

Optimal Control
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Lecture - 22
Solution of Quadratic Programming Problem Using Simplex Method

This is the primary problem given, let us assume that the solution is given x study is x_1 is 20, x_2 is 16. We want to use whatever the theorems we have studied it used as theorems to find out the dual solution directly from the primary solutions.

(Refer Slide Time: 00:37)

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∴ Dual problem.

$$\text{Min}^m f_d = 80y_1 + 100y_2 + 40y_3 = d^T y$$

Subject $\begin{cases} y_1 + 2y_2 + y_3 \geq 3 & (d_1) \\ y_1 + y_2 \geq 2 & (d_2) \\ y_1, y_2 \text{ and } y_3 \geq 0 \end{cases}$

$$A^T y \geq d$$

$$A^T = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}, d = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Show that $x^* = [20, 60]^T$ is an optimal Sol^m to the primal problem.

We have seen using this equations a and b, we have ultimately we have solved and got these values. That y_1^* is equal to 1, y_2^* plus 2 solutions, it will come 1, so our solution is y_1^* is 1, y_2^* is 1, and y_3^* is 0. So, if you find out the objective function value in dual problem that will come if you see the expression for that one.

(Refer Slide Time: 00:51)

(a) $y^T(Ax - e) = 0_{1 \times 1}$ (b) $x^T(A^T y - d) = 0_{1 \times 1}$

\downarrow

$$\begin{bmatrix} y_1^* & y_2^* & y_3^* \end{bmatrix} \begin{bmatrix} x_1^* + x_2^* - 80 \\ 2x_1^* + x_2^* - 100 \\ x_1^* - 40 \end{bmatrix} = 0_{1 \times 1}$$

$$y_1^* (x_1^* + x_2^* - 80) + y_2^* (2x_1^* + x_2^* - 100) + y_3^* (x_1^* - 40) = 0$$

$\geq 0 \leq 0$ $\geq 0 \leq 0$ $\geq 0 \leq 0$

$\therefore y_1^* (x_1^* + x_2^* - 80) = 0$ $y_3^* (x_1^* - 40) = 0$
 $y_2^* (2x_1^* + x_2^* - 100) = 0$ $y_2^* = 0$ since $x_1^* = 20$

(Refer Slide Time: 01:20)

From (b)

$$x_1^* (2y_1^* + 2y_2^* + y_3^* - 3) = 0 \rightarrow 2y_1^* + 2y_2^* + y_3^* - 3 = 0$$

$$x_2^* (y_1^* + y_2^* - 2) = 0 \rightarrow y_1^* + y_2^* - 2 = 0$$

$\begin{cases} 2y_1^* + 2y_2^* - 3 = 0 \\ y_1^* + y_2^* - 2 = 0 \end{cases}$ Solve these equations.
 We get $y_1^* = 1, y_2^* = 1$

$\therefore y_1^* = 1, y_2^* = 1, y_3^* = 0$

$z_p = 3x_1^* + 2x_2^* = 3 \times 20 + 2 \times 60 = 180$

$z_d = 80y_1^* + 100y_2^* + 40y_3^* = 80 + 100 = 180$

$z_p = z_d = 180$

They have d 80 to y 1, this is 80 into y 1 star plus and then you 100 into y 2 star 100 into y 2 star plus that is 40 into y 3 star 40 into y 3 star that y 3 star value is 0. Now, y 1 star value is 80, y 1 star value is 1 and y 2 star value is 1 again, so this value will objective function dual problem which you come on that 80. Similarly, if you see the primary problem objective function values that object of a primary function objective function is z p is equal to z p is equal to you see 3 x 1 plus 2 x 2. Again, this is our objective function of the primary problem and we have given x 1 star is you see here the value of x 1 y is 20.

Solution of primary problem is maximization of the objective function at what point will get the maximum value of the function x_1 is 20 x_2 is 60. If you put it here, 30 into 20 plus 2 into 60, so its value is 180, so objective function value in dual problem that makes a problem. Value is same as that dual problem minimization; minimum value of the objective function value is 180. If both are same at optimal conditions minimum optimal point in what is called primary domain and as well as in what is called in dual domain or dual domain again.

(Refer Slide Time: 03:37)

Example:

$$\begin{aligned} \text{Minimize } f(x) &= (x_1 - 3)^2 + (x_2 - 3)^2 \\ &= x_1^2 - 6x_1 + 9 + x_2^2 - 6x_2 + 9 \\ &= x_1^2 - 6x_1 - 6x_2 + 18 \\ &= \frac{1}{2} x^T \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} x + \underbrace{\begin{bmatrix} -6 & -6 \end{bmatrix}}_{c^T} x + \underbrace{18}_{d} \\ &= \frac{1}{2} x^T H x + c^T x + d \end{aligned}$$

subject to $x_1 + x_2 \leq 4$ where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$A^T x \leq b \dots$$

So, today we will discuss about the solution of what is called quadratic programming problem using the simplex method. So, let us check in one example and two example will explain that how we will solve the quadratic programming problem using the simplex method. So, let us consider that what is called function is our function is minimized of x that is we have to minimize is equal to x_1 minus 3 whole square plus x_2 minus 3 whole square. If you expand this one, we will get it here x_1 square plus minus 6 x_1 plus 9 from this vector and from this vector x_2 square minus 6 x_2 plus 9.

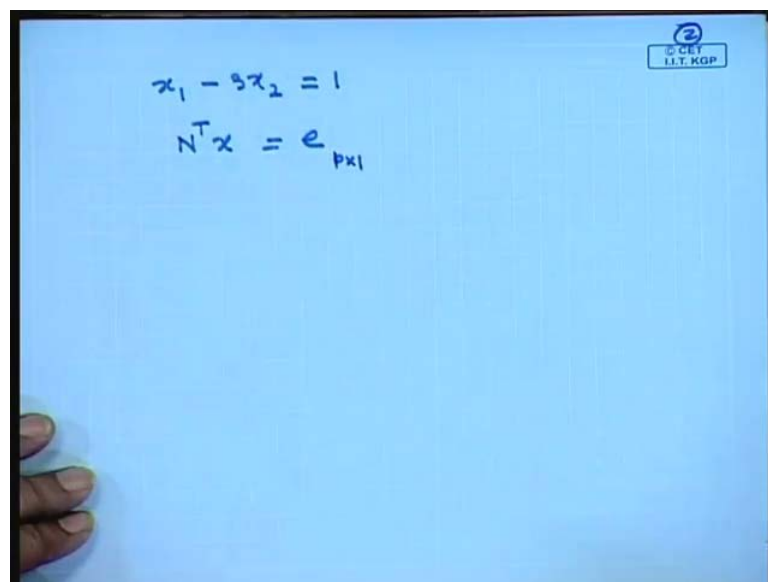
If you simplify this one, x_1 square minus 6 x_1 minus 6 x_2 plus 18, so you can easily write in quadratic form like this way. This one I can write it half 2, 0, 0, there is no cross product term $x_1 x_2$, so I will write it this is this way and this before that one will be that x transpose and this multiplied by x . Then, your x is 0 plus minus 6 minus 6 into x plus 18 which you can write it half x transpose this if you consider as an 8 matrix h which is

in symmetric. In general, it is symmetric matrix if you convert into quadratic form into x plus this if you consider as a c transpose that c transpose x plus what is the remaining term is left in this expression, 18 plus this one is denoted by d .

So, this is one the objective function which is expressed in quadratic form that one and a function is in quadratic form and this function is a objective function is a quadratic and convex. If it is convex, what is called Asian matrix of this function should be positive semi definite. So, this if you take the what is called Asian matrix of this one second deliberate of this function then it will come only H . This term this term will come in 0, now this is the problem which is in we have converted into a standard what is called quadratic form which is a convex function in our problem.

So, subject to the in equality constant and what is called equality constant x_1 plus x_2 is less than equal to 4. So, all in equality constants if we have only we have one in equality constants, in general if you have a number of in equality constants is m and number of you what is called equality constants is p .

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$$x_1 - 9x_2 = 1$$

$$N^T x = e_{p \times 1}$$

Then, we can write number of inequality constants in matrix and vector form, let us call these we have represented into eight transpose into x is less then equal to our b that it is the in equivalent terms right hand side, what we get it that is b . So, where our inner case x is what there are two decisions variables x_1 and x_2 , so this is our equation in equality

range. So, if you have a inequality constant of m such constants are there, I can always write in terms of $x^T x$ is less than equal to b.

Now, if you have equality constants are there, let us called we have equality constants $x_1 - 3$ $x_2 - 3$ is equal to 0, so that we represent this one equality constants by n transpose x is equal to e and that inequality constants.

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Example:

$$\begin{aligned} \text{Minimize } f(x) &= (x_1 - 3)^2 + (x_2 - 3)^2 \\ &= x_1^2 - 6x_1 + 9 + x_2^2 - 6x_2 + 9 \\ &= x_1^2 - 6x_1 - 6x_2 + 18 \\ &= \frac{1}{2} x^T \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} x + \underbrace{\begin{bmatrix} -6 & -6 \end{bmatrix}}_{c^T} x + \underbrace{18}_{d} \\ &= \frac{1}{2} x^T H x + c^T x + d \end{aligned}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Subject to $x_1 + x_2 \leq 4$

$$A^T = \begin{bmatrix} 1 & 1 \end{bmatrix} \quad A^T x \leq b_{\max}$$

If you consider p cross p number of equality constants, you consider inequality constants is m cross 1. So, we can write it here what is a transpose in your case in your case a transpose if you say see this one is nothing but A 1, 1, this is our A transpose and what is our n transpose here, n transpose in this case you will be 1, minus 3. So, our e is equal to in your for our example is one and in our case b is equal to b is equal to 4. So, we have identified what is eight, what is our c transpose, what is g, what is 8 transpose, what is b, what is n transpose, what is e, all these things. So, our in general our problem we will write it this way minimize a function f of x which is a half x transpose h x plus c transpose x plus d is the constant subject to a transpose.

(Refer Slide Time: 09:19)

$x_1 - 3x_2 = 1$
 $N^T x = e_{p \times 1}$
 $N^T = [1 \ -3], e = 1$
 Minimize $f(x) = \frac{1}{2} x^T H x + c^T x + d$
 Subject to $A^T x \leq b_{m \times 1}$ $A^T x \leq b$
 $N^T x = e_{p \times 1}$
Lagrange function:
 $L(\cdot) = \frac{1}{2} x^T H x + c^T x + d + \lambda^T (N^T x - e)$
 $+ \mu^T (A^T x + s - b) \quad \mu \geq 0$

All equality A transpose x is less than equal to b and b is the dimension of vector is m there are m such in equality constants are there which is combined together is this an equality constant n transpose x is equal to e that we consider $m \times p$ cross 1 . Now, these problems can be solved by using simplex method, first what is the necessary condition that this function should have a minimum value at what point. This function has a minimum value this function means quadratic problem that function of quadratic which is a convex in nature. So, what is the necessary, so one can easily find it by using k t condition, so let us write it first that lag range function.

If you recollect, we have discussed in details that lag range function how to generate this lag range function. So, lag range function l is equal to our objective function what is the objective function x transpose h x plus c transpose x plus d , this is the objective function. Then, that equality constants, so you have equality constants let us call we have equality constants λ transpose the lag range multiplier. I am just following what we have discussed earlier that k t condition to find out the optimum value of an objective functions.

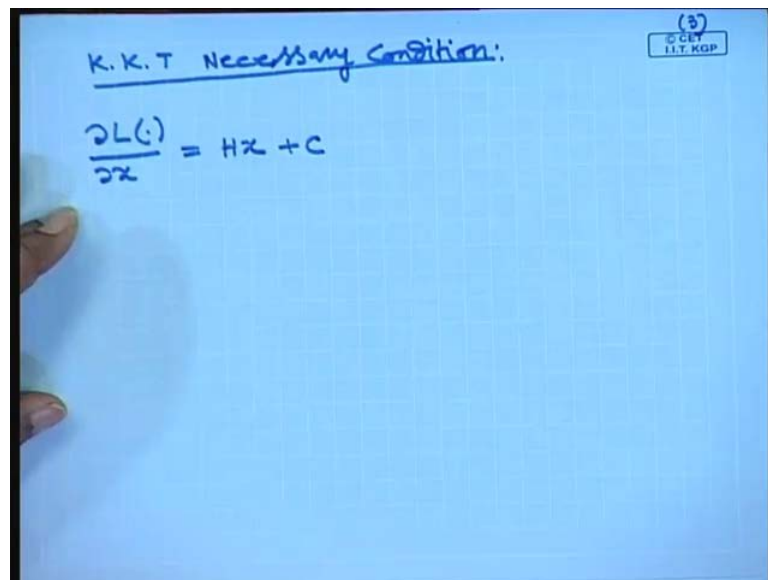
So, our equality constants is n transpose x minus e , so this quantity g 2 had what part will you get the optimal value or fusible solution of this one, it must decide this side will be 0 . So, l λ transpose of this plus, then you have A μ transpose the minimal display is a vector μ transpose that multiplied by a transpose x plus s minus b . So, A transpose

x minus b is less than 0 , so you have to add some what is called variable and that variable is called the variable what we have considered.

That variable should be some positive value, so that this equal to 0 , so this variable is called as the slight variable again. So, that means you have to you have to say this is the surplus variable that means we have to add this one x transpose x minus b is less then equal to 0 , so you have to add some value of this one. So, this is the you would call what is called slight variable of this one and when there this will be greater than equal to 0 that means this is a positive quantity, you have to subtract something.

So, that is called surplus variable, so in your case it is a slight variable subject this and this value is greater than equal to 0 , but this lag range multiply x of z with the equality constants that sign is unsigned. This value may be positive or negative for all types of inequality is x transpose x minus b less then equal to 0 . We have proved that μ should be greater than equal to 0 that means multiplier would be greater than equal to 0 in reverse type of inequality.

(Refer Slide Time: 15:11)



K.K.T Necessary Condition:

$$\frac{\partial L(\cdot)}{\partial x} = Hx + C$$

For reverse type of equity means a transpose x minus b is greater than equal to 0 that type of things then our μ should be less then equal to 0 . That means non positive numbers in that situation, but our case is a x transpose a minus b is less than equal to 0 . So, it is a non negative numbers, so keeping let us call this equation is equation number

1. Now, you can easily write the necessary condition k t, k t necessary conditions that have the minimum value of the function necessary conditions.

So, what is the necessary condition of KKT that is the function with respect to x, you have to find out. Now, this is our objective function is that is this one, now I am differentiating these things with respect to x. So, x transpose each x if you differentiate with respect to x that will be coming twice h, second different if you differentiate this one with respect to x, then it will come twice h x. So, twice x h and h is the half is there, so it will be coming h of x, the next term you see c transpose x if you differentiate this one, then it will come c only.

That is what we have discussed earlier if you recollect this things, then similarly if you this x term is involved first of this bracket first term. So, lambda transpose n transpose x this is no x variable lambda transpose e no x is involved, so that term will be 0, so here this will be A, I can write this one.

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$$x_1 - 3x_2 = 1$$

$$N^T x = e \quad p \times 1$$

$$N^T = [1 \quad -3], \quad e = 1$$
 Minimize $f(x) = \frac{1}{2} x^T H x + c^T x + d$
 Subject to $A^T x \leq b \quad m \times 1 \quad A^T x = b_0$
 $N^T x = e \quad p \times 1$
Lagrange function:

$$L(x) = \frac{1}{2} x^T H x + c^T x + d + \lambda^T (N^T x - e) + \mu^T (A^T x + b - b_0) \quad \dots (1)$$

$$(N^T)^T \lambda, (A^T)^T \mu$$

If you see this one, I can write n lambda whole transpose x this quantity, I can write it this one, now differentiate with h to x that will be n into lambda.

(Refer Slide Time: 17:08)

K.K.T Necessary Condition:

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$$\frac{\partial L(\cdot)}{\partial x} = Hx + C + N\lambda + AM = 0_{n \times 1} \quad (2)$$

$$\frac{\partial L(\cdot)}{\partial \mu} = A^T x + s - b = 0_{m \times 1} \quad (3)$$

$$\frac{\partial L(\cdot)}{\partial \lambda} = N^T x - e = 0_{p \times 1} \quad (4)$$

$$\frac{\partial L(\cdot)}{\partial \lambda_i} = 2\mu_i \lambda_i = 0, \quad i = 1, 2, \dots, m$$

$$\mu_i \lambda_i = 0, \quad i = 1, 2, \dots, m. \quad (5)$$

$\frac{\partial x^T P x}{\partial x} = 2P x$
 $\frac{\partial c^T x}{\partial x} = c$

N.T.E.: $s = \begin{bmatrix} \lambda_1^2 \\ \lambda_2^2 \\ \vdots \\ \lambda_m^2 \end{bmatrix}$

So, that will be n into lambda, similarly this x coming involving the first second and third that is no x comes. So, if you partial differentiation of that term which is to x, we can write it this one this mu transpose s transpose x. We can write it equivalently into this form a mu whole transpose x form, then you differentiate with x square 2 x if you differentiate with it x you will get a mu is equal to 0. Now, see this one and how many equation will get it there are n equations like this way. We have used this property when you differentiating this put together we have used this poverty x transpose p x.

When differentiated with respect to x, we will get it twice p x, then we have a x c transpose x when differentiate with respect to 2 x. The result is c that is we can verify by simply visible of what is called differentiation of a scalar quantity with respect to vector way a b t. So, this is the first, we got it next equation necessary condition property differentiation this with respect to mu assigned to 0.

Now, mu term is involved, you know only in the this term, so if you differentiate with respect to mu that will come what will get it for this one mu is a vector of that one you will get it. This whole quantity is scalar quantity that will come a transpose using this poverty, you can write a transpose x, a transpose x plus s minus b because there are different with respect to mu a transpose x plus s plus b is equal to 0. This term is constant or you can say this one, because this is a scalar quantity, you take transpose both side it will come whole transpose that mu.

Now, differentiate with respect to mu using that property it will get that one, so note here that are s is equal to defined as how many inequality constants are there. We have a in equality the each component of this, we define like this way if you recollect or earlier case this one and how many this equation you will be this is m cross 1 equation. Next is to differentiate this with respect to lambda there is lag range monitor to play with respect to lambda. So, lambda is associated with this term again, so since it is a scalar quantity take the transpose, then you differentiate with respect to lambda partial differentiate with x.

So, that will come it will be n transpose x minus e and how many equation will be there equality constants are there? There are p equality constants, so we have altogether n plus m plus p equation, so this equation we have in addition to that we have a switching functions that you have seen all here. Then, you have differentiate with respect to del s again, so this if you differentiate with respect to s this is a s square this s each m component of this one is h square. I am differentiating with respect to s, so this will get it twice mu, I see this one this, so each component will be mu 1 s 1 square plus mu 2 s 2 square plus mu 3 s 3 square.

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$x_1 - 3x_2 = 1$
 $N^T x = e$
 $N^T = [1 \ -3]$, $e = 1$
 Minimize $f(x) = \frac{1}{2} x^T H x + c^T x + d$
 Subject to $A^T x \leq b$
 $N^T x = e$
Lagrange function:
 $L(x) = \frac{1}{2} x^T H x + c^T x + d + \lambda^T (N^T x - e) + \mu^T (A^T x + b - b)$
 (N^T)^Tx, (A^T)^Tx
 $\mu \geq 0$
 A^Tx = b so No restriction in sign.

So, I am differentiating with respect to i, so there will be twice mu i s i is equal to 0 and i is equal to is equal to 1 2 dot this is s z to the inequality constants how many equality constants are there, m. So, this I can write further I can write it this mu i s i whole square

both side multiplied by s_i is equal to 0 is equal to 1 2 dot m again. So, if you give the equation number this is equation number 2, this is equation number 3, this is equation number 4 and this is equation number 5, and note that lambda that lambda is free in sign, no restriction.

Earlier, we told is beginning of the statement, so here five equations are there in addition to that if you want to solve by you solve this by using a simplest method that we have, A lambda also one of the variable in the expression. So, this I can write it that, next is note lambda is free in sign.

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Note: λ is free in sign.

$$\lambda = y - z$$

$$B X = D$$

Dimensions: $B: (n+m+p) \times (n+2m+2p)$, $X: (n+2m+2p) \times 1$, $D: (n+m+p) \times 1$

So, you can write it lambda is equal to lambda what is lambda is equally constant how many what is the dimension of this one p cross 1. So, lambda you can write it y minus z each is p cross 1 and each component each component is greater than equal to 0, this is also greater than equal to 0 for that difference of this one may be positive negative and 0. So, this is the our mu variable which have been in place of lambda, I will write it y which is greater than equal to 0 minus z which is greater than equal to 0, so we can reformulate the problem.

Now, if you just see, we can write it this equation number 2, 3, 4, 2, 3, 4, I can write it. Now, see this one equation number 2 what you can write it H, so what is the vectors in bulb x unknown lambda is unknown mu is unknown. I will rearrange first x the mu, and

then λ and you have a s also this s also you have and s what is the dimension of s , you have a $m \times 1$.

So, I will just re arrange in this way first x then μ and then s s and μ in the same expression s , then y which is λ is a two parts. If you see the λ , we have two parts y and then z that could be I will write it, so first expression each into x , so I am just writing it here. So, I am arranging this one x and what is a how many decision variables of x are there $n \times 1$ side by side. I am writing what is called the dimension also, that it is easy for us to track these equations. So, next I will write μ what is μ dimension $m \times 1$, then I will write it as s is a vector whose dimension also $m \times 1$.

So, I will just partition, then I will write it our λ has a two parts y and z , so y I am writing is dimension is $m \times 1$, then another is $0 \ z \ p \times 1$. So, this is our all unknown variables how many unknown variables are there $n \ m \ plus \ m \ n \ plus \ twice \ m \ plus \ twice \ b$ this is the unknown variables are there. So, from equation number 2 what we can write it see this is the constant this is we know from that description of the problems. So, I will take that is in the right side of the equation, so $h \times h$ multiplied by x , then you see next variable is μ multiplied by a multiplied by μ is a vector column vector, so a multiplied by μ a multiplied by μ .

So, this I have written it then $\lambda \ n$ multiplied λ is y minus z , so there is no s here, so I will write this value of s is here 0 and that is λ is n into y minus z . So, y is in bulb $n \ z$ also will be in bulb with minus n , so I will write it n , then partition minus n . So, let us write this dimension of this one and what is the how equations will get it from the first equation number two how many equations there are n equations are there. So, we have here to here n equation and this is the A into μ , A dimension is n and μ dimension is m .

This is also same as s dimensions same as μ , so this is also will be $m \ n$ into $p \ n$ into p this dimension n dimension is n and this is a p and this is also p . So, we know this block, so this is the corresponding to I can write this corresponding to equation number 2 is equal to 2 what is the right hand side right hand side is equal to c . If you take from minus c , so minus c and what is the dimension of this c what is the dimension of this one it is a n row $m \ 1$ column. Now, write it is a second equation a transpose x , so x correspondence

to a transpose, then you see that is a s and this is the constant term from the description of the problem, you take it right hand side b .

So, s is next component is 0, then it is a s_i because s , I can write it as i into s , this I can write i into s and other terms is 0. This term is equivalent to b and b is the number of inequities right hand side of the b is the right hand side of the inequalities that dimension is your m . So, this dimension is also will be m , now what is that this equation 2 corresponding to equation number 3 we have represented. Now, see equation number 4 see equation number 4 that this is the e_n transpose equity constants n transpose x is equal to e it is a constant that may take it right hand side. So, then if you have a already n transpose x , so first block will be this first block will be n transpose and other elements are 0, 0, 0, 0.

If you take the right hand side of this one it is the e and how many equation equality equation are there p , so this also will be p . So, you know the whole description of that one that that means if you consider the whole matrix is b the b dimension of this one is how many rows are there n into m plus m p rows.

If you see this one, the rows of that one is n plus m plus p , so this dimension is n , sorry n plus m plus p this one into number of columns n twice m plus twice p . So, this is the matrix, the whole matrix which we received which we got it obtained from KKT necessary conditions. That dimension of this one, let us call this, I am calling as X and that X dimension n first n components of x .

Then, m components of μ x components of x then y component y p components z p components, so this x is dimension n plus twice m plus twice p . This is equal to i consider this one is your d and this d dimension is if you see how many rows are there n plus m plus p . So, this dimension is n plus m plus p in to 1 into 1 that number of rows is 1 number of rows is 1. Now, if I write it this equation, now I can write it h h into x 1, h into x 1 plus a into μ n into that is y minus n is equal to y . You can say this one directly what is this equation stands for put the value of h put the value of c put the value of n .

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From (6).

$$Hx + N(y-z) + \lambda \mu = -c$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} y - \begin{bmatrix} 1 \\ -3 \end{bmatrix} z + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mu = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

$$2x_1 + y - z + \mu = 6$$

$$2x_2 - 3y + 3z + \mu = 6 \quad \underline{Bx = D}$$

From (3),

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \lambda = 4$$

From (4)

$$\begin{bmatrix} 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 1 \Rightarrow x_1 - 3x_2 = 1$$

Put the value of a in this expression, then you will get this expression, let us call I am writing it first necessary condition H into x plus n into lambda is y minus z that one agreed plus a mu is equal to minus c. So, h is what if you recollect this problem that our each matrix is that one our h is if you write it h 2, 2, 0, 0. The diagonal matrix that is our 2, 0, 0, 2 our x is what x 1 x 2, this is from equation necessary condition from 2, and then n is what in our case n if you see this our n is where is the equality constants e equality constants.

This is our n, n is n transpose 1 minus 3, so this will be a n is n will be 1 minus 3, so n will be you are writing n is 1 minus 3. Then, it is y that our y is that is a only one that is this is a lambda that is that is only one what is called equality constant so that dimension is in our case is 1 for our particular example is 1. So, I can write y that minus sign 1 minus 3 z is dimension is 1 cross 1 because we have A 1 equality constants and A matrix is you see a transpose is a 1 1. So, it will be a column vector 1 1 and mu we have a only one in equality constants this equal to c value.

If you take that 6, transpose minus 6 minus 6 if you take the transpose of that one, it will be a what is called minus 6 column wise is equal to minus six and that is equal to minus x, so it will be a 6, 6. So, write in more details 2 x 1 plus y minus z plus mu is equal to 6, 1 equation. The next equation twice x 2 minus 3 y plus 3 z plus mu is equal to 6, so this is this two equation we got it from the equation number 2 of KKT conditions or directly if

you write it this one directly. After that notation, you are familiar, you directly from this equation, you write it you will get two set of equations that is n is equal to n is equal to 2, we have a two equations will get it here.

Next, n is equal to 1, number of inequality constants is 1, we will get one equation from this one what is this one if you say a transpose x plus s . So, A transpose x , A transpose is what a transpose our A transpose is 1, you see the A transpose, A transpose is 1. So, if you write it A transpose from this from third equation is 0, you can write it a transpose x A transpose is y 1, A transpose is 0, 1, 1 into x , x has a two components x_1 and x_2 .

So, next is you s x is the that is a only one inequity constant, so this is be a 0, i into s is equal to c is equal to b . That is equal to this equal to b or second equation of this one is a transpose x plus x is equal to b , so b value is what from the problem of the b value is 4. We have A , this equation this equation this equation, then another equation will get from 4 KKT condition from 4 and four condition is n transpose x is equal to n transpose x is equal to e . So, what is n transpose n is what n transpose is what one see this problem one three into n transpose x n transpose x is x_1 x_2 is equal to our e value is what from the statement of the problem.

It has to see the statement of the problem our e value is given to one e value is 1, so this is equal to 1, so if you make it this 1×1 minus 3×2 is equal to 1. So, our KKT condition all are if it is a quadratic programming problem that our KKT conditions is coming in linear form all equation is linear in more general is b into x is equal to d form. Here, we have seen also it is a linear, now instead of using y z μ s , we can just continue the variables in terms of x_1 . So, we have already have x_1 x_2 , so if you consider next variable, you see the way we make it argument of this one.

Next variable will be μ , how many components have been μ m components, so I will consider up to x_1 x_2 dot x_n . Next is x_{n+1} is equal to μ_1 , next is x_{n+2} is equal to μ_2 in this way last element of μ will be x_{n+m} .

Similarly, these variables how many variables are n m variables are there, so the first element of the s , I will represent as a x_{n+m+1} . Similarly second element of this one, I will define as a new variable x_{n+m+2} in this way and similarly, we can continue this way. So, if you re define it, you define this one, now then what we can

write if you define this one what you can write it you see x and we have a another constant error switching constant μ_i or μ_j j is equal to 0.

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$$\mu_j \delta_j^2 = 0, \quad j = 1, 2, \dots, m.$$

$$x_{n+j} \cdot x_{n+m+j} = 0, \quad j = 1, 2, \dots, m.$$

Assum

$$x_{n+i} = \mu_i, \quad i = 1, 2, \dots, m.$$

$$x_{n+m+i} = s_i^2, \quad i = 1, 2, \dots, m.$$

$$x_{n+2m+i} = y_i, \quad i = 1, 2, \dots, p.$$

$$x_{n+2m+p+i} = z_i, \quad i = 1, 2, \dots, p.$$

$$B X = D$$

Dimensions: $(n+m+p) \times (n+m+p)$ and $(n+2m+2p) \times 1$ for B ; $(n+m+p) \times 1$ and $(n+2m+2p) \times 1$ for X ; $(n+m+p) \times 1$ for D .

That is what we got it μ_j s_j is equal to 0 that what we got it and j is equal to 1, 2 dot dot m . Now, what you can write it from this this equation what is the position of μ , see the position of μ if i x if i express all the term variables in terms of x then first element of μ will be x_{n+1} second element of μ x_{n+2} . So, if it is so I can write it x_{n+j} multiplied by see this one element of this one, the elements of this one is s_1^2 square s_2^2 square. All these things what is the first element of s in terms of x , x_{n+m+j} means first element is $1 \times x_{n+m+2}$ will be the second element of s .

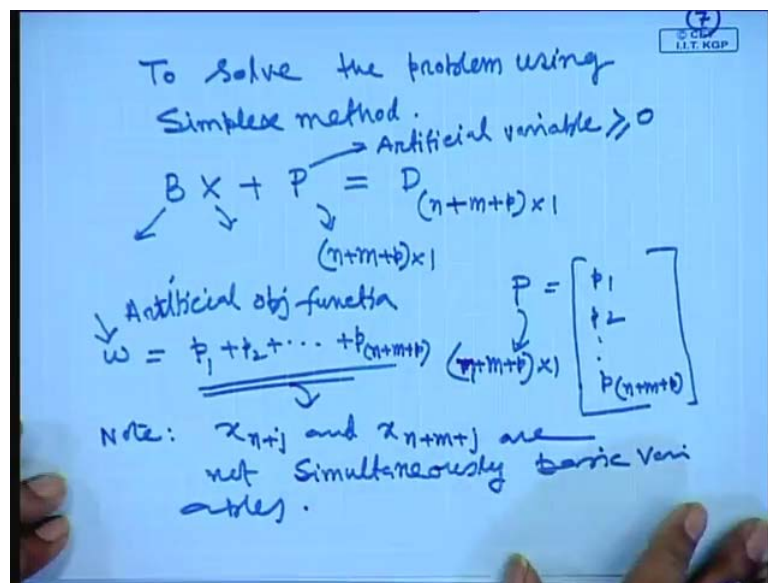
So, if we in similar method, I can write it x_{n+m+j} is equal to 0 for j is equal to 1, 2 dot dot m , so this I can write it this one what we do we assign. Now, I am writing, we assign that x_{n+i} is equal to μ_i and i is equal to 1, 2 dot dot m that x_{n+m+i} is equal to s_i^2 and i is equal to 1, 2 dot dot m and what we assign is it is $1 \ y \ z$ and x_{n+m+2m} . So, I can write it x_{n+2m+i} is equal to y_i is equal because we have a how many in general p equality constants. So, next we can write it that $x_{n+2m+p+i}$ is equal to z_i is equal to i is equal to 1, 2 dot dot p .

So, this are the variable were defined in place of μ x y and z , so our problem now boils down to solve a what is called linear equation that this linear equation we have to solve it. So, recall this equation recall this equation $B X = D$ and this dimension of

this I have just mentioned you already. Also, this dimension is n plus m plus p , this multiplied by z n plus twice m plus twice p this into 1. So, this dimension is this into twice n plus this this dimension is n plus m plus p this is a p plus twice m and what is the dimension of that one n plus twice m plus twice p into 1. I have written it here explained here same thing, I am writing here in this case because it is a column vector that is one will be there.

So, what is our d dimension is n cross m cross, sorry plus this cross 1 and this one we have to make $b \times$ is equal to in such a way all right hand side of this one is positive quantity. Now, how to solve this one using that simplex method, so first you see first you see that our quantity programming problem using KKT necessary condition. We can convert $v \times$ is equal to d from linear set up equation, now question is this linear set up equation. We have to solve by what is called a simplex method, we have introduced new variable in place of μ s y λ is replaced by y and z that we replaced in new variables x .

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Then, the question is how to solve that one, next question, so this problem solved to solve them to solve the problem using simplex method. So, you do not have any objective function here because already from KKT make it necessary condition. We got set up linear equation and we have to make in such a way that it is a standard helping problem form, but still now it is not a standard helping problem. I can make right hand

side of the equation b always positive by some manipulation, so our b into x plus p is equal to d .

So, I have introduced from artificial variables which is a vector that the dimension of this one is same as dimension of d , so this dimension is n cross m plus m plus p plus 1. So, this dimension, you know this dimension, you know this dimension. This is this p is a vector which is vector a artificial variable which elements is greater than or equal to 0. Artificial variable of dimension this p is a vector whose elements are the artificial variables and each element is greater than equal to 0.

So, you can write it the p is nothing but our p_1, p_2, \dots, p_n plus m plus p this one and this dimension of this one this dimension of this vector is that one m plus n plus m plus p cross 1. Now, we have just introduced from artificial variable, so this equal to that ultimately artificial variables below must be 0. So, when we have introduced the artificial variable, we know the sum of the all artificial variables is our objective function. So, w is nothing but our objective p_1 plus p_2 plus \dots p_n plus m plus p element, this is our objective function that objective function is called artificial objective function.

Then, how will you get that function below in terms of x permits that p, u and I can always write the first element of p_1 . I can always write in terms of x and d elements, so $p_1 x$ plus in terms of take all, take $b x$ in the right hand side and element twice p_1, p_2, \dots, p_n is equivalent and then you add p_1, p_2, \dots, p_n all these. So, ultimately this you get a function of all and this is variables and this is an variables not only this is a decision variables let lag ranger multipliers μ, λ, s is equal to this one when s with this one.

So, you know how to solve this one, let us call the 1, 1 what is called precaution or 1, 1 observation one must note that we know these we know μ into x square is this is equal to 0. So, both these product of this one both product cannot be non negative that means if one is non visible other must be both the elements cannot be what is called basic variables. That means if basic variables value is what is called non zero, so this if it is non zero, both the things, then it cannot be product cannot be 0.

So, one can see the product of these two things for j is equal to 1 on this both variables this one and this one cannot be a what is called both basic variables. Either, one of these must be non basic variables in general, so note that x_{n+j} and x_{n+m+j} are not simultaneously the basic variables.

So, the problem given in this one, I can easily solve by using the simplex method if you just solve this one, you can solve it in simplex method. So, in your case how many variables are there if you see in our case this one how many variables are there into that μ s or μ x y z n twice m twice p n is 2 m is 1, 2, 2, m 2 plus 2 4 plus 2 6. So, we have a six variables are there and six variables are there, so if you just prove put the tabular form.

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Basic/Artificial	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	b/RHS
x_7	2	0	1	0	1	-1	1	0	0	0	6
x_8	0	2	1	0	-3	3	0	1	0	0	6
x_9	1	1	0	1	0	0	0	0	1	0	4
x_{10}	1	-3	0	0	0	0	0	0	0	1	1
Artificial Obj. Func.	-4	0	-2	-1	2	-2	0	0	0	0	w

I can easily write it now $x_1, x_2, x_3, x_4, x_5, x_6$, this x_6 , and then we have a how many we have, what is called vector of artificial variable. There are four artificial variables are there because we have a four equations are there the set of linear equation artificial variable is four x_7, x_8, x_9, x_{10} . Then, b then you receive you write receive the standard way and our basic variables are what basic variables in this equation the basic variables is x_7, x_8, x_9, x_{10} . Then, our artificial objective function you can filled up according to our problem if you filled up this one that I am just writing it this 1, 2 from the equations from that KKT equation what we got is and we have written this equation.

If you see this equation, this equation if you write it in terms of z μ in terms of x you write it you will get it 2, 0, 1, 0, 1 minus 1, 1, then 0, 0, 0. Next is 0, 2, 1, 0 minus 3 plus 3, 0, 1, 0, 0 and this b is 6, this is also 6, then next is 1, 1, 0, 1, then 0, 0, 0, 0, 1, 0 and last equation 1, minus 3, 0, 0, 0, 0, 0, 0, 0, 1. That right hand side is 4, this right hand

side is 1 and the cost function if you see minus 4, 0, minus 2, minus 1, 2, minus 2, 0, 0, 0, 0 and that is w . So, next class I will complete, not complete I will explain how you have to proceed and get the solution of linear quadratic regular programming problem using the simplex method. That is basically from KKT condition we have converted into a linear set of equation for quadratic programming problem only, so here I just stop this one.