Electrical Engineering Optimal Control
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Lecture - 18
Solution of LP Problems with Two Phase Method

So, today we will take an example and show, how to solve the simplex method that LP problem by using simplex method by what is called the tabular form.

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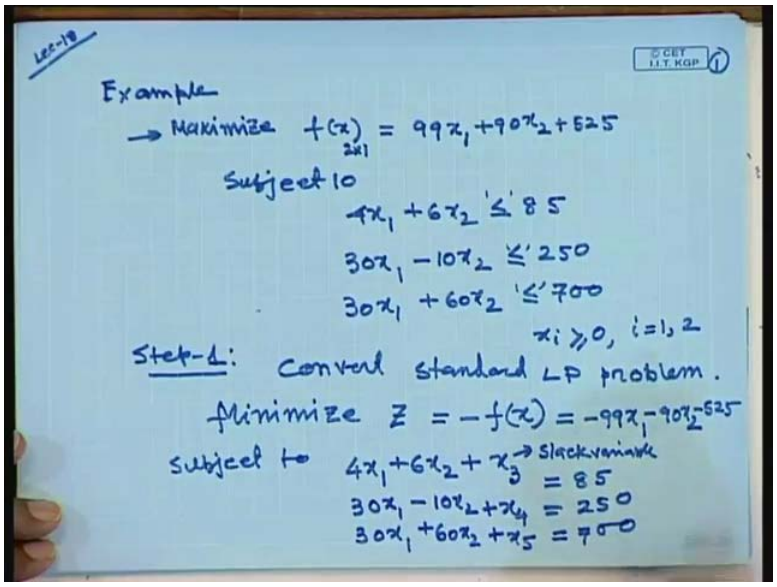

Let us considered this example maximise the f of x, x is a variable there are two decision variables as there is dimension is 2 cross 1 x. So, we have 99 x 1 plus 90 x 2 plus 525 is the objective function subject to subject to inequality constant that is 4 x 1 plus 6 x 2 is less than equal to 85 and 30 x 1 minus 19 x 2 is less than equal to 250 and 30 x 1 plus 60 x 2 is less than equal to 700. Here, x i, i varies is greater than 0 or you have it 2 i 1 and I 2 x 1 x 2, these are the constraints are given way to solve to maximise this function. Last class we have solved a problem of linear programming problem using what is called a algebraic approach.

So, today we will solve the LP problem using the tabular form the same basic concept is exactly same as before only we are representing the all operation in tabular form. So, let

us take this LP problem to convert into a standard LP problems, so first we convert step 1. We convert this problem into a standard LP problem, convert standard LP problem or standard LP problem is we have considering the beginning that makes to minimise the function, so a given problem is maximization.

So, we will solve this problem that optimization problem LP linear optimization problem by considering the minimisation of an objective function. So, f minimise the function let us call z the function which is nothing but a minus f of x f of x is the maximization considering minus means it is minimisation that we have explained earlier. Also, this is nothing but a 99 x 1 minus sign minus 90 x 2 minus 525 subject to the constant, what is the constant we have the standard LP problem. These inequality constants, which is of this type less than equal to all are less than equal to of this type. There may be some greater than equal to some expression is greater than linear expression greater than equal to some numerical values.

So, will see later how to tackle such that type of problem for the time being we are considering the inequality of less than equal to type. So, this you have to convert it equal to sign. so 4 x 1 plus 6 x 2 and this quantity left hand side is less than 85, so we have to add some variables let us called adding some very venue variables x 3 and which will call the slide variable this equal to it 85. Not only this, right hand side equal to sign, we have to convert we have to take care so that the right hand side quantity constant quantity is positive quantity non 0 company positive quantity.

This is we have converted first one and second one is 30 x 1 minus 10 x 2 plus this is less than 250, the new something some variable you have to add it to this one with expression. That means let us call this is x 4 is equal to 250 and we have a last equation in equality equation 30 x 1 plus 60 x 2 is less than 700. That means we have to add another variable which is slide variable this is also slight variable x 4 x 5 is a set variable it slight variable, value is greater than equal to 0. So, that is your 700 with the constraints that x i is greater than equal to 0 for i is equal to in our case.

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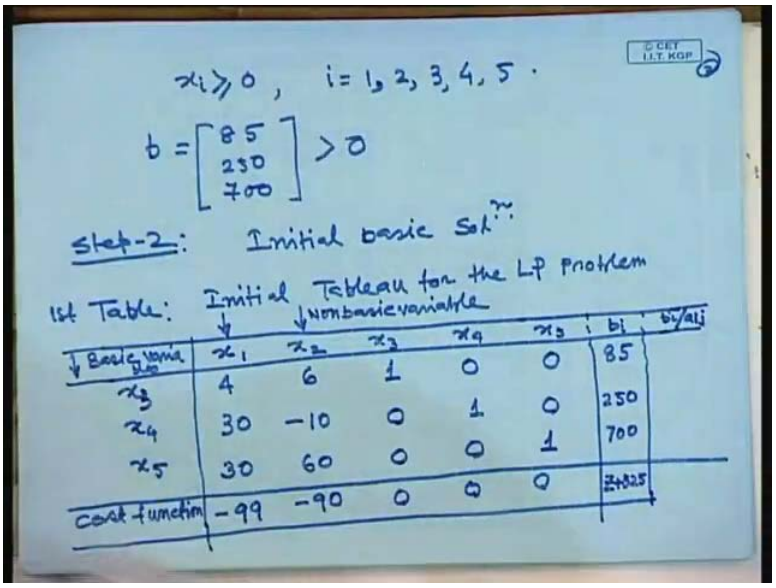

Now, 1, 2, 3, 4, 5 and if you say the right hand side of the equality constant that is b denoted by vector v earlier that is the region's of this 85 first equation equality sign is 85, 250, then 700 at each component of these vector is greater than 0. So, this problem we have to solve first we convert into standard LP problems, next is start, so initial basic solution the way we did it for algebraic approach the same we are doing. So, you can identify which are the variables are non basic variable which variables are basic variables.

That means another words non basic variable will assign to 0 and basic variable, hence basic variable will calculate from the expressions. Now, I am writing into the way we did it for algebraic approach, now I imported in tabular form exactly same manner, so this is initial table of the LP problem the basic variable table 1. So, you can write in these, so you write it this first table initial table and tableau for the LP problem for the LP problem. So, first column you write it the first basic variable of basic variables, I put it this down.

Then, I am writing all the variables assigned in the standard LP problem x 1, we have an altogether five variables x 1, x 2, x 3, x 4 that x 5. Then we will write right hand side of the equality constants that is the b i, we can say then we write it the ratio of b i divided by a a i j. So, let us call this equation first impression, I am writing the first equation four

x 1 plus 6 x 2 plus x 3 is equal to a table, x 1 that is 4 into x 1, next is 6 into x 2 value x 2 6, then x 3 1 into x 3 x 3 value 1 and there is no x 4 x 4.

So, I will put this coefficient 0 and right hand side is what 85, similarly, this equation x 1 30 into x 1, so x 1 coefficient is 30, then minus 10 into x 2 minus 10 no x 3, x 4 1 into x 4 x 3. There is no x 3 means in other words I can say x 3 coefficient is 0, x 2 coefficient is 1, there is no x 5, so I can write x 1 coefficient is 0 is equal to 250. Similarly, the third equation 30 x 1 30, 60 x 2, then know there is no x 3 x 4, so their coefficient I can write 0, the x 1 coefficient is one that is equal to your 700.

So, this is the standard LP problem, this is the equality constant I have written into tabular form, next is your cost function objective function expression function and writing so see the objective function. Here, z minimise z is minus 99 x 1 x 1 coefficient is minus 99, so minus 99 into x 1 value x 1 minus 99 and you see this minus 90 x 2 x 2 coefficient is minus 90. So, value x 2, I will write minus 90 and there is no other variables are here, so the coefficient I can write it these into this into this now look at this expression.

If you recollect, I mentioned earlier that if you add a constant term with the objective function than the function value maximum that minimum or maximum value, the function at what point it occurs if you add a content constant to the functionality. It will occur in the same point, only you can say the whole objective function curve is lifted up or down depending upon the value of that constant term either positive or negative.

So, I can easily put it this into the side z, which are now objective functionality till z plus 525, this is our objective function value in our z plus 525. So, I will tell z plus 525, this function you have to minimise. So, first equation you see I have written this one, now which one I will consider the objective from what is called non basic function and non basic variables and basic variables.

If you see this metrics itself that you were according to our definition in a definition this the a metrics the x 3 involved in equation number 1 only no other x 3 is there. So, the x 4 involved in equation number to this equation only an x 5, it involves in equation number three or other equation. So, I can easily consider the x 3, x 4, x 5 are the basic variables, so I am writing x 1, sorry x 3 x 3 x 4 x 5. This is the basic variable I have consider and non basic variables are the remaining. If you recollect and are very equation, the

inequality equation or equality equation of the a what is called LP problem, there are m equations are three n variables are there.

So, there will be basic variable is m and n minus m variables will be the non basic variables. Similarly, here also we have a three equation five unknowns are there, so three are the basic variables and two are the known and remaining variables are non basic variables. So, I identified with an arrow x 1 and x 2 these are the non basic variables, now you say on this table if x 1 x 2 are non basic variables means it is value is assigned to 0, its values is assigned to 0.

Immediately, from this expression I can find x 3 is 85, you see 4 into 4 into x 1 value is 0 plus 6 into x 2 x 2 value is 0. This is 0 plus x 3 into 1 0 into x 4 0 into x y 0, so our x 3 non basic variable is 85, similarly in this expression, you see our x 4 is equal to 250. There are x 5 is equal to 750 because 30 into x 1 x 1 value is 0, 60 into x 2, x 2 value is 0, only x 5 is equal to 700.

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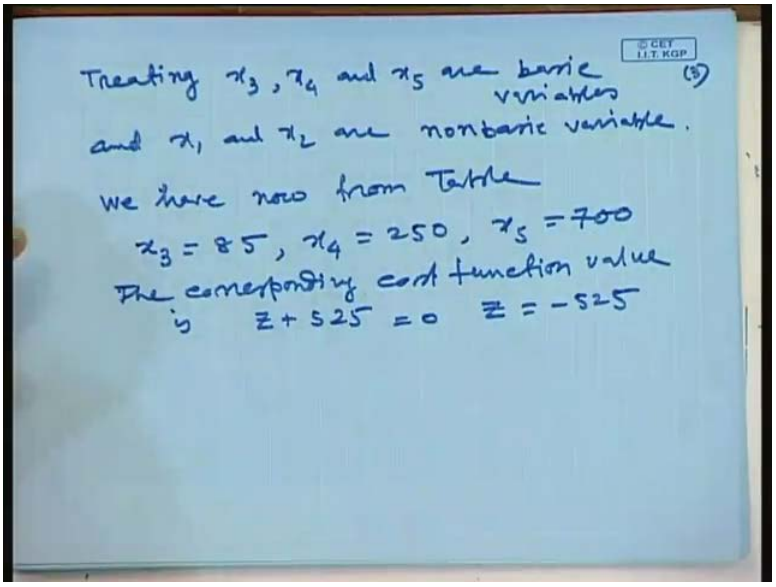

So, from this we can straightway write from this table that what is call our treating x 3 x 4 and x 4 are the basic variables and x 1 x 2 are the non basic variables whose values are 0 non basic variables. Then we have we have now from the table if you say, now we have from the table I can write we have, now on the table x what is our x 3 x 3 value is this one. This coefficient is 0, 4 into x 1 6 into x 1, 6 into x 1 x 2 value is 0.

So, our x 3 is equal to 85 x 3 value is our 85 similarly, x value is equal to 250 and x 5 value is 700 so this is the basic solution of that one. Now, corresponding objective function value is what see this one x 1 in 29, 0 plus x 2 into 90 0 x 3 into 0 plus x 4 into 0, 0. So, our objective function value is this left hand side, I did 0, right side is this, so z is equal to minus 525. If you see and the corresponding the corresponding objective function value cost function value is z plus 525 is equal to 0, z is equal to minus 25.

Now, our search we have to search to we have on to the adjacent vertex so that the function value is further decrease or the rate of the decrement must be negative with a negative decrement. Now, we find out where to select which one there are two non basic variable x 1 x 2 which one go for a basic variable means entering as a basic variable which one x 1 or x 2.

Similarly, from the basic variable which one will leave as a non basic variable which one lead as a non basic variable either x 3, x 4, x 5 leaving or you can say leaving basic variable which one is living the basic variable living basic variable. So, let us see with this one you make it concentration on this one that our actual 99 into x 1 minus 90 into x 1 x 2 have only explained that the largest negative coefficient will be our choice of pivot column. That means which element which variable will consider as the basic variable of non basic variables, so largest coefficient in the objective function is that one this sign largest.

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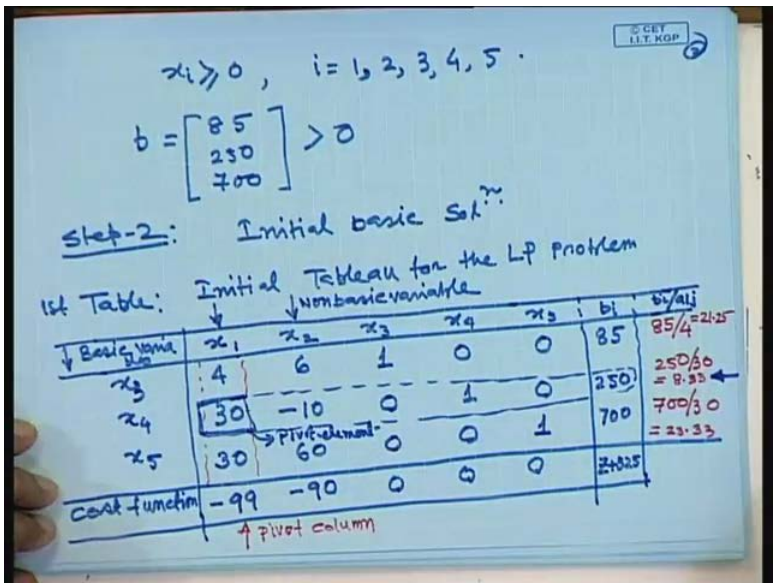

So, this is I will call as a pivot column, so you can concentrate on the dark cost function expression what is the expression 9 minus 99 minus 90 plus minus 90 into x 2 0 into x 3. Only these are seen the objective function or cost function only non basic variables are there and out of these two non basic variables which variable will consider as a basic variable. The variable which having what is called largest negative coefficient that will be treated as a the non basic variable is treated as a cable that means it is living from the nonvisible entering as a basic variable.

So, you write as a EBV, entering basic variables this one, so if you well, so this column and this column belongs to that x 1. So, this is the pivot column is the x 1 will go as basic variable, now who is variable that means x 3, x 4, x 5 which variable now will act as a non basic variable. So, that we have seen we have told you that how the check it this one take the ratio b this is b 1, b 2, b 3, b 1 this first row we want divided by a 1 which column first column. So, you are write it this ratio you calculate 85 divided by 4, then this is a b 2 is 250 divided by 30, then it is 700 divided by 30.

Suppose, f i you have a negative sign here that things will just ignore because that will different get it negative sign this one what will happening minus 30. If it is a negative minus 30 x 1 plus 60 into x 2 plus x 3 plus x 5 is equal to 700, now you see that x y what we are telling that x y is equal to what minus 30. If you take that side right hand side, then plus 30 x 1, then minus 60 x, so you have already considered is the pivot element of this one. So, if your value is increased if you take that side, it will be with 30 x 1 x 1 value is increased from 0.

Then, the function values are increasing instead of decreasing is increasing, so that is why if there is a negative sign is that you should not consider that quantity again. So, you cannot make what I am telling you cannot make x 5 look at this cannot make x 5, 0 by entering this one if it is negative. So, every negative that corresponding ratio you ignoring now we are all are positive we got it this one out of this ratio which was is the minimum list that will consider as a pivot row.

That is also we have when we are discussing in a algebraic approach, so if you find this one this will be a 21.4, 21.5; and this ratio will be 8.33 and this ratio will be your 23.33. So, it will take the ratio which one is the list, so this is the list one, so this arrow according to arrow will give you the pivot row.

So, this correspondent to this you see x 4 is a pivot row that from here you are corresponding to one is x 4. Now, so out of x 3, x 4, x 5 that one is identified as a non basic variable that means living basic variable which variable is living x 4 is living as an non basic variable. So, you just identify these columns, so which element is the pivot element, now this is our pivot element pivot column and pivot row corresponding to the element. The column element is the pivot element this call the pivot element, now you can see the x 1, I will treat as a basic variable, this is pivot column refunded an x 4, I will take as a non basic variable.

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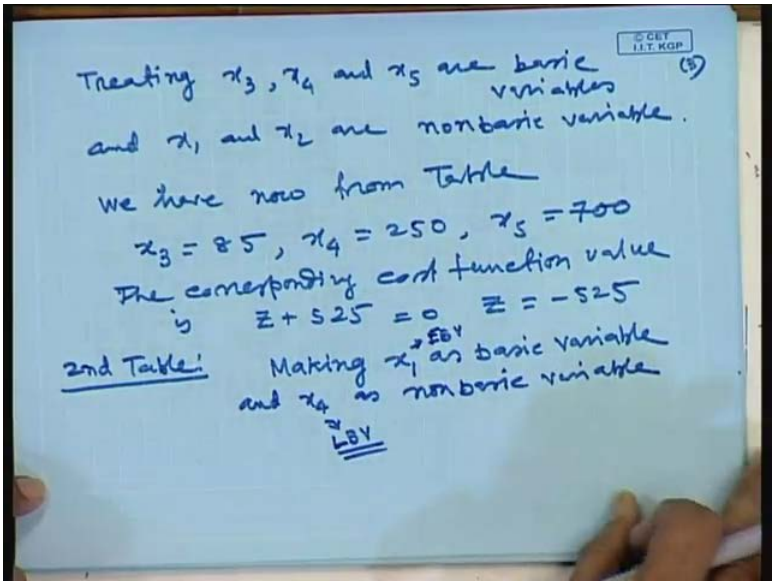

Now, generate that what is called new table suitable to second table is a second table in the second table. Now, you mention making x 1 as basic variable and remaining variables are same previously x 3 x 4 x 5 is the basic variable was there. Now, we are identified we have identified x 4 is living from the basic variables and acting as a non basic variable will x 4 place that is x 1 will take place non basic variable will act as a basic variable x 3 and x 5 in result.

We can say our basic variables are now x 1, x 3 and x 5 and which basic variable will treat as a non basic variable that is your x 4 and x 4 as non basic variable. So, you can call the x 1 is the x 1 is equal that entering basic variable x 1 entering as a basic variable this one. You can say that this basic variable living basic variable living basic variable which variable is living x 4 is living basic variables this one.

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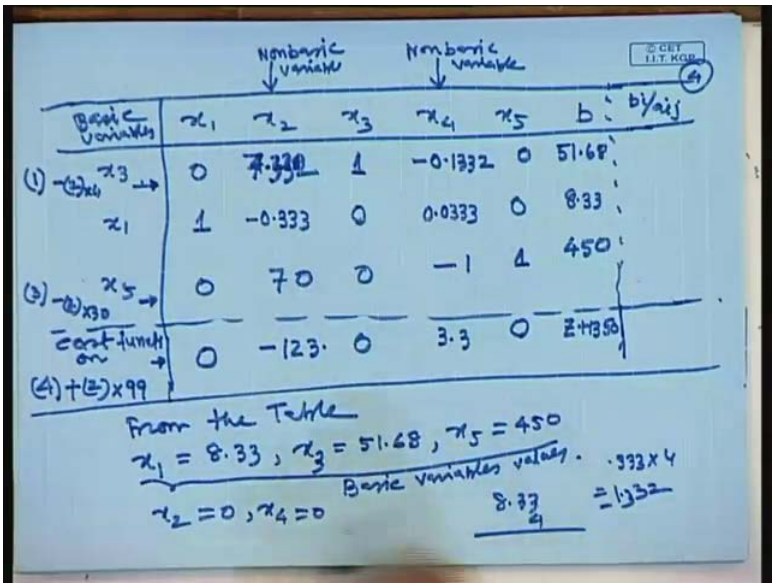

Now, keeping this thing in mind we just write the second table from the second table say how I am forming again you write basic variables in again x 1, x 2, x 3, x 4, x 5, then b then our ratio what is called b i divided by a i ratio. So, you previous that table which one is basic variable pivot element this is and that element is coefficient is x 1 coefficient is 30, but I had to make it one coefficient because when you convert into canonical all coefficient was 1. You see x 4 as 1, now this is a basic variable acting as x 1 is a basic variable, so that coefficient must be 1 what is mean that whole equation and dividing by 1 scaling by 30.

So, you prove the whole equation divided by 30, 30 by 31 minus 10 by 30 by 31 by 30, 0 by 32, 50 by 30. So, if you do this one that corresponding to that our second row, so I am writing the second row, you see one than minus 10 by 30. Then what is this variable will become one third was minus 0.333, then 0 by 30, this will be 0, then 1 by 30, this will be 1 by 30. That means this and we have 0.0333, then 0 by 30 is 0 and 250 by 30 that is 0 and that is two 50 by 30 will be a 0.833.

So, this and since x 1 is the basic variable and it is another basic variable, here you are previously using x 3 very variable than x 5, but x 4 is going as a non basic variable. So, this plays will be replaced by x 3, so x 3, then x 1, then x 5, this indicates which one are the basic variable and non basic variable use what previously x 2. Now, it is x 2, now x 4 is treated as a non basic variable will be treated as with arrow non basic variable. So, this

basic variables you say you see the x 1, x 4, x 5 this debate will in one equation x three first equations the x 3 is involved.

So, other equation x 3 is not that they that mean the basic variable involving one equation only x 4 visibility involving secondary question only. Other two equation with the first and third is no remember, so here I made it, but you see in the first equation the first division x 1 is involved 4 into x 1. So, I have to remove it that can remove it by elementary operations, now what I did it after scaling this equation. I multiplied by 4 desire to remove that 4 the first equation. Again, if you remove if you multiplied by 4 I am just writing this this is the equation number 4 each equation number 1.

So, you do the operation equation number 1 minus equation number 2 multiplied by 4, so if you multiply by 4, it is 4 before whole row at A. So, see this one I multiplied by 4, so 2 multiply by 4 and then subtract from 1, so it is now divided for 4 minus 4, 0, so this is a much 4 means 4 is into this it will be 1.3 something and the coefficient is 6. So, 6 minus of this one and what is this one if you say this value will come 4.668 and this 6, I multiplied by this.

I multiplied by this 4 that means for whatever multiplied by 4, if you multiply by 4, this will be a did this will be how much just a rough calculation. You see this will be 1.43, just 0.333 multiplied by 4, this will be 1.33 minus your subtracting that one. So, that value will be what subtracting means it will be it is already minus is subtracting.

That is not 7.33, just check it this one, so I divided by 30 first, so this is one, these one third this is 0. This one by 31 by 30 than 0, 250 by 30 that is the now i am doing elementary operations, I multiplied by this equation after scaling by 4 and subtracting from this operative from one, so 1 into 4, 4 minus 0. Then this is minus already agreed to 1, I separated from this 16 plus 70 minus 1 that means this will be 1. Then this is subtracted this is multiplied by 4, means just not we have calculated is 0.03 that value will be 1, 3, 2, 2 and x 2 value is what 0.

So, pointed this, this will be minus 0.1332, this is 0 and x 5 value is 0, so that will be also 0, then this multiplied by 48.3 multiplied by 4 and subtracted by 85. So, this will be near about 32 something and you see 8.33 divided by 4, 33 something subtracted from 85. Then you subtract this one, it will be coming 51.68, now you see this equation first equation x 1 is eliminated. So, x 1 is involving equation 2, but here x 1 also there so that

from the row 3 x 1 must be eliminated, so how you do it? This is already you have scaled by 1, so this equation multiplied by 32 second equation, second this equation you will get it 3 minus 2 multiplied by 30, if you multiplied by 30 is nothing but that one.

So, what will you subtract 3 minus 3 minus 3 is zero 3 minus this one is this 70, so it will be 0, then 70, then you say 0, 0, then this 0 minus 1. Then you see 0, 1 minus 0, 0 then it is you see this one this is 700 minus 250, it is a 450, so I got it this one, now you see there is no harm, now if you see in this expression x 1. If you see x 1 multiplied by 99 x 2 minus 90, there is no harm if you keep it like this way, but you can see easily because I know the expression for x 1.

I can easily eliminate x 1 from this equation from this second equation with the help of second equation; I can eliminate what is called our basic variables from the objective function. There is nothing wrong if you just eliminate away how to eliminate that this coefficient is you see in 99 into x 1. If want to eliminate x 1 from an objective function I have to multiply the normal equation of the equation 2 by 99 and then add it.

So, if you are multiplying by this order to this is a cost function in cost function our equation is fourth equation. So, fourth equation minus equation number 2, this is our equation number 2 into 99, so if you multiply by 99 and subtract from this one multiplied by 99 this one and subtract from this equation not subtract, add this.

You have to add it, so I made it add because already it is minus sign is there, so you have to add it if you add it. Then you can write it this is 0, this is you multiplied by 99, add it with this one 99 means approximately 100 that is 30.33 and add with this one. So, it will be what near about 30 three so 37something if added this this one, you will get it how much how much you will get it this is minus 99. So, you divided by this, you got this one, so you add this one, so this I am adding with this one, I am adding with this one that is one that is minus 99 and this is 33.

That means 123 near about 123, now this 0, 0 and third position is 0, so 0, 0, then this multiplied by 99, so let us call 3.33 add with the corresponding element that corresponding element means x 4, x 4 is 0. So, it will come 33, 103.3, so 3.3, then this 99 and this position is 0, so this will come 0 and this is what your multiplied by 99 added that add with what z plus 525. If you added this one, it will come as z plus 1350, so what

originate here are exactly same thing you are doing here. Again, this way this is for corresponding to inner to get this row this is corresponding inner to get this row.

Then, this is corresponding to did this row, now look at this objective functions the coefficient nonzero coefficient non zero coefficient objective function related with the non basic variables is x 2 into this one x 4 into this one. Previous table literacy exactly same and the basic variable coefficients are 0 and A. So, what is this cost function values x 1 into 0 plus x 2 into this one x 2 value is also 0 non basic x 3 value is non zero, but is value is a coefficient is 0, 0 plus x 4 into 3.3, but x 4 value non basic value is 0 to x 5 into 0 x 5 is not equal to 0. It is a basic variable, but this coefficient is 0, so our objective function value is that one z plus 1350, so I can easily find of this one.

Now, you see this one, so from the table immediately can write what is the basic variables value x 1 value is a x 1 value is 8.33, how x 1 into one x 2 into this quantity plus x 2 value is 0. So, 2, 0, 0 plus x 4 into 0 x 4 value is 0, 0 x 5 into 0, 0, so x 1 value is only 8.33, similarly I can write x 3 value. So, x 3 value is 51 x 3 value 51.68 and x 5 value is 450, so these are all basic variables value non basic variables is what x 2 is equal to 0 x 4 is equal to 0.

Now, look at this objective function this objective function you see 123 into x 2 and you have x 3.3 into x 4, so if you want to make increases non basic variable to basic variable x 4 value, you have to increases if you increase the function value will also increase. So, that will not disturb the positive quantity that will be act as a non basic view, but here you say this is minus 123, if you increase the value from 0 to some quality value, the whole function value is now decreased again. So, this one can do it, so this I can take it, now which will be our pivot column, I told you if really at this one you look at the coefficient of objective function which is most negative coefficient that will be considered as pivot column.

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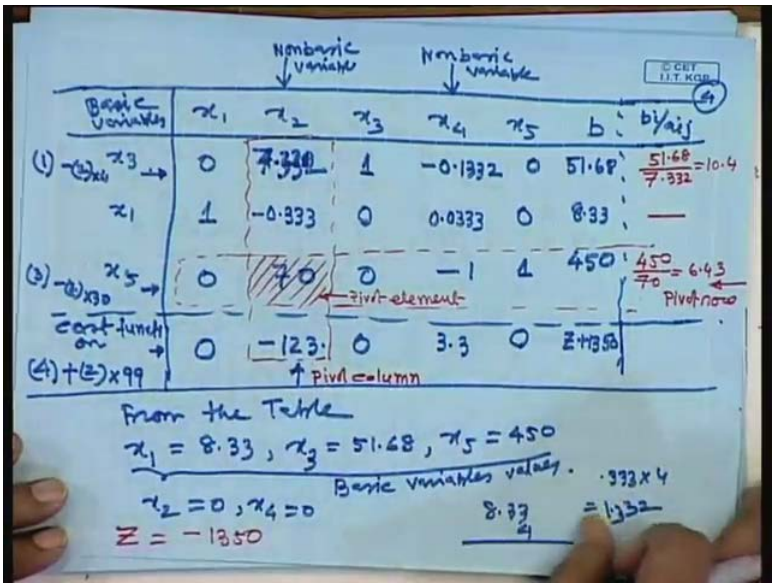

This is the pivot column for pivot column for the second table, so this is the our pivot column, so that means now the x 2 value as a basic variable to find out if x 2 go as a basic variable. Then which variables basic variable will enter in place of x 2, so that will be decided by the ratio of that one that we have explained when discussing the endemic approach. So, this ratio will be 51.68 divided by 7.332 and that value is coming in point and this negative, I told you do not care value that will not help you to decrease our function value.

So, these are 450 divided by 70, so this value is coming to 6.43 and lowest positive ratio is this one. So, this row will be traded as a what is call row pivot row means in this row we have whatever the basic variable we have that will be that will go as a non basic variable. You see this coefficient of this one, this is x 5, here, also if we just draw a that row and this will treat as on the pivot column, sorry element. So, to now start our operation with this element, so with this in that means this element is which one x 2. x 2 go as a basic variable and x 5 will go as a non basic variable.

So, basic variable if it is x 2 is a basic variable the coefficient you see you have so that coefficient I have to make it 0. Similarly, the corresponding coefficient in the objective function that also your to make it 0 by using elementary row operation. The same procedure if you follow now you say what is the form this table what is our objective function value is minus 13.25. Previously, we got it, if you see this one previously

function value you got it z is equal to some function minus 525. Now, you got minus 1350, now function value decreased is further we can decrease, it is the function value since the coefficient of objective function one coefficient is 0, so that is a possibility to increase that one, so let us call in table three.

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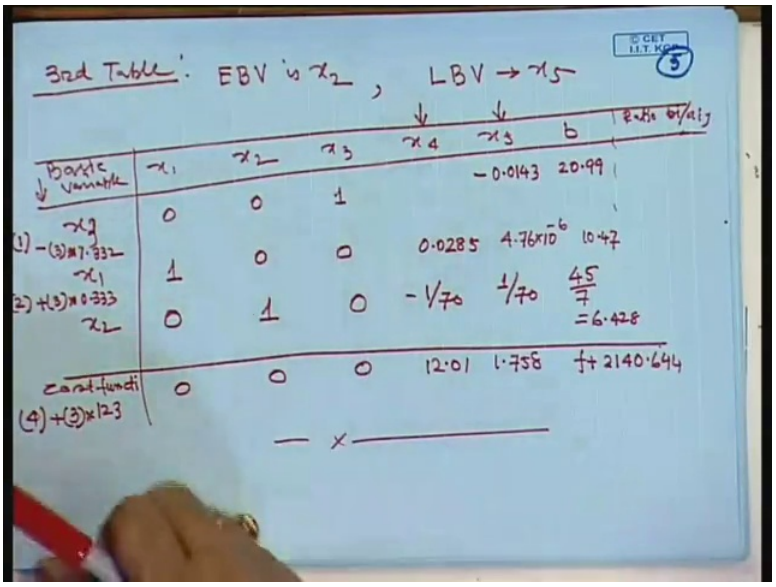

Now, third table, so third table can you tell me which one will weigh your entering basic variable, our entering basic variable is this basic x 2 is entering as a basic variable. So, in short I write it entering basic variable is x 2, so living basic variable which value which basic variable is living that means x 5. So, x 5 living basic variable is x 5, now you can easily generate our table same way x 1 x 2 x 3 x 4 x 5 b than ratio and here is basic variables variable. So, if you recollect this on what I told you here just now this is the pivot element, so I have to divide that one by 70 that third row. I have divided by 70 because I have to make its coefficient 1, so if you divide by 70 this equation, it is 0, 0, 1 by 70.

So, I am writing third last equation is 0, 1 then 0, then 1 by 17 minus 1 by 70, then this is our if you see our 450 by 70, 450 by 70 is a 45 by 7 in other words. This one value is 6.4 to 8, this one side, so I have to remove at to eliminate x 1 from this equation from this equation and as well as from this equation. So, what will do this one equation number 1 equation number 1 multiplied by this equation after normalising what we got.

This you multiplied by 7.33 and subtract from equation 1 and subtract from equation 1, this equation after normalising, you multiply by 7.33, then subtract from equation 1. Similarly, to get you eliminate x 2, you multiplied by this equation after nomination by 3.33, add with equation number 2 to eliminate that one. Then this equation you multiplied by after normalization of third equation after multiplying by this by one, 23 add with the objective function. Then you will get what you will get it, so our nonbasic variable are which one we have identified non basic variable, our non basic variable says x 4 and x 5 x 5 is going as a non basic variable.

So, x 5 and then our x 4 is the non basic variable and our basic variable are x 1 previously our x 1 this is x 3, x 1 and x 2, x 3, x 3 was the x 3, x 2 is entering as a basic variable. So, I have written x 2, so this is our basic variable, now you filled up with this one, then you will get this the similar manner. If you do it as 0, 1 as I told you the same method, if you follow it, third equation I multiplied by this third equation is I multiplied by this one. What I am writing equation number 1 minus equation number 3 into 7.332 in order to get equation number 2, equation number 2 minus equation number 3.

It is already plus minus that I have to add this multiplied by 0.333, so then next cost function however that the cost function. Then you will get that equation number 4, this is the equation number 4 plus equation number 3 into 123, it will get this finally, if you do this operation equation number 1. Then your 2 and 4, then you will get finally, the results of that 1, 0, 0, 0 12.01, 1.578, then you will get this will come f plus 2140.64, this will come that is x 4. That value this will come 0, then this will become 0.0285, this will come 4.76 into 10 power 6 and 10.47 and this will come minus 0.013 and 20.99.

Now, immediately can say x 3 value is what 20.99 x 1 value is what 10.47 x 2 value is what 6.42, so these values is will be coming, see this value 1 by 70 into 1 by 70 into this will be 0. So, the next class I will complete this table, now time is up, so just complete this in next class, so will start from this table next class, so if you see this last two of the objective function totally all coefficient are positive. That means there is no chance to the function value means decrease the function value by changing one of the basic value is a non basic variable and non basic variable is a basic variable, so we will continue next class.