Optimal Control
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Lecture - 23
Interior Point Method for Solving Optimization Problems

So, last class we have discussed the problem formation of quadratic optimization problem using the simplex method. If you recall we have this is the quadratic function our problem is to minimize f of x which is in quadratic form and we have a inequality constant and equality constants.

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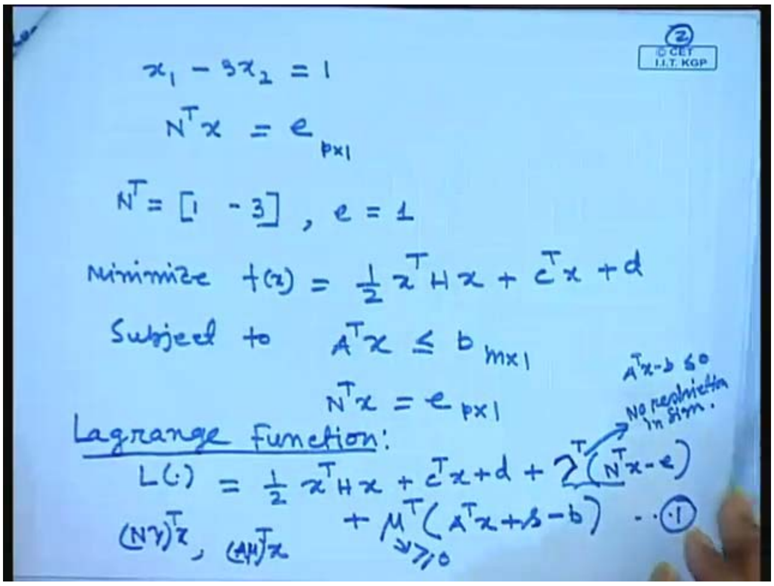

But this inequality constant and inequality constant they are affine function linear functions. So, we have formulated this thing into standard Lagrangian function inequality constant we have convert into equality constant then it is what is called a Lagrangian function is formed. The mu lambda is your Lagrange multiplier which is associated with the equality constant and that equality constant that lambda is associate with equality constant and the lambda value is not restricted that its value can be positive and negative 0.

Whereas, this factor is associated with the inequality constants and these Lagrange multiplier corresponding to inequality constant, we considered as a mu that is greater than equal to 0 when these in this type of inequality constant we have. That is we have discussed earlier in details that why mu value should be greater than equal to 0. So, if you now write the what is called KKT necessary condition we have shown that these are the equation will get it.

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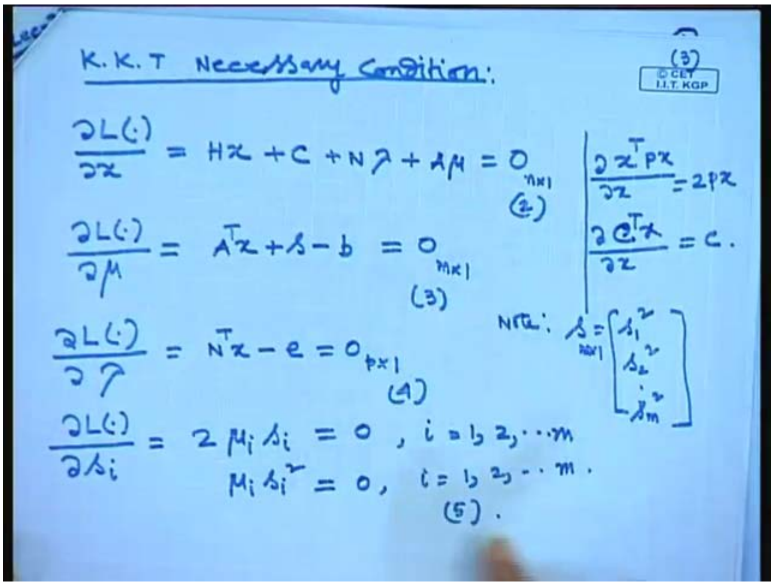

Equation number 2, 3, 4 and 5 so this equation we can write into matrix and vector form which will be equal to into this from. The first is experiment than we have considered mu Lagrange multiplier is with the inequality constant than Lagrange multiplier the equal to what is called that.

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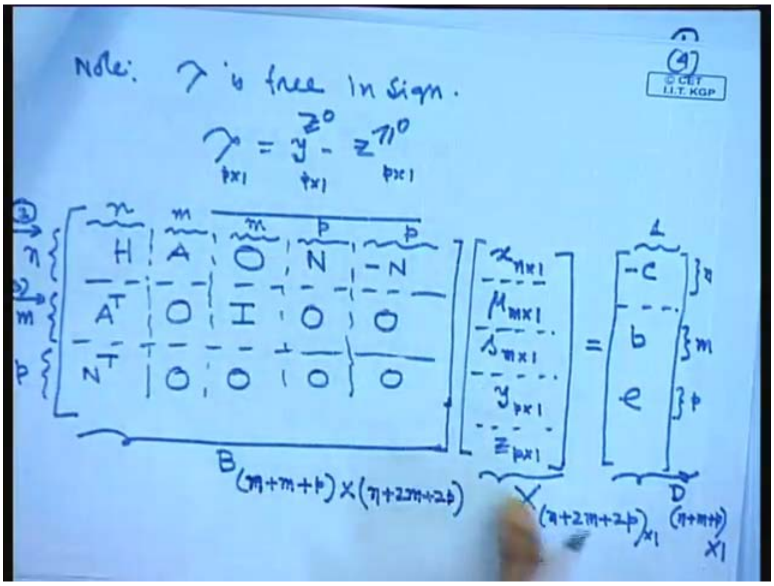

The next is variable we have consider the s associated with the that is your slide variables then we consider lambda this two are lambda. Lambda is since unsigned we have split up into 2 positive quantities. So, in this equation, equation number 2, 3, 4 and 5 if you write in matrix form equation number 2, 3, 4 if you write equation number matrix and vector form it will come into the structure, v this whole matrix and v into x minus x is the all variables Lagrange multipliers and the you decision variables and right hand side is the d we have defined as d so we have to solve this one. Now, this problem is simply a what is called algebraic equation b x is equal to b so this problem we are in we cannot solve directly what we consider introduced, we introduce what is called a artificial variable along with this one b x is equal to d.

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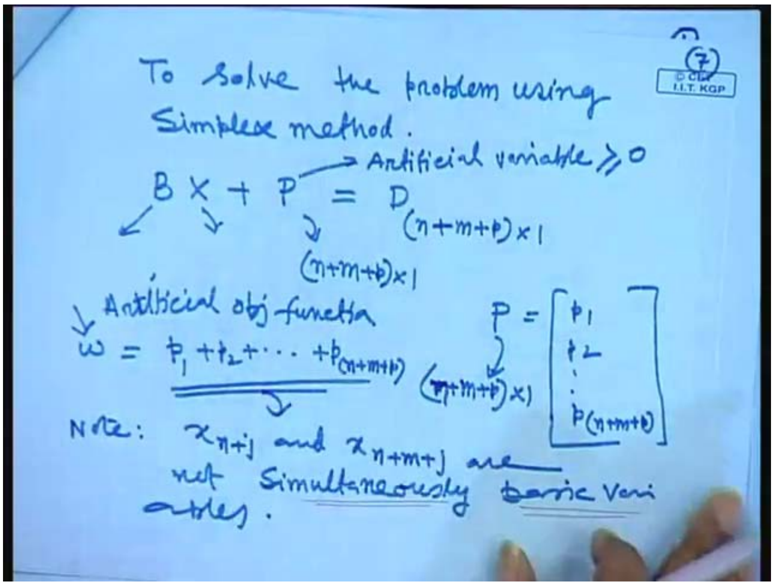

We have consider artificial variable p and p how many that this is a vector of dimension n cross m plus p into 1. So, there are m plus n plus m plus p variables are there so this artificial variable all elements of each artificial variables which is a p is a vector we had together and that w is called artificial cost function. So, look at this expression our problem is minimize this artificial objective function subject to the constraints whatever constants we have, we have constants these equations which is converted into a standard lp problem, is a standard lp problem. Our problem is minimize these artificial objective of the product subject to this objective function. Now, we know this b matrix if you.

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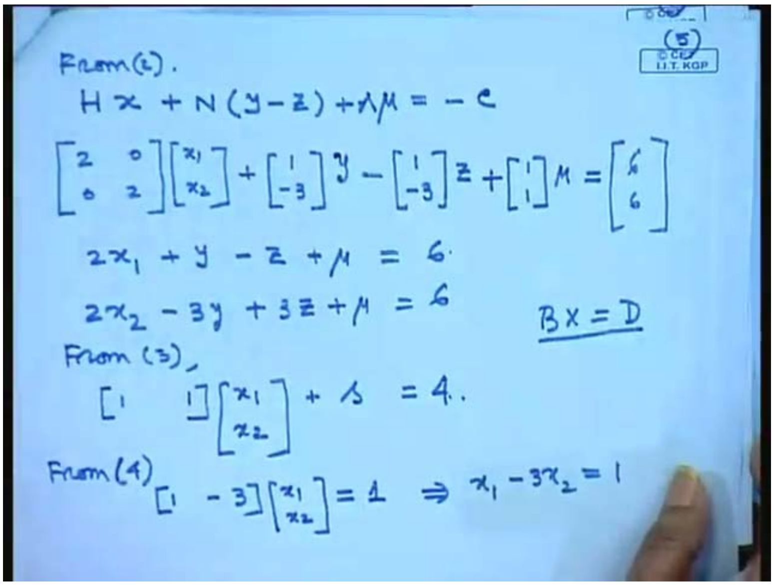

Now, I write in tabular form to solve indiscipline simplest method you can see carefully that how I formed these matrices. So, that equation number 2, the equation number 2 we have taken a simple example we have seen were h and c a b e and all these things are, see all this things are described in the examples. So, I am just writing the equation number 2 this it is equation number 2 this one which in turn it will come this and this equation. Now, this is our only equation number 2 this is we got from equation number 3 and this is we got from equation number 4 with each equation there is an artificial variable, with this equation p 1 is that with this equation p 2 is the with this equation p 3 is there with this equation p 4 is there.

If you write in tabular form now it will be like this see x 1 x 2 our the 2 original decision variable then x 3 is corresponding to our mu then x 4 is corresponding to our artificial variables then x 5 is corresponding to, x 5 is corresponding to y and x 6 is corresponding z. And y and z combinedly they formed is a Lagrange multiplier is associated with the equality constants. So, remaining x 7 x 8 x 9 x 10 are the, what is called our artificial variable introduce in the system equation that is b x plus p this one.

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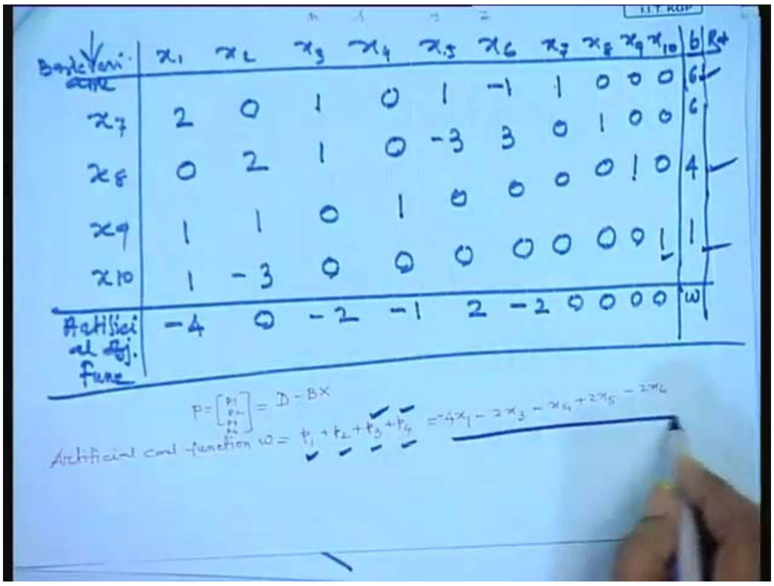

So, this s is a our slide variables I mentioned earlier artificial is slide variables x 4. Now, how is formed this table you see 2 x 1 coefficient of x 1 is 2, I have written 2. Then you see there is no coefficient of x 2 that x 2 is 0 then there is a this is where we have denoted is x 7 and what is called that y is we have denoted in x 5. So, x 5 coefficient is 1, so x 5 coefficient is 1 then z is x 6 we have defined x 6 that coefficient is minus 1, I have written minus 1. Then mu, mu is the mu that variable is defined as x 3 that coefficient is 1, see x 3 is 1 and remaining coefficient is 0 except the what is our artificial with there is our artificial variable will p 1. You can see from this expression that p 1 so we have written this x 7 is 1 and it is equal to 6 that is the first equation we have written.

And similarly, second equation you can write it which you will get it this equation and there are artificial variable is x 8 coefficient is x 8 is 1. And from equation 3 you see the x 1, x 1 plus x 2 plus x s, s is defined by variable x 4 so it will be x 1 coefficient 1 x 2 coefficient 1 x 4 coefficient 1. See x 1 coefficient 1 x 2 coefficient 1 x 4 coefficient 1 and this is third equation than p 3 will come and p 3 coefficient is p 3 correspondent x 9 is 1 and this equal to 4 that 4.

Similarly, last equation one can write it is nothing, but x 1 minus 3 x 2 plus a artificial variable that will p 4 which is we defined mu variable will x 10. So, it will be you see this 1 then minus 3 and tenth position is equal to 1 so this. Now, how you will get that artificial variables that is the artificial were now what is a artificial cost function that is we did sum of p 1 p 2 upto p n in our case upto p 4. So, if you just p 1 p 2 p 3 p 4 and what is p 1, p 1 from this equation p 1 is nothing, but a 6 minus if you take that side right hand side 2 x 1 minus x 3 minus x 5 plus x 6 is equal to x 7 and x 7 is nothing, but a p 1.

Similarly, p 2 is equal to x 8 is equal to p 2, I can write is 6 minus 2 x 2 minus x 3 plus 3 x 5 minus 3 x 6 if you take that side is equal to p 2. Similarly, I can write p 3 and p 4 if you add all this things I will get this one so the artificial objective function I can write it now 4 x 1 minus 4 then 0 there is no x 2 x 3 minus 2 x 4 coefficient minus 1, x 5 coefficient 2, x 6 coefficient minus 2 is equal to that is we have denoted w so w we have written. Now, we have to follow our standard simplex technique to minimize these objective artificial objective function w equal to 0 that is earlier.

So, next step is what you find out the pivot column this you from the you just look the what is the maximum negative coefficient associate in the artificial objective function here. So, this column is the pivot column and what is the pivot 2 you divide this coefficient along this column the 2 divided by 6 divided by 2 is this is 3 you can write it this is 3 then this is 0 ignore. This is 4, 4 divided by 1 that 1 divided by 1 is 1. So, this is the minimum ratio you got it so this is our pivot element and this is our pivot row.

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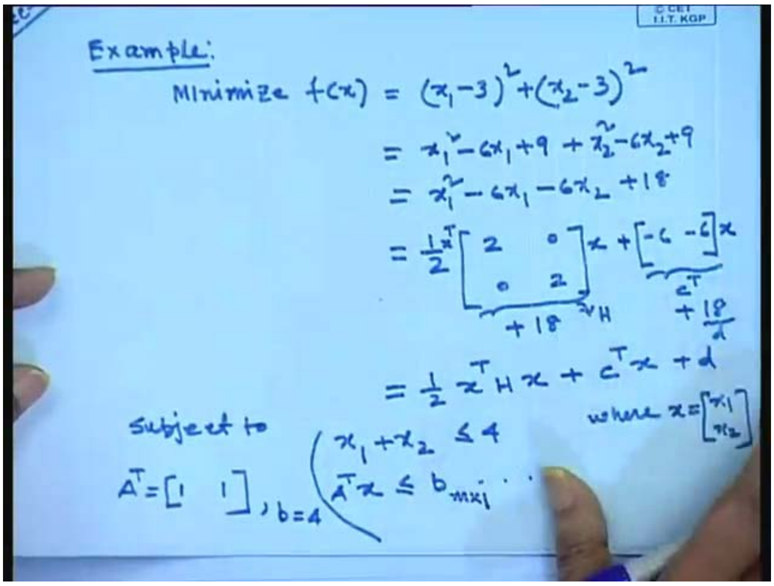

So, this way we have identified than after that you know that x 1 will be entering as a basic variable and x 10 will lead as a non-basic variables and then was in your whatever we have discuss earlier repeat for few iterations then you will get the final solution of these problem. That means initially problem is given if you see initially problem is given a quadratic optimization problems to solve that by using what is called simplex method.

That is this quadratic objective function is quadratic and what is our that our inequality constant and equality constant and affine function this, so this is called quadratic optimization problem that can be solved by using what is called simplex method before that you to convert what is called KKT necessary condition by convert into Lagrangian functions. So, this is we have discuss last class up to this.

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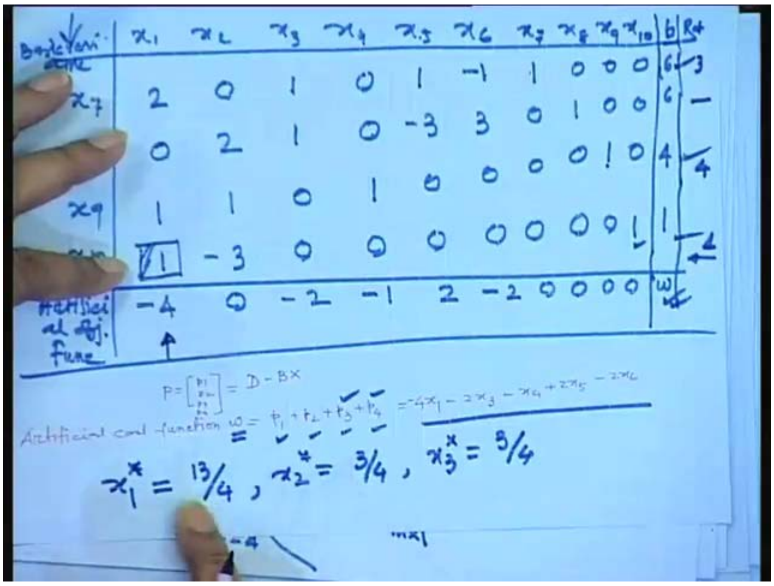

So, please solve this problem complete this problem you will see after few iteration the solution is given x after 4 iteration see what is the value of x 1 is coming this is 13 by 4 x 2 is coming 3 by 4 then x 3 is coming, 3 by 4. So, our basic importance variables are x 1 and x 2 so this solution of you will get optimum value of the function at x 1 when it is 13 by 4 and x 2 3 by 4. So, today we will start that is the solution of what is call optimization problem using interior point method. So, our next topic is interior point method.

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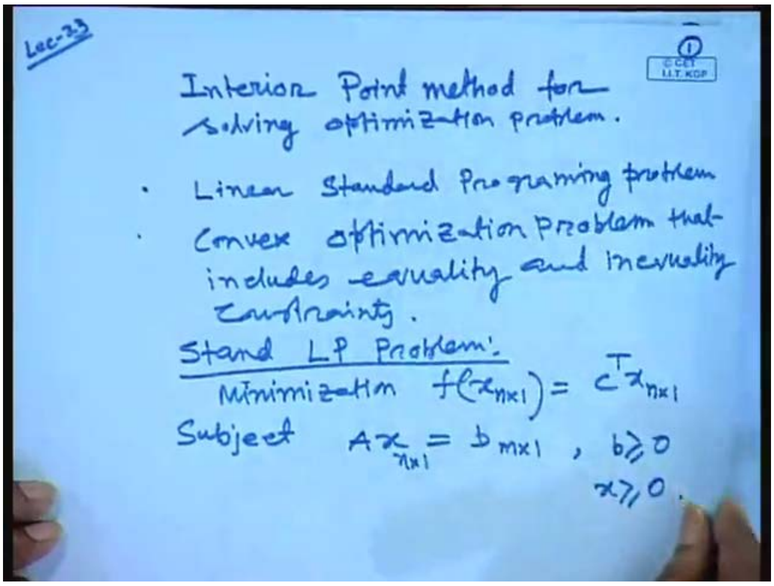

Interior point method for solving optimization problems, so interior point method can be used for solving the linear programming problem. First, it can be the Interior point method can be used for linear standard linear programming problem, linear standard programming problems and also we can solve the convex optimization problems, convex optimisation problems that includes inequality and equality constant, that includes inequality and inequality constant.

So, you through interior point method you can solve two types of problems one is linear problems, linear standard lp problem another is convex optimization problems along with the inequality and equality constant. Standard lp problem may be equality and inequality constant also they are in relation to this. So, let us see that what is our standard lp problem, standard lp problem recall our standard lp problem is minimization f of x whose dimension and decision variable is written c transpose of x n subject to the standard lp problem I am writing. If you can recollect this thing that what we have discussed earlier a x is equal to b, b dimension is n, m cross n this and b is greater than equal to 0 and x is greater than equal to 0.

And we define now feasible 0 we have to find out the solution of them, solution of this problems such that the function value is minimum not only minimum is must satisfy this equation. That means that point at which the function value will be minimum that point must be inside the feasible region. Then what is feasible region.

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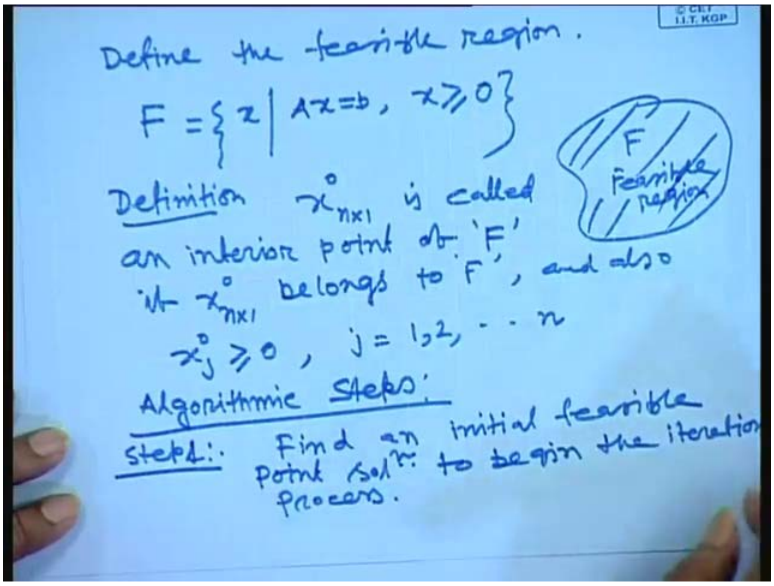

The feasible region as we define like this way define the feasible region f, f is the feasible let us call this is the feasible region that f we have denoted, this is the feasible region. Feasible region f is denoted by x any point in this feasible x it must satisfy all equality condition of standard lp problem that by a x is equal to b it must satisfy for x greater than equal to 0. So, this if for any value of x in this region if it is satisfy this one then f is called that feasible region of the corresponding problem.

So, simply definition is like this way x whose dimension n cross n is called an interior point of f, f is what feasible region x is called interior point this of f. This means if any point in the feasible any point inside the feasible region is there you consider that point is the interior point of f. If x superscript this belongs to f and not only this every component of x must be greater than equal to 0. And also x j superscript 0 means any point in the initial point is must be greater than equal to 0 for j is equal to 1 2 dot dot n. So, this is the definition of interior point so if you have a feasible region if the point is belongs to this in the feasible region and it satisfy that the all values of x is greater than equal to 0 then this point is called interior point, interior point of the feasible region.

So, let us see that how one can solve this type of problems whether it is a linear programming problem or it is a convex optimization problem what is the basic steps are there to solve this problem. So, our algorithm steps, algorithmic steps are like this way so what is algorithmic steps first you find out their interior point again of the feasible region of the problem. Then you move from this point in such a direction so that the function value the objective function value decreases.

That objective function value is decreases that means we are in the interior point from their we move in such a direction so that function value is decreases and then test it whether it this point has reached to the optimum value of the function or not. If it does not reach to the optimum value of the function then move to another point. So, that the function value is decreases from the previous value of the function and in this way we can reach to the optimum value of the function starting from the initial starting point should be inside the what is called interior point. So, our algorithm steps first, first step is find an initial, find an initial feasible point solution to begin the iteration process this is first step.

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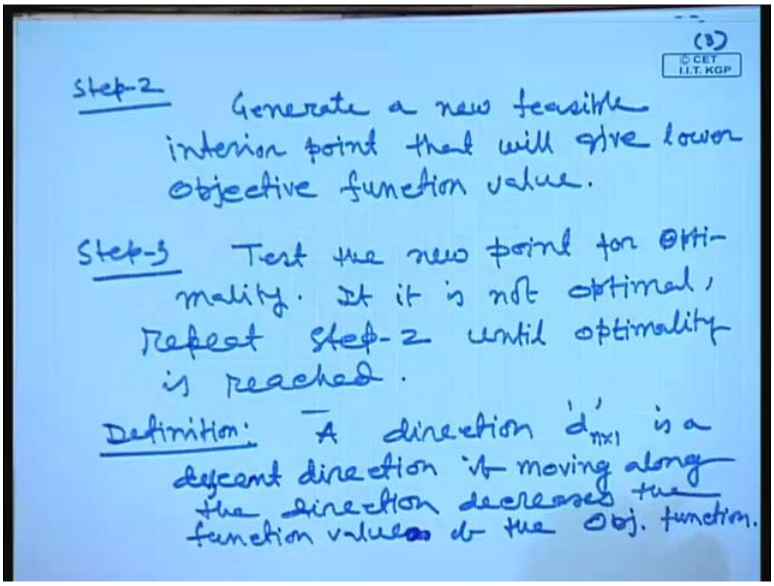

And next step, step 2 generate a new point, next is generate a new feasible interior point that will give lower objective function value. That means from the initial point you move in such a direction so that function value is decreased. Step 3 tested for optimality, test the new point for optimality if it is not optimal only if it is not optimum, if it is not optimal move to the if it is not optimal repeat step 2 until optimality is reached, optimality is reached.

So, this is the 3 basic steps where we will adopt for interior point method is. So, let us see that how out to find out the what is called the feasible direction. So, next is definition a direction d whose dimension is same as the feasible decision variables direction d whose dimension n cross 1 is a descent direction if moving along the direction, if moving along the direction decreases the function values, the function values, function value of the objective function. That we have discussed and details when you have discussed the how to find out the optimum value of the function by using stiffens descent method if you recollect our earlier discussion.

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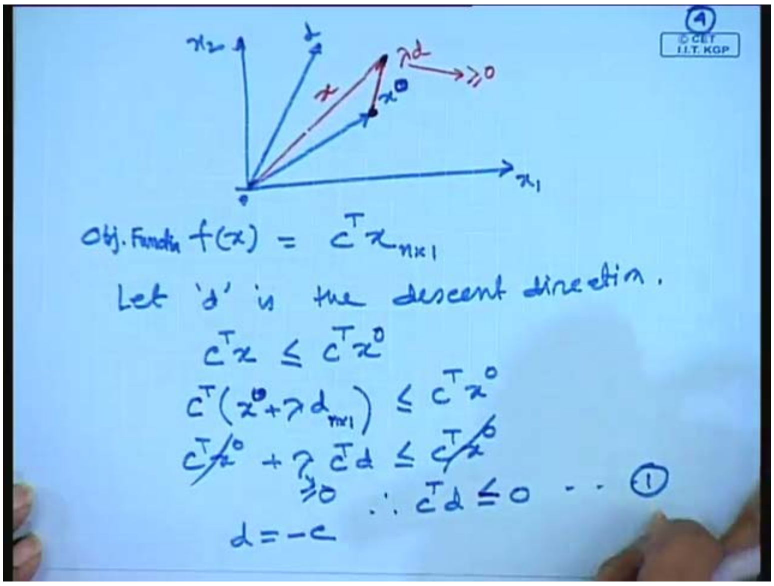

So, let us see this one suppose we are in the feasible region interior point of the feasible region and we are here let us call first that is x superscript 0 we are here. And will move in such a direction so that the function value objective function value is decreases from the previous value of the function. So, let us call this is our direction that you move in this direction then this function value will decrease. So, parallel to this one we are here now parallel to this one, so this one is parallel to this one I moved in this way from initial from this state and I moved this way.

So, this is our we can say lambda into d where lambda is greater than equal to 0 greater than equal to 0, if it is 0 then this is this point. Now, what where is the our new vector of x this is the our new vector and that is our x so we moved from initial vector which is in the feasible region in such a direction so that the new vector, new decision variable with the help of new decision variable the function value should decrease. That is our basic so this is our x 1 let us call for two dimensional case, two decision variable case that can be extended for n dimensional decision variable.

Note, now see this is one our objective function value is what, objective function value, objective function f x is equal to c transpose of x and let d is the our descent direction that d is the descent direction. And in this direction if we move from this initial position if you move in this direction and we are in if we are here then our vector is x is here. So, our we can write it c transpose x must be equal to c transpose of our initial vector initial decision value of this vector x this must be your. Then we are moving in the right direction to minimize the functions.

So, that is the direction what is called descent direction and from this one with parallel to this one we have drawn this one. Now, we can write it c transpose x is what this vector plus this vector so x of superscript is x of 0 plus lambda into d and d is a vector of n cross 1 is equal to less then c transpose x of 0 and then c transpose x of 0 plus lambda is a scalar quantity we can take it out and we can write c d is equal to c transpose x 0 so this, this cancelled. Since lambda is greater than 0 greater than equal to 0 so you can write that c transpose d is equal to 0.

That also have discussed in earlier in descent stiffens descent method. Now, what is your conclusion which direction will move if you can move in this direction and this direction how you will decide, if you decide d is equal to minus c then form equation 1. That is this condition indicates, this condition indicates that our function value will decrease if you move from x to x 0 to in this direction and this will be function value will decrease and this condition must satisfied. And when this condition must satisfy this one sorry this is greater than equal to less than equal to when this condition will be satisfied if one of the choice of d is if you consider d is equal to minus c then from equation 1 we can write it.

From 1 we can write is c transpose into d is minus c is equal to let us see what that is minus c transpose c this is nothing, but a equivalent norm of that c vector and square and this quantity if you see this quantity always greater than 0 greater than equal to 0. So, this indicates this quantity since it is minus is there presented with minus sign this is presented with minus sign that whole quantity is less than 0 this quantity. So, this I can write it less equal to 0 so this is the condition and one of the choice of descent direction is will be with minus sign of the cost function coefficients.

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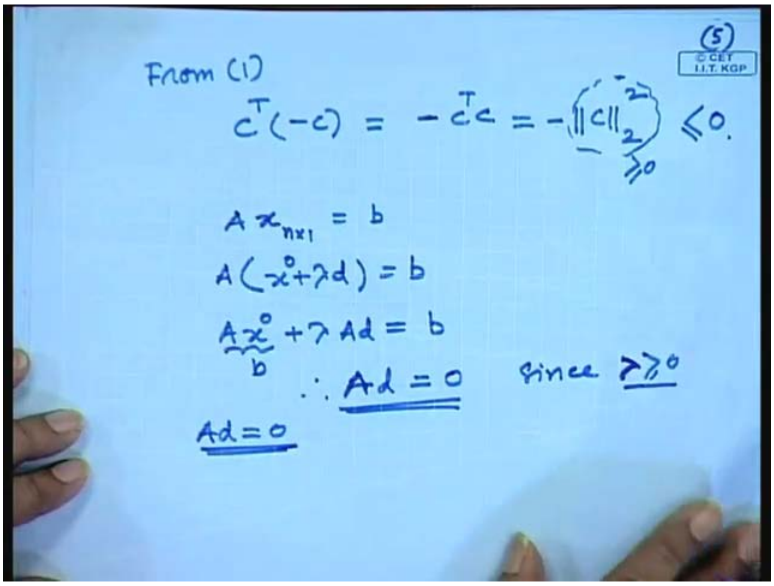

Now, we have and if this is the direction and this x is belongs to this feasible region if you move in this direction and that final vector is x and if it is belongs to what is feasible region then this new point, this is the new point it must satisfy our equality constant if you see it must satisfy our equality constant is equal to b. Let us see what is this one x is what x is 0 plus lambda d is equal to b and this x 0 plus lambda into a d is equal to b and this quantity since x 0 is belongs to the feasible region means interior point of the feasible region this quantity is b.

Therefore, this condition is a into d is equal to 0 since lambda is greater than equal to 0 lambda since when we are moving from this point to some point than lambda quantity is positive you can omit 0 also if you write. So, this is the condition so a and d must be 0 this is the conditions we got it so remarks.

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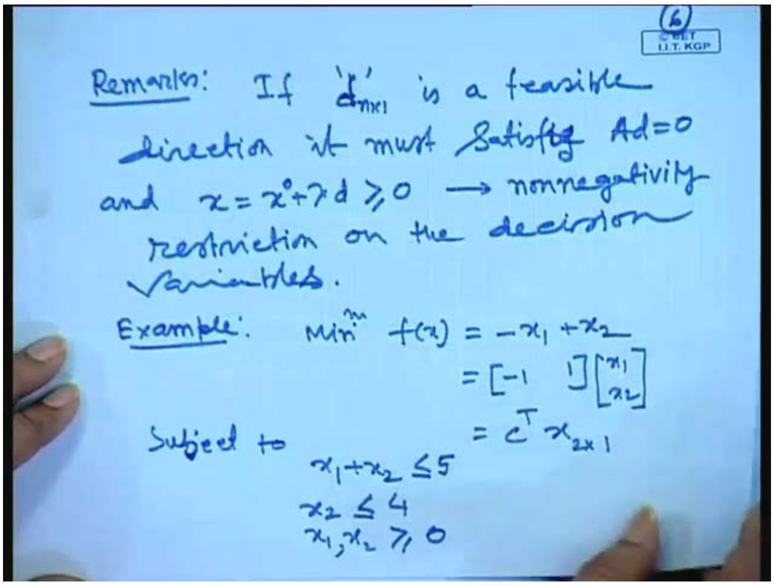

What is a remarks we can write it for this one if f if d, d is the descent direction is a feasible, is a feasible direction it must satisfied, it must satisfy it a d is equal to 0 if d is a feasible direction that it must satisfy feasible direction means what feasible descent direction you can write it feasible descent direction means that, the function value will decrease if you move in this directions and it must satisfy the our equality constraints also, satisfied this one.

And not only this that x which is equal to x 0 plus lambda d must be greater than 0 to satisfy the non-negativity of the decision variables this is the non-negativity restrictions on the decision variables. In short if d is the descent direction then it must satisfy a into d is equal to 0 this is the, what is called equality constant must satisfy. Let us take one simple point and see the effect of interior point then how to choose the interior point of this one. Example, this is where given till now some basic definition of this one let us call minimize f of x is equal to minus x 1 plus x 2 which you can write c transpose minus 1 x 1 x 2 which is nothing, but c transpose x, x dimension 2 cross 1. Subject to x 1 plus x 2 is greater than is less than equal to 5 then x 2 is less than equal to 4 and x 1 and x 2 is greater than equal to 0 this is our problems of this one. Now, let us see if we represent this one in graphically what it looks like this is it to represent.

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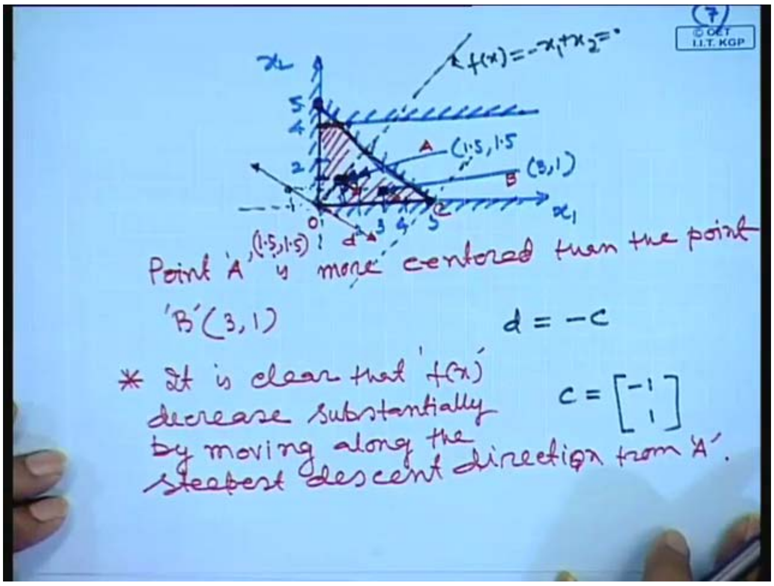

First, we have a coordinate x is x 1 and this is x 2 and this since x 1 is greater than equal to 0 then this indicates the upper half of this from this line. Since, x 2 x 1 is greater than equal to 0 this part of this one will be our feasible region. Another constant is that x 2 is less than 4 the x 2 suppose this is 4 this is 2 2 4 so that should be less than equal to this. Now, we have a x 1 plus x 2 is less than equal to 5 so x 1 is 0 x 2 is equal to 5, let us call this is the 5 some 5.5. And similarly, when x 2 is 0 x 1 is 5 so let us call this is 2 for 2 4 2 4 this is 5.

So, you draw this sorry you draw this line and this region below this region is a feasible region so our affective feasible region is this portion this from here is the feasible region. Let us take we take there are 2 points are there inside the feasible region may interior point we take 2 points let us call one is let us call we take 1.5 and 1.5 this coordinate is let us call 1.5 and 1.5 sorry it is here somewhere not here this point, this point this 1.5. Let us call this coordinate is 1.5 and this coordinate is 1.5 here. Then another coordinate you consider that 3 1 let us call this is the 2 this is a 3 and 1 this is 2 our case the 1 will be somewhere here.

So, this coordinate is 3 1 so this point we will call the point b and this point we will call the point a this point. So, there are 2 points at there we have considered inside the interior region the interior point. Now, we can say that point a is more centered, point a is more centered then the point b we can say point a is more centered then the point b and point b coordinates are 3 1 and point a coordinates are 1.5 and 1.5, more centered in the means this point a point is equi distance from the coordinate axis this is one is 1.5 from here also 1.5 equidistance. And this here of centered you can say this point b is more of centered compared to a.

Now, clearly if you see our objective function is what if you see the our objective function is this one this function you have to minimize when f x this equal to 0 when objective function will be equal to 0 we can write x 1 is x 2 is equal to x 1. That means our objective function expression is that one with a 45 slope is 45 and this is the our objective function value f x which is given minus x 1 plus x 2 is equal to 0 this one and it is passing to the origin.

Now, you see this objective function line if you move in this side the function value is decreasing, decreasing, decreasing, decreasing and at most you can go at this point when it is crossing this one at this variable function value is minimum. This is graphically one can see that one so if you move in this direction the function value is increasing and from this one your function value is decreasing. Now, if this point a is the our you can say starting point of our iteration of interior point this then which directions we have to move it just now we have proved it.

That choice of d must be equal to choice of d what is called direction vector will be minus c and what is c just say c is our case is minus 1 and 1 is c. So, with minus sign I have keeping with multiplied by with minus sign 1 what about the vector you will get in that direction you move it. Let us call c is what the c what is called our objective function this is the c transpose and minus 1 plus 1 so take the c, c is minus 1 plus 1 this. So, our c is minus 1 in the directions and plus 1 it is a 2 plus 1 somewhere here so this. So, this is our c vector so our d vector will be which one if you see our d vector our d vector is opposite to this one so this is our d.

So, which direction from this point which direction a parallel to this direction you have to move it, parallel to this direction from this point a I have to move parallel to this directions and this direction is in this direction. So, this if you move in this direction the function value is decrease and physically also or graphically also you can see if you just move in this direction you will get optimum value of the function at this point. Let us call this point is c you will get this and here if from a point you to move in this direction.

Let us call if you are at b point you have to move from this point parallel to the d in these directions.

So, this point and difference between this point and this point, if you move in this direction it will hit the wall at some iterations, so you can say this one that is if both the cases function value is decreasing no doubt. So, we can make in remarks like this way it is clear that f x function value decreases more, decreases substantially by moving along the steepest, along the steepest descent direction from point a. The other end point b the off center point b is the off center point will take us towards the wall of the feasible region, the feasible region before much improvement can be made, before much improvement can be made.

Now, question is how to centered because our initial guess maybe anywhere in the feasible region. Now, question is how to centered the off center point in a feasible region the simplest way is do some transformation on the original decision variables and that will make that will transfer the original point into a what is called transformed point which will be the center point. Center point means from the axis from the axis's the distance will be same.

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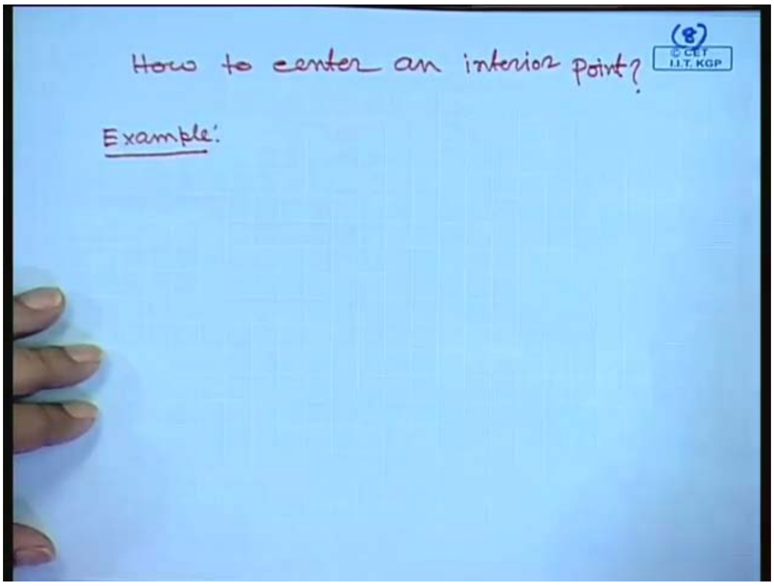

So, next our point is how to how to centered an interior point. Suppose, in this case suppose we are at this point now this point how to make it centered so you have to do the initial variables x 1 x 2 some transformation which will convert into new variable in new variable will get the center point from the x center point from the both axis's. The point will be in the transformed axis the point will be equi distance from the coordinate axis. Transformed systems so let us call we have taken one example and is explained what is the meaning of this one.

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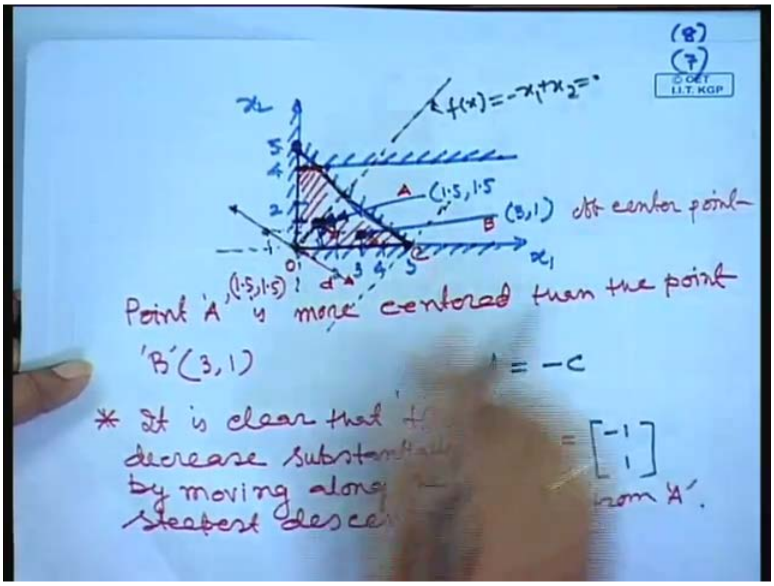

So, let us call we considered our point b point which is off centered the point b is off centered point.

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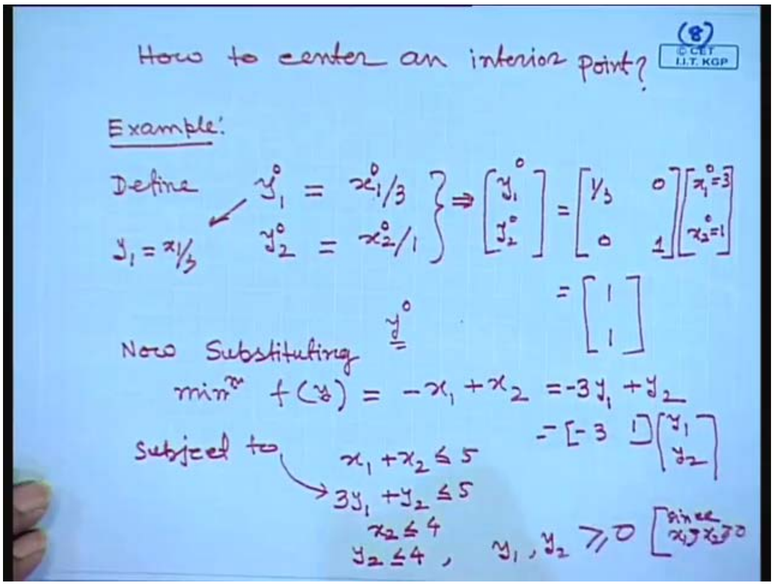

Suppose, this we take it so if you take it now we do transformation like this way after the transformation you will get new coordinate axis and that point should be equi distance from the coordinate, transformed coordinate axis. So, we define new coordinates axis y 1 is equal to the old value of x 1 divided by 3 a direct then the coordinate y 2 is a x 2 divided by 1. Now, you say whatever that is coordinate is I got it the x 1 of 0, x 1 of 0 divided by 3, x 2 of 0 divided by 1 this what we did. So, it is better to write x super script 0 0 then 0 this the transformation we made initial point which transforms.

Since, it is the off center point that point is not a equidistance from the axis is one is 3 x axis is distance, from y axis distance is 3 from x axis distance is 1 so it is off center point so you have made it this one, which equivalently we can write into matrix form y 0 y 1 of 0 y 2 of 0 this equal to write it that one is what we can write 1 by 3, 1 by 3 that is 1 0 0 multiplied by x 1 0 x 2 0. Now, you see if you substituted this is the transformation so what is this is coming if you do the x of 0 x 1 of 0 value is what 3 that is 2 of 0 value is 1 then if you this into this, this value is 1 and this is 1.

Now, see this one the transformed coordinate axis the point y 0 point, y 0 point is equidistant from the transformed coordinate axis's 1 1 is. Now, see the our can transform system what is our what is called our optimization problem will see. Now, substituting the values of x and x 0 in terms of y 0 by substituting minimize the function which will be in terms of y is equal to minus x 1 plus x 2 x 1 is what just see x 1 is 3, x 1 is 3 y 1 3 y 1 in general. Now, if I write it in general this of 3 y 1 then x 2 value is y 2 so this which you can write it this is minus 3 1 this is minus 3 1.

So, I can write minus 3 1 y 1 y 2 and corresponding to this this value this point x 0 that will be 3 y 1 0 y 2 0 in general this is that transformation you made it to this one. Then subject to our is x 1 plus x 2 is less than equal to 5 which we can write it x 1 is what see this one relationship between these x 1 is 3 x 2 in general that relation is what y 1 is equal to x 1 by 3 and that corresponding point we can write in this one. So, this is that one and we can write 3 y 1 plus y 2 is less than equal to 5 now it is converted this is in transformed coordinate axis or problem in minimization f of y, what is f of y minus 3 y 1 plus y 2 and subject to this condition.

Another constant is they are what is called x 2 less than equal to 4 which equal to nothing, but a y 2 is less than equal to 4 and then y 1 and y 2 is greater than equal to 0.

So, if you transform coordinate axis we can say minimize f of y what is a objective minus 3 y 1 plus y 2 subject to constraint equality constant 3 y 1 plus y 2 is less than equal to 5. Another equality constant is y 2 is equal to less than equal to 4 and y 1 y 2 are greater than 0. Since, why it is greater than 0 since x 1 is x 1 and x 2 greater than equal to 0, x 1 x 2 greater than equal to 0 that is why y 1 y 2 is greater than.

Now, you see if you just read of this optimization problem in what is call graphical, graphically if you plot the optimization problem this looks like this and you will see that point in coordinate new coordinate system that point is in the center point. So, I am now plotting that one you see this constraint this is the one constant this another constraint 2 and this constraint.

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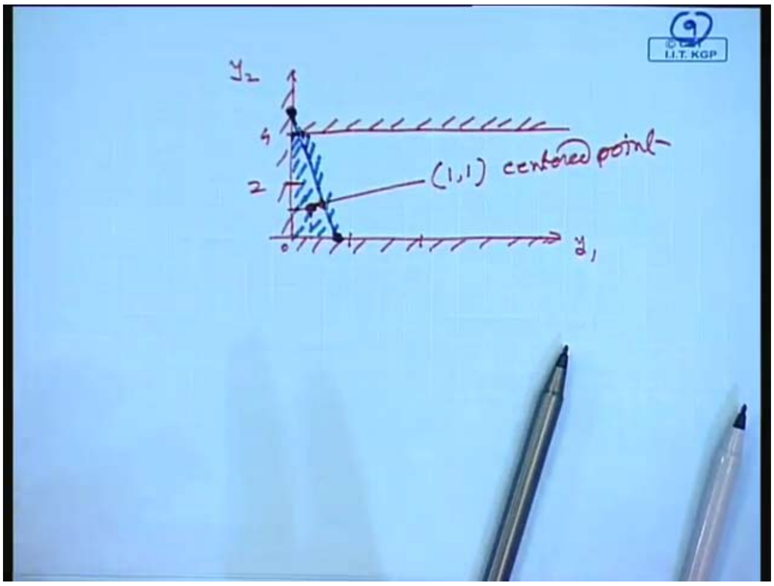

So, our new variables and transformed variables are y 1, y 1 y 2 and y 1 greater than equal to 0 that means it indicates it is this side and y 2 is greater than equal to this indicate this is that side and we have a another constraint in transformed coordinate axis y 2 is less than equal to 4. Suppose, this is 4, 1 2 3 4 suppose this is 4, this is 2 so this is one thing is that one and our inequality constant you see this one 3 y 1 plus y 2 is equal to 5.

Let us call y one is 0 y 2 is 5, y 2 is 5 here then y 2 is 0 then y 1 is 5 by 3 that means 1.66. So, this is 1 1 2 then it is 4, this is 1 1.6 may be here so our that constraint is that one. So, this so our feasible region is that this shaded blue shaded portion this is the (( ))

and our what is our initial interior point 1 1, so this is 1 and that is 1 so this is our initial point in transformed unit initial that is 1 1 which is a centered point this is a centered point, so in general now will show how to convert if it is if the point is half centered point how to convert into a centre point in transformed coordinate axis. That will discuss next class in details this one how to convert into that that portion, and so will stop it here now.