that value after scaling, that transformation and in what is called, in transform coordinated axis, this value is, if you see it is nothing but 1. So, y j both side i multiplying by minus so it is lambda j, d j of 0 is less than equal to 0.

So, now you can write it that one, minus lambda j d j superscript 0 is equal to y j 0. So, lambda j is equal to less than equal to y j 0 minus d j 0. Now, you see the d j 0 is negative, negative and negative positive, this is already positive quantity and it is greater than 0. So, this quantity lambda is positive quantity.

Now, you have a, how many components of y and d j we have? We have a n components j is equal to 1, 2, 3 this, when anyone of the component of d j is positive, we ignore, we do not need it. Because, the sum of y 0, sum of this one will be, sum of this one will be always positive, if d j is 0 here positive, then this will be always 0. So, when d j is positive quantity, that we do not consider we ignore this, if it is a negative then see this one.

Now I am choosing that, what is of value of lambda then lambda max is equal to minimum of lambda j superscript 0 less than 0. For negative value of lambda, see this ratio will be minimum, which case it will be minimum. And that will consider lambda max because this value is positive and this value is d j value is negative, positive ((Refer time: 40:08)).

So, this ratio which that d j component, let us call in d vector there are five components are there is negative, out of these out of these which one is minimum, this ratio because this minus minus of this ratio will be positive, which one is minimum that will consider as a lambda max. And this will ensure, this will ensure that y of, that is y of y j will be always greater than equal to 0. These ensure, this will be greater than equal to 0.

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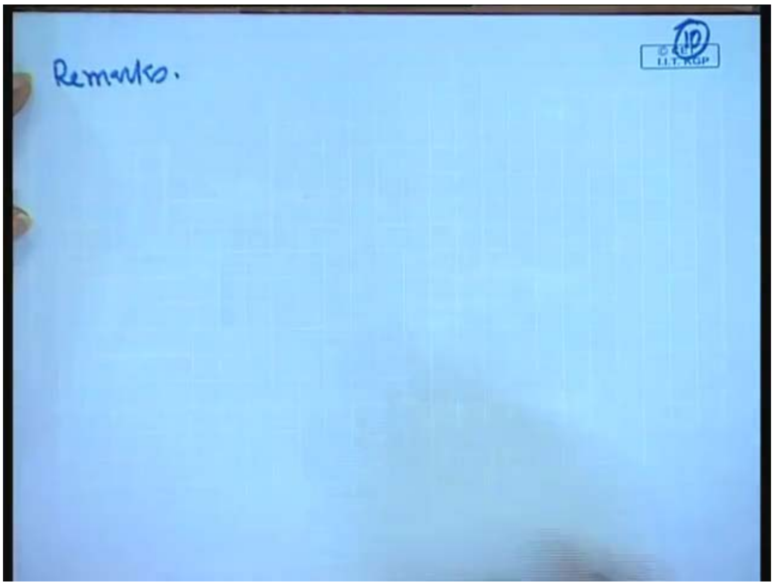

Now, that remarks I am writing. Can you tell me, when it will be the 0?

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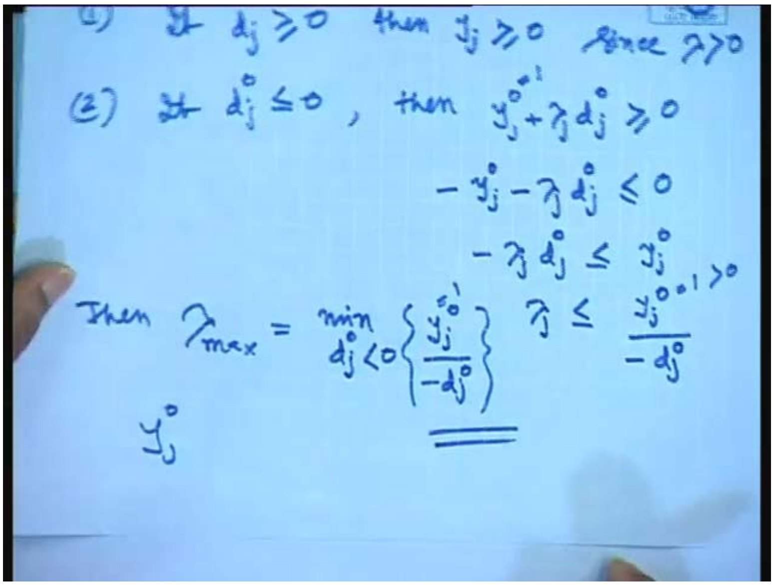

y, y j will be 0 when because I am taking the minimum of this one, which case it is the minimum. Let us call fifth position of d j, j is equal to 5, this is I got it minimum. And corresponding that y j element will be 0 because it will be cancelling that one lambda j value is this one, multiplied by d j jth component 5, jth component 5 denominator cancel, this will be cancelled, then what is call it is same as your y j 0. That means, it will be cancel ultimately will be 0.

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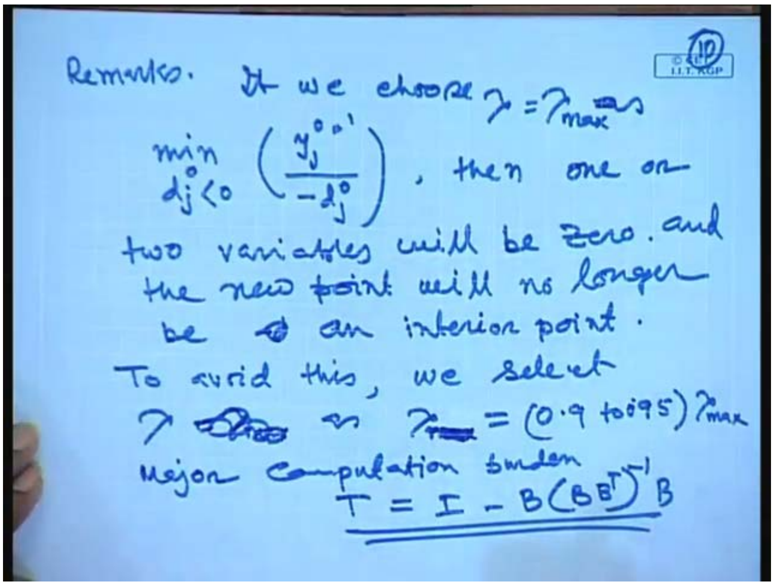

So, our remarks is that if we choose lambda is equal to, lambda max as minimum of delta j superscript 0 less than 0, there is a need for negative quantity only jth component of d is equal to y j 0, y j 0 divided by minus d j 0. And this value is 1 indicates this.

Then one, at least one will be 0 in y j component, at least one it may be more than one, one or two variables will be 0. If it is 0 our new point, our point will no longer be at interior point because it is boundary on the feasible region, it will be boundary on the feasible region, but it is not the interior point of the feasible region. So, you can write it and the new point and the new point will no longer be an interior point.

So, that is why because I told you it will be 0, because if you take lambda is in this lambda max as like this way. So, whatever that problems, because at least one will be 0 when one coordinate axis value will be coordinates of y, will be 0 that means it is on the boundary. Then y is 0 means it is on the boundary, then what wherever that one, what you did it, we whatever the lambda max will get it, to avoid that situation you multiplied by these thing by a 0.9 to 0.95b within this range you make it. If you multiply by lambda max with 0.9 to 95, 0.95 then corresponding that element will not become 0. That is our main aim, before it reaching to the optimal point it will not be 0.

So, to avoid that one, we select lambda, is equal to lambda max as, lambda max is equal to or you see we select lambda ultimately, lambda as a, as lambda is equal to 0.9 to 95 multiplied by lambda max. What is the lambda max? We got it here lambda max, as this one, that you multiplied by 0.9 to 95 and then you consider, that is the our step length in the direction of d will move. And that will overcome that problem, that means that no component of y, new point y will be 0, that means it will not reach to the boundary point of this region. So, if you look at this one our major competition burden to compute that one I minus B, B into B transpose whole inverse B. This a major competition involved in this process.

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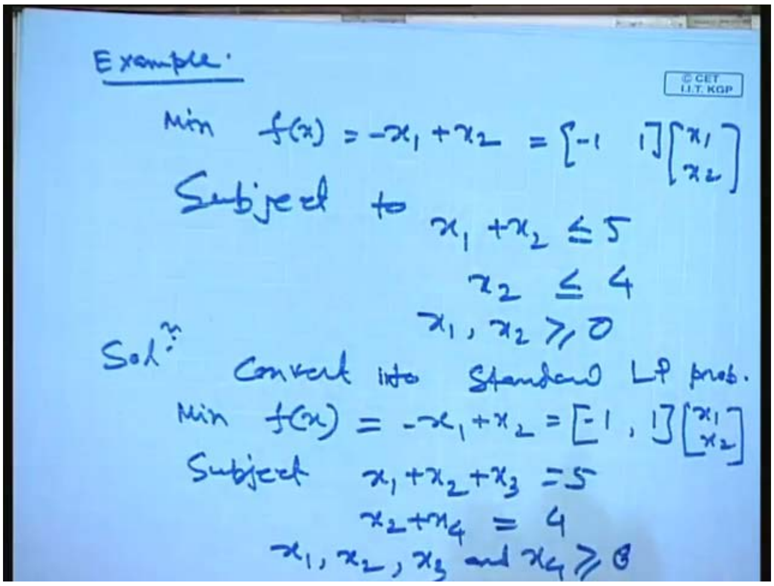

Let us call, we solve the same problem, what we consider earlier and briefly and quickly. I will solve this problems because our earlier problem you have consider, minimise f of x is equal to minus x 1 plus x 2 and subject to this is equal to minus 1, minus 1 1 that x 1 x 2 and our subject to, subject to x 1 plus x 2 is less than equal to 5. Then x 2 is equal to less than equal to 4 and x 1 x 2 is greater than equal to 0. So, it is our LP problem, but it is not a standard LP problem. So, you have to convert first the standard LP problem, this so solution convert to, into, convert into standard LP problem.

So, without much discussion it is a, we have discuss so standard LP problem minimise minus x 1 plus x 2 and it is our minus 1. If you see 1, x 1 and x 2 and subject to, we have to convert into equality constant. So, this quantity is less than 5, that means we had and what is call slag variables, let us call we added slag variables is x 3 is equal to 5 and in this case x 4 x 2 plus x 4 is equal to 4 so our x 1 x 2 x 3 and x 4 is greater than equal to 0. This has now converted into our standard LP problem. Now we identify which one is our a b and c.

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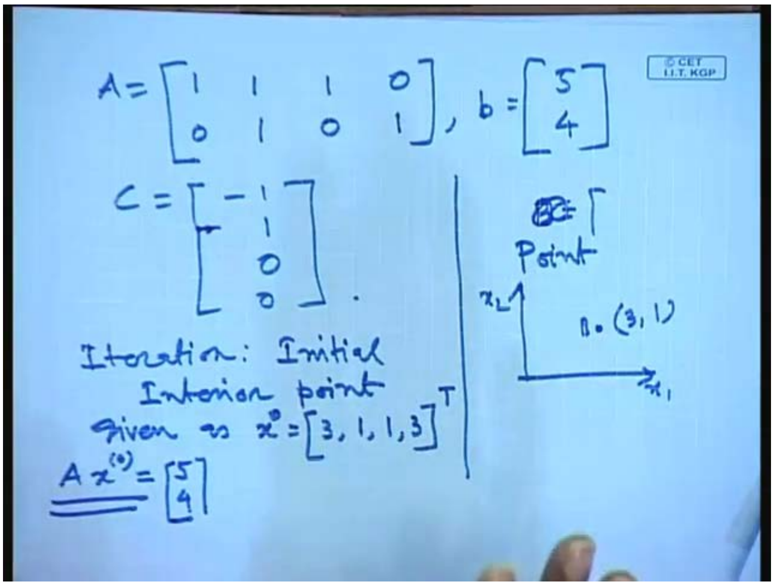

So, if you see our A matrix from this, from this two equation I can write it A matrix quickly I am writing at a coefficient of is a, sorry this, this 0 coefficient of x 4 is 0, then this is 0, coefficient of x 2 is 1 and coefficient of x 3 is 0. Then this is 1, this is A matrix this is B matrix is 5, 4. And our C matrix is what? You see this our C matrix x 1 x 2 coefficient x 3 4 coefficient is 0, x 4 coefficient is 0. So, we will write it, minus 1 1 that is I am writing 0 0 the, this is our C matrix.

So, our basic state if you recollect this one, that first you convert into a what is call our, what is our initial coordinates was there, if you see the, our earlier problem. We have consider the point B is our coordinates, sorry point B coordinates in x and x 1 and x 2 coordinates of the original problem B coordinates was 3 1 and it is half centred. So, what you have to do, then iteration before starting iteration, what you have to see the, our initial interior point, our how many coordinates points are there, we have a four coordinates are there x 1, x 2, x 3, x 4.

So, initial interior point given as or you have to find out as now, I know 3 1 axis x superscript 3 is 3 1. What is x 3 and x 4? We have to select x 3 and x 4 in such a way it satisfy the equality constants. So, in order to check the equality constraints, you have to see A into x of superscript is equal to our case is 5 and 4.

So, first equation is you see here, the first equation x 1 is minus 3 and x 1 is your, sorry x 1 is 3, x 2 is 1, x 3 must be 1, then only it will be 5, x 4 0. So, our things is that is 1, for the time being I can make a x 4 0, but here you see come to this point x 2 is what 1, but I have to get the right inside 4, then x 4 must be 3, then only 1 plus 3 will be 4. So, our, this will be 3 so this, our initial guests, which satisfy the, our equality constant, that we have to check first.

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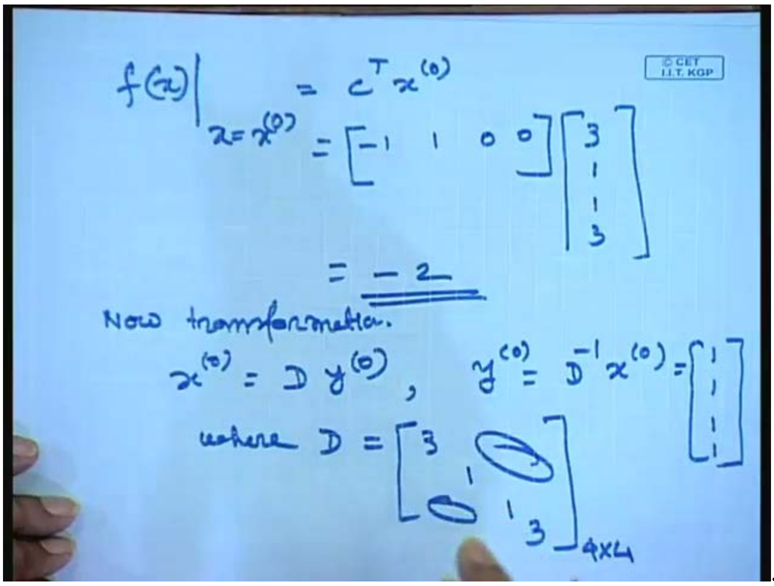

Once you get it that one, then your first see what is the objective function value, at x is equal to x interior point of that one. This is our interior point, we have verified, if you consider ,how you have selected this just we have mentioned it, this one. So, this equal to this, this value if you put it C transpose into x of superscript 0, you have to put the value of C minus 1 1 0 0 and x 0 is 3 1 1 3. If you multiplied by this it is a, is a minus 2 2. So, our at this interior point, the objective function value is minus 2 so what we have to do next.

We have to transform and our new transformation is, now transformations quadrant of transformations, then our x of 0 u 3 is equal to d y of 0 and you now, what is y of 0? y of 0 is our D inverse x of 0. What is our D? If you remember our D is what, 3 1 1 3, this all other elements, this D matrix is 4 by 4, other elements is did so D inverse will be 1 by 3, 1 1 1, 1 by 3 multiplied by this, this will be all 1 1 1 1. So, it is now centred a new coordinate axis, it is in centred means, that point is, that new point y 0 is equal distance from all coordinate axis.

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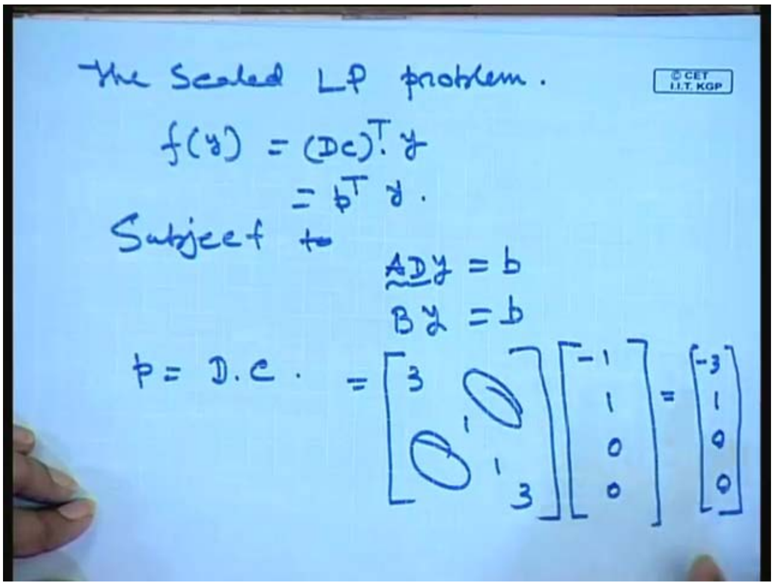

So, now see this one, the scale problem, the scaled LP problem now. What is this scaled problem, f of y is equal to if you, we have done it this things D C whole transpose into y, this is p transpose y. And subject to, if you see A into D y is equal to b and this is I considered B, B into y is a small b.

Now, let us complete what is p, p is D into C and D you know, it is a 3 1 1 3, multiplied by C, C is what minus 1 1 0 0. If you multiplied by this, this is a diagonal matrix other elements are 0, if you multiplied by this I will get it minus 3 minus 3, then this, this 1, this and this if you multiplied by these will get 0, you multiplied by this you will get it 0. So, our p matrix is minus 3 1 0 0 because I need the information of p in order to get that the, what is called directional vector.

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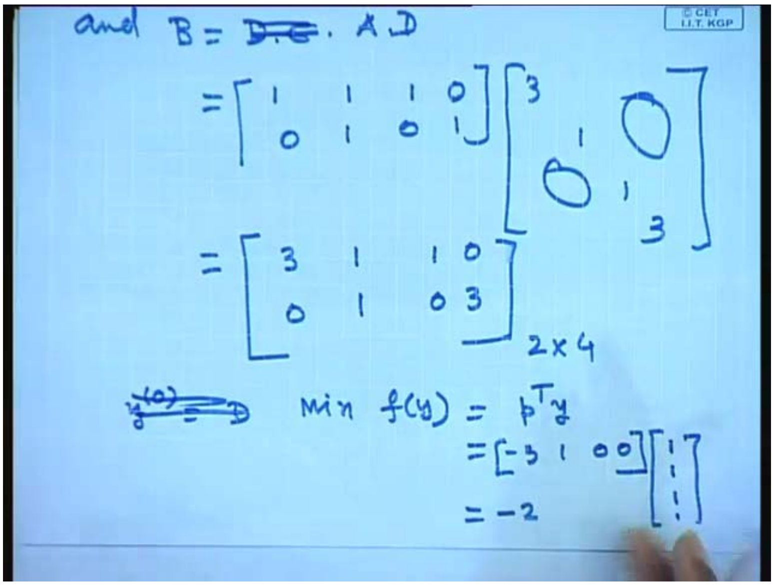

And B, B is what D into C so you put this value of d, sorry this is A into D so A matrix is what 1 1 1 0, 0 1 0 1 and D matrix is 3 1 1 3. So, are all other elements is 0, if you multiplied by this I will get the matrix 2 row 4 columns 3 1 1 0, then 0 1 0 3. Just multiplied this one so this is our matrix B matrix is this one, which is in transform coordinates axis, equality constant will got it, this, this one.

So, now what is our y coordinates, y coordinate is D inverse, that is where calculated or no, I think you have calculated here, we have calculated that y inverse this is what we calculated, we need not to do once again this. So, now let us see in the transformed coordinate axis what is the function value f of y, is nothing but a p transpose of y, p value just now we have calculated value of p. If you see the value of p, what is calculated minus 3 p transpose p this one is a minus 3 1 0 0, then y value what we got it, the y value we got it if you see y of 0, y of 0 value what we got it 1 1 1 1. So, will see this value is minus 2, that is same as our original problem, is get it. So, once you just check the original.

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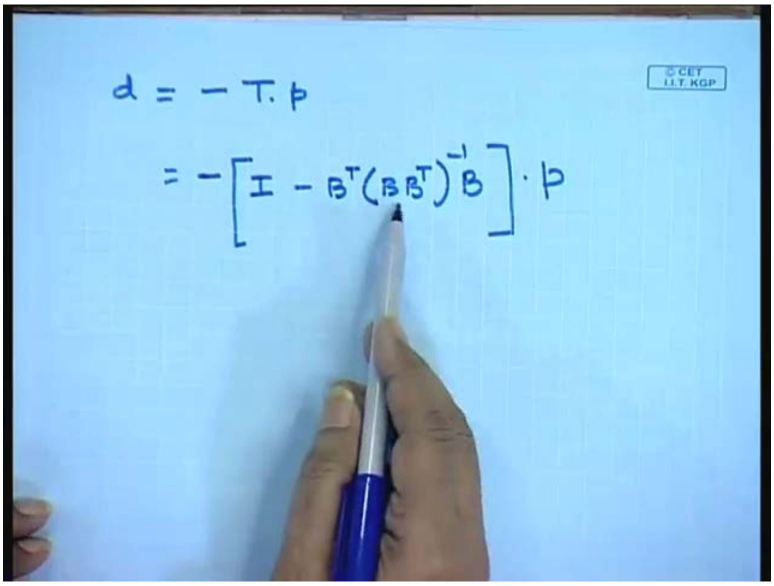

Now what is our p, sorry our directional vector directional d, if you choose minus T into p then this direction it will satisfy simultaneously two condition. One is the feasible direction it will go and not only that, the function value, that objective function value will be less than the previous objective function value. So, this what is T, T is nothing but a if you see I minus B transpose B into B transpose whole inverse B, this is our p into p. So, you put the values of D b you know, p you know you will get the direction of vectors, for this one. So, I will continue the next class after this.