Optimal Control
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Lecture - 15
Matrix Form of the Simplex Method (Contd.)

So, last class we have discussed regarding the basic variables and non variables, in context with the solution of algebraic, what is call equation linear equation.

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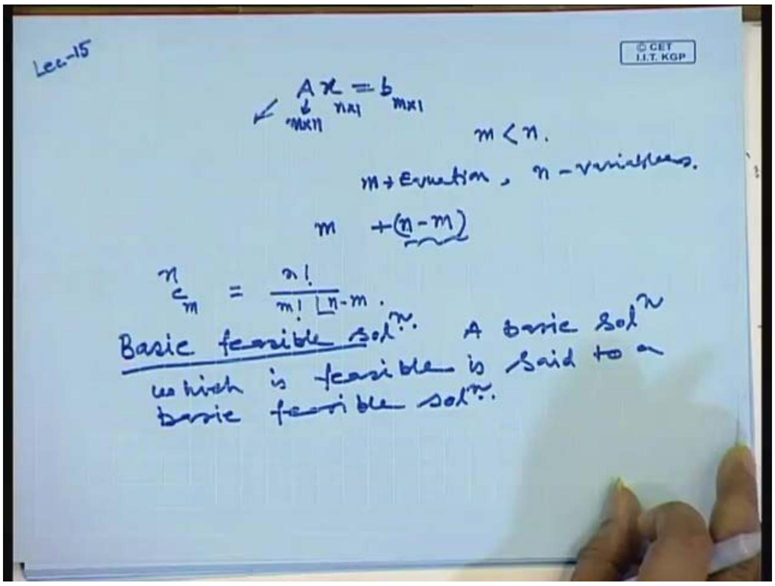

Let us call our algebraic linear equation A x is equal to b, x is the number of variables whose dimension is n cross 1 and variables are their b is the right inside of the equation which is m cross 1. So, immediately we can find out what is the dimension of A, this indicates that we have a m sets of linear equation again and n unknown are there. So, if there are m equations, m equations are there and n variables are there out of n variables we can in this n variables, we can split up into 2 parts m plus n minus m variables. If we assigned that n minus m variables, variables value assigned arbitrarily then easily we can find out the m variables that x m variables we can find out.

So, we have a in fine number of solution of this one for arbitrary choice of n minus m variables, which in turn we can get it m x variables. Now, question is if that n minus m variables say last n minus m variables of x, we are assigned to it 0 then first m variables of x, we can find out and in that situation when we have assigned n minus m of x of last n minus m variables of x, assigned to it 0. And in turn would we have found out the first m variables of x then that solution is call basic solution. And number of basic solution in turn will get it n c m, that is equal to factorial n divide by factorial m and the factorial n minus m that we have seen.

The variable associate with the non-basic variable we have assigned 0 again and variable with what is call the variable which is not equal to 0 and a non-0, that we call is the basic solution basic variables of this solution of this equation. So, we have a now basic variables at basic variables and non-basic variables, when we solve this equation will get a what is called a set of variables in the basic solution, a set of variables in the basic solution and in the basic area set of variables which is non-0 will call the basic variables. And which variables are 0 then we will call it is a non-basic variables.

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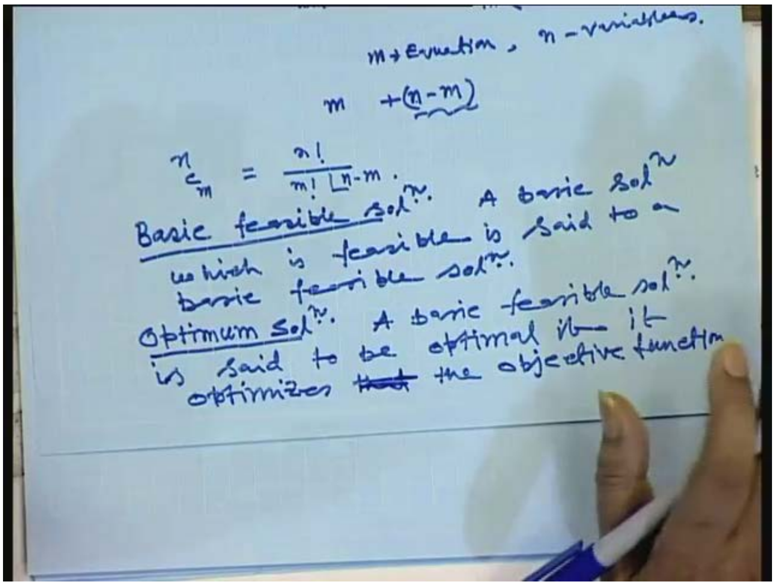

So, we have discussed up to this one then what is meant by the basic feasible solution? The basic feasible solution, a basic solution a basic solution which is feasible a basic solution which is feasible is said to be basic feasible solution. That means basic solution whose satisfied all the constant associate with the optimization problem, all the constant associate with the optimization problem if it is satisfied then that solution is called basic feasible solution. A solution maybe basic solution, but it does not satisfy the all constant associate with the optimization problem, then this solution is called basic non-feasible solution and that solution is not acceptable. so next is your optimum optimal solution.

A basic feasible solution is said to be is said to be optimal, if it optimizes that objective function optimizes the objective function that is suppose, that we got the basic feasible solution multiple of basic feasible solution we get, out of this all basic feasible solution and actually its basic feasible solution, which basic feasible solution will give you the optimal value of this objective function will call that is the optimal solution of the problem. That mean linear programming problem.

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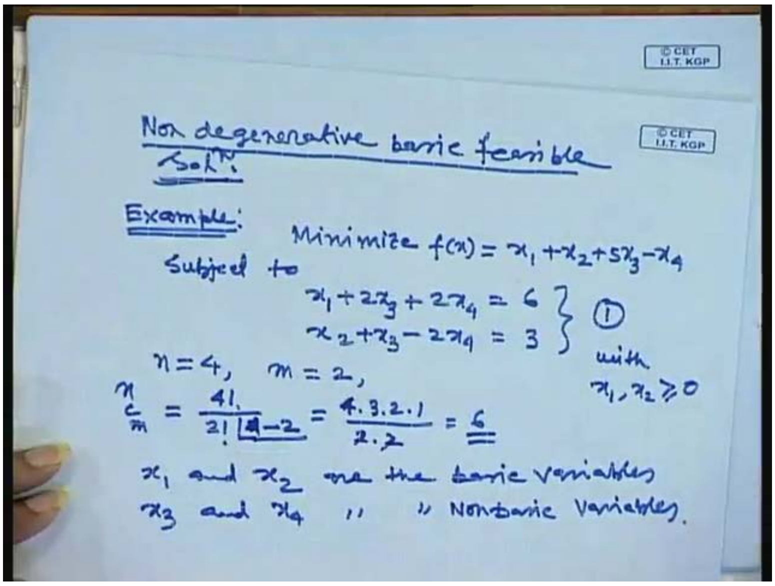

So, next is non degenerative the basic feasible solution, suppose if it got the basic solution of this what is call a linear programming problem is about the if we got all basic variables, all the basic variables are positive again than it is called the non-degenerative basic solution. All basic variables values are positive that it is called non-degenerative basic solution. And let us see that how we can introduce the solution of this one solution of what is call linear programming problem by using matrix method.

So, let us take one example minimize f of x plus x 2 plus 5 x 3 minus x 4 subject to the constant x 1 plus twice x 3 plus twice x 4 is equal to 6 and x 2 and x 2 plus x 3 minus twice x 4 to is equal to 3. So, let us call this is the equation one and with x 1 and x 2 greater than equal 0. So, our problem is our objective function is linear and our constant here in this problem is all are equally constant are also linear. So, it is a linear programming problem, then how to solve this one in matrix form that in other words finally, will say simplex method basic background involved in this one will just discuss.

Now, see this one how many unknown variables are there x 1, x 2, x 3 and x 4 so n is equal to 4 and how many equations are there m is equal to our equation m is equal to 2 used. So, naturally there are 4 variables are there 2 equations are there so n minus 2 is the number of non-basic variables that the variables the variables, which we will assign you will assign to 0 corresponding other variables we can find out and that solution is called basic solution. And we already know we already know the number of basic solution involved is n c n c m in this case in this problem we have factorial 4 factorial 2 then factorial n minus and m is a 4 minus 2.

So, this is equal to 4 into 3 into 2 into 1 than 2 into 2 so we have a 6 basic solution will get it. Now, here if you see our basic variables are there at this point I can assign the our basic variables that this and this that x involving this equation, but x does not involved in this equation. Similarly, x 2 involve in this equation only second equation, but it does not involved in first equation. So, this is already in canonical form that what we have discussed earlier. So, you can write it that our x 1 and x 2 are the basic variables and x 3 and x 4 are the non-basic variables.

Now, you see if we assign x 3, x 4 is 0 in this equation immediately can find out x 1 x 2 and corresponding objective function below you can find out. Now, we look for this is the one basic solution, now will see which of the which one of the basic variables will move out of this tool move to some other point, agree? And which of the basic variable will move to some other point that means that either of x 3 or x 4 which is now 0 one of them will be non-0 and in this case x 3 x 1 x 2 which is non-0 values will be shipped will go to the 0 values. That means one of the basic variable will act as an non-basic variable and one of the non-basic variable will act as a basic variable, in turn whether we are getting the function below is reduced from the previous case or not, so let us call in this case if you see our corresponding to this one, I can write it that our solution.

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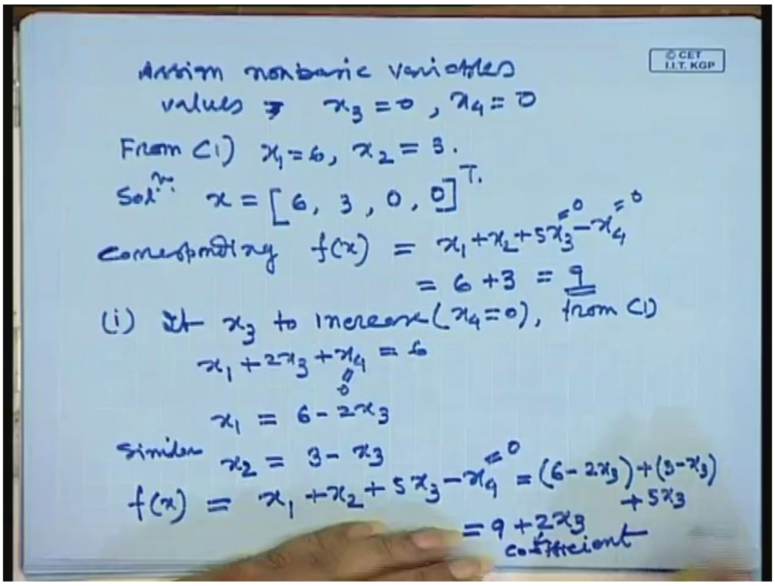

Assign non-basic variables values is equal to x 3 is equal to 0 x 4 is equal to 0 then from this equation from equation one from one, one can easily find out x 3 0 x 4 0. So, x 1 is equal to 6, so x 1 is equal to 6 and x 2. Similarly, x 2 equal to 3 so our solution now coming x is equal to x 1 is 6 x 2 is 3 x 3 is 2 x 3 is 0 x 4 is 0, this is our solution and see what is the corresponding objective function value. So, corresponding function value objective function value, what is this objective function? If you see this our x 1 plus x 2 plus 5 x 3 minus x 4 and this equal to 0, these are these are the non-basic variables and so we will get it 6 plus 3 is equal to 9.

Now, our problem is that some of the basic variables will enter as a non-basic variable and some non-basic variable will enter as a basic variable, so which one will enter among x 3 and x 4 which will enter as a basic variable. Similarly, x 1 and x 2 which will be entered as a non-basic variable, let us investigate or in see this situation case. First situation if let us call x 3 to increase because previously x 3 value was 0 you see this one 3 value is 0. Now, I want to increase x 3 keeping x 4 same if x 3 is increased an x 4 remains same keeping same than one from one equation one, what we can write it x 1 plus 2 x 3 plus x 4 is equal to 6 and our x 4 is equal to 0, but our x 3 is not 0 because we are increase this value this value was 0 now we have increased from 0 some value. So, that we can write it the x 1 expression is 6 minus 2 x 3 this is one equation.

Similarly, from equation two from equation one sorry, x 2 is equal to 3 minus x 3, agree? So, what I did it whatever the coordinates was there x 1 x 2 x 3 x 4 now a part of x 3 to some positive value and see whether the function value is going to be decrease or not, if it is decrease than will accept that one, but simultaneously to see which basic variables we have to change it to a non-basic variables. So, let us see the objective function is what, now objective or cost function is what x 1 plus x 2 plus 5 x 3 minus x 4, but we have not change its value x 4 is 0. So, what is coming, x 1 value we write in terms of x 3 so minus 6 twice x 3 this is x 1, x 2 is see x 2 value I am writing x 2 is 3 minus x 3 plus as it is 5 x 3, which will come if you say I manipulate this one 6 plus 3 9 than minus 2 x 3 minus x 3 3 x 3 plus 5 x 3. So, it is a plus to x 3.

Now, look at this expression this important point you see the function value, now previously function value was 9 with this that our non-basic variable was x 3 and x 4 and basic variable which value is 6 3 correspondingly we got it 9. Now, we are get getting 9 no doubt where plus some other quantity 2 plus x 3. So, 2 is positive and x 3 value is greater than 0, we are increasing this value does not 0. Now, this value is increasing function value is now increasing, so that x 3 we cannot select as a basic variables. So, this coefficient is called this coefficient associate with these variables, which is converted into a non-basic basic variable this coefficient is called…

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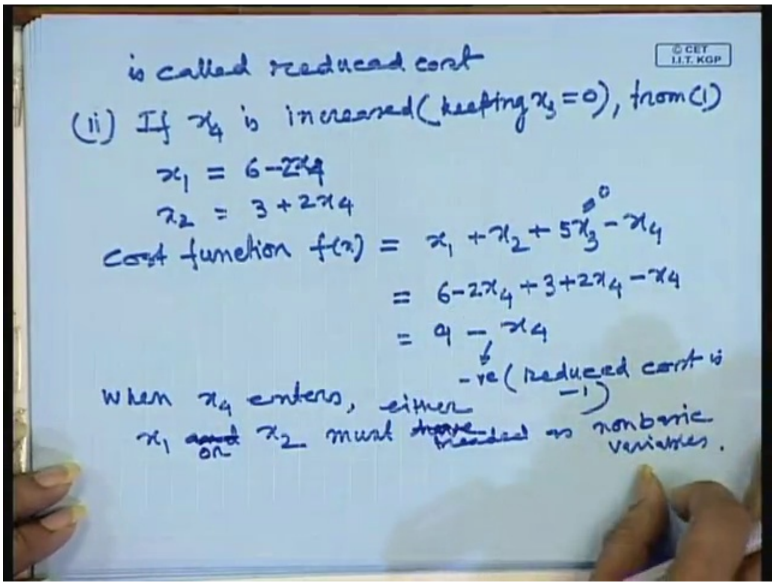

is called reduced cost, now look at this expression this is not reducing the cost when it will reduce if the coefficient associated that constant m associated with the variable, if it is a negative quantity than it will reduce the cost in that sense it is called reduce the coefficient associated with this one is called reduced cost. So, our conclusion if you consider x 3 from non-basic variable to basic variable will not able to reduce the function value, we were not able to reduce the function value from the previous value. So, this cannot be a choice for basic variables. So, what is the option we left x 4.

Next is our x 4 is 2 if x 4 is increased keeping x 3 is equal to 0 then form similarly, form one putting the value of the x 1 x 2 x 3 x 2 now I have increased that means x 4 now I am increasing x 4, now entering as a basic variables. So, from equation 2 one can write x 1 is equal to 6 minus x 2 x 4, see this one here from equation one this case. So, our x 4 is non-zero x this is 0 so x 1 is equal to 6 minus 2 x 0 minus 2 x 0 similarly, here you see x 4 is non x 0 x 2 will be 3 plus 2 x 4 so x 2 will be 3 plus 2 x 4 then what is our cost function? Is f of x is equal to x 1 plus x 2 plus 5 x 3 minus x 4.

Now, you see our basic variable as we have not change only the x 4, we have increased the 0 that x 4 that what is called non-basic variable it is not change that x 4 is the non-basic variable, we are now changing means it values is increasing it is now entering as a basic variables non basic variables is entering as a basic variables. So, this what is this value you see x expressed in terms of this x minus 2 x 4 the next 2 is 3 minus 3 plus 2 x 4 then minus x 4. So, what is this 9, 9 minus x 4 and the coefficient is a constant term associated with this one is negative, this quantity is negative at a reduced cost is minus 1, reduced needed that a reduced cost the coefficient is minus 1 a negative minus 1.

So, what is this possibility that if you previously x 4 is 0 now if you increase it than this objective function below will decrease, agree? So when x 4 enters when x 4 when x 4 enters either x 1 or x 2 must have 0 value that means they must, must either x 1 or x 2 to either x 1 or x 2 not n and or x 2 must be treated as a non-basic variables, of a must be treated as non-basic variables. We mentioned earlier if you remember that way if you have a n variables they are n equations there that n minus m is the non-basic variables and m is the basic variables.

So, there are two basic variables that way consider, the value is becoming objective function is reducing, which of the 2 x consider till now x 3 is entering as a basic variable.

So, one basic variable earlier that x 1 and x 2 out of this 2, 1 will enter as a non-basic variable out of this, that is when we to investigate which one will enter as a non-basic variables. So, this is decided by looking again, what is your you see this expression x 1 is this expression you look at this expression, which out of this see is clear from this exposure x 1 and x 2 out of this x 1 and x 2 which variable I can make it. Previously it was not 0 now I can make it 0 with variable it is clearly that these variable, I can make it that x 1 I can make it 0, because x 2 is positive if I put x 2 value x 4 value is 2, 3 then it is variable, but this is not possible, so our basic variable which one is x act as a non basic variable.

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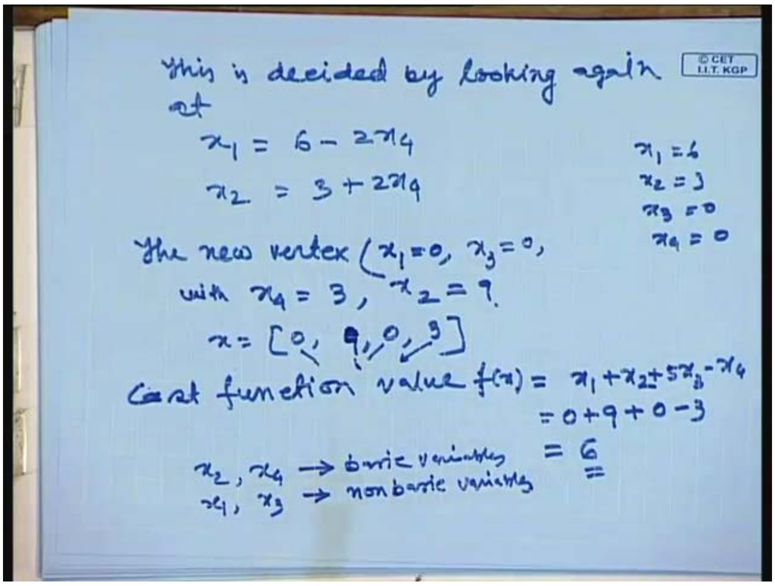

So, that is I am reviewing this is decided by looking again at this expression at x 1 is equal to 6 minus 2 x 4 and x 2 is equal to 3 plus 2 x 4, and look at this expression if x is in case if x we increase into 3 x for is equal to increase to 3, then this will be 0, but there is no chance x 4 is becoming 0. So, our new vortex ultimately our vertex is coming our new vertex, previously if you remember we have started with the vertex x 1 equal to I think you got it, x 1 is equal to 6 x 2 is equal 3 x 3 is equal to 0 x 4 is equal to 0.

Now, our new vertex is x 4 is entering at the basic variable and so on new vertex is now coming x 1 is 0 because I and it is possible to make it this 0, then x 3 is 0 and immediately this non-basic variables are assigned than we can find out with x 4 is equal to 3 and x 2 is equal to 9. So, our new vertex is 0, 0, 3, 9 that means our x is equal to 0 0

0 3 0 9 0 9 0 3 this new vertex. And now what is our function value just say new that objective cost function value cost function f of x is equal to x 1 plus x 2 plus 5 x 3 minus x 4. Say our new vertex is coming x one is 0 0 x 3 is 9 9 x 3 is 0 x 2 is 9 x 3 is 0 that 0 then this is minus 3 so our value 6 is coming.

So, previously at this our function value was 9 if you see our function value that previous vertex or point or 9 so that 9 value now, it has become cost function is seen to function below is reduced. So, this function value next step that our now you see this and these are the non-basic variable and these are the basic variables. So, our basic variables which one x 2 and x 4 are basic variables now and x 1 and x 3 is non-basic variables. Now, we have to see which non-basic variable will act as a basic variable following the senior procedure, which non-basic variable out of x 1 and x 3 will act as a basic variable and which one of x 2 and x 4 will act as a non-basic variable in next iterative process.

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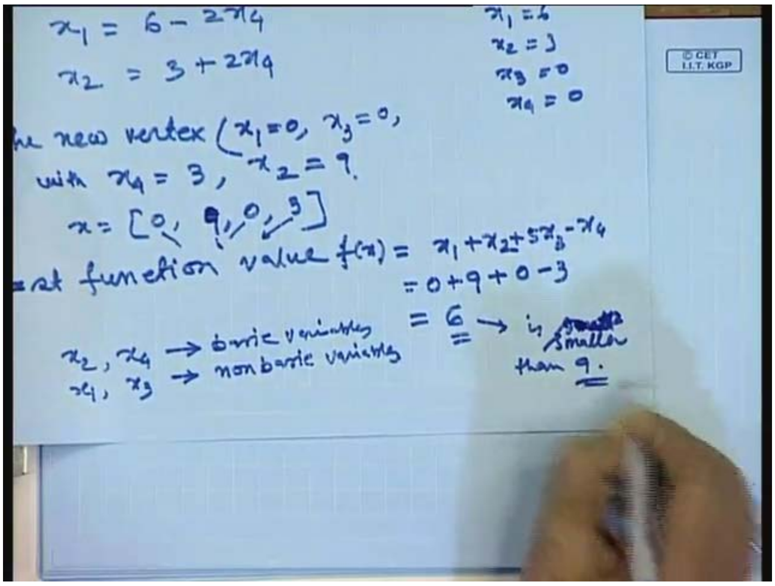

So, let us see next you can see this 6 is smaller 9, what we got it is smaller than smaller than 9. So, you can further check that what we can proceed now this whole process I can do with the matrix operation, if you see this whole process I can do matrix operations. So, far I did it up to this what we will do this, you see the our basic equation our a matrix it takes if you see our a matrix is what the in beginning I have written possibly, I am not written so let us call this situation that this equation I can write into matrix vector form. So, if you write it in matrix and vector form you see this one.

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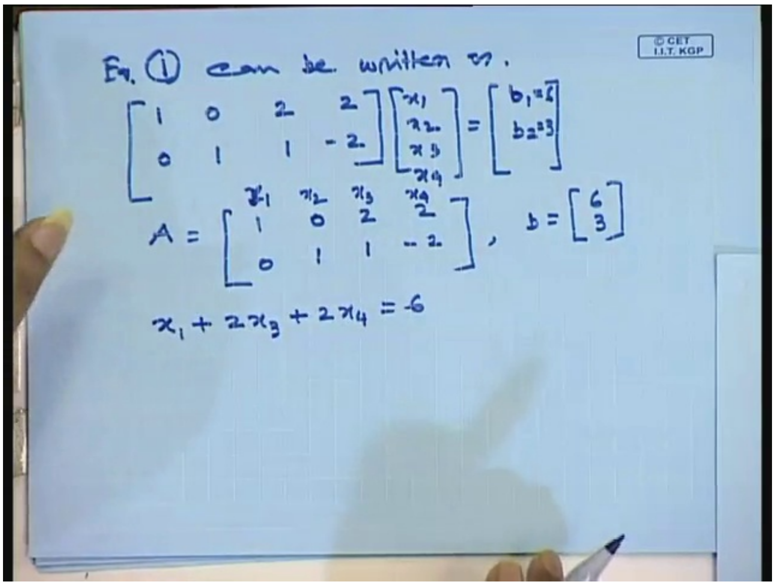

That our equation one can be written can be written as a is 1 0 2 2 than you 0 see I am just writing from this equation matrix and vector form from equation one, 0 1 1 minus 2 into x 1 x 2 x 3 x 4 is equal to b 1 or our that matrix you can write it b 1 b 2 and what is this b 1 b 2, b 1 is our case is 6 this is our case is 3, 6 and 3 this is our a matrix is we write or assume 1 0 2 2 0 1 1 minus 2. So, first situation this, this co-efficient corresponding to your x 1 is coefficient corresponding to x 2, this coefficient corresponding to x 3, this coefficient corresponding to x 4. And our b is what 6, 3.

Now, you see this one what we can write it for this one for first we have this is already we see if I we see whatever we have done it that if you want to represent into matrix, and vector form in other words in the elementary row operations, we can do it like this way. See this one what I am writing integer x 1 plus twice x 2 plus twice x 4 is equal to 6, so clearly you see first column and your second column corresponding to the our basic variables. First column coefficient x 1 x 2 visibly, because this is already in anti matrix form and the column 3rd and 4th column are the non-basic variables, agree?

Now what we have to do with this than this one that operation that what you did with if you want to do that operation that what you did that x 4, we have seen that x 4, this x 4 is entering as a that x 4 is entering as a basic variables. And x whatever seen it that x 1, x 1 is entering as a non-basic variables. So, there is a interchange between x 4 is going as a basic variables and x one is going as a non-basic variables. So, corresponding matrix is what?

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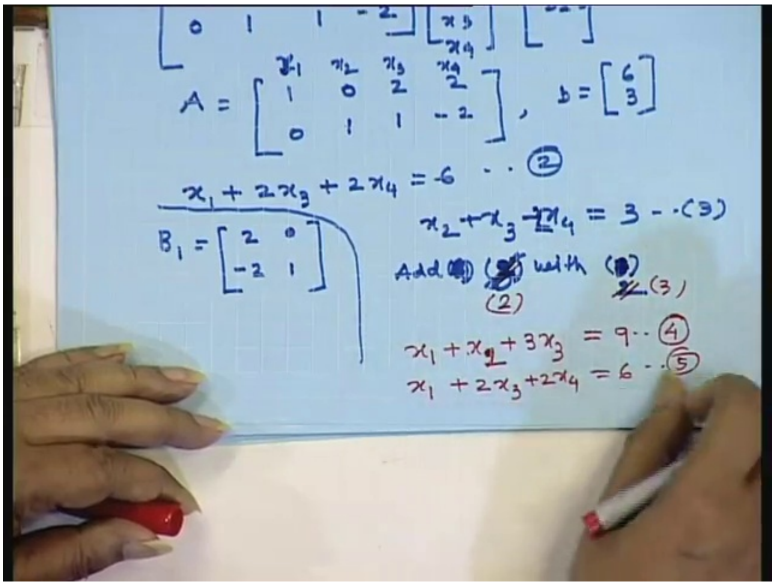

B 1 you see B 1 I am writing is 2 minus 2 0 1 and corresponding to 2 you see 0 one and corresponding to 4 it is 2 2 so this. Now, you do the what is call elementary operation of this and this one, how will do it? You see this equation I written it and another equation I can write it here that x 2 plus x 3 minus x 4 is equal to 3, just say this on. So, what we have to do this I have to consider as a basic variable that means x 4 will be in one equation their x 4, now it is 2 involving this equation. Let us call this is equation number 2 risk of this education number 2 and this is equation number 3 so x 2 x 4, which is going as a basic variable now involving 2 and 3.

So, x it x 4 should be involved into one of this equation and our next basic variable is what x 2 x 2 is not there, but it is here so this will not disturb x 2 only x 4 will remove from this place then how we will remove from this place you to do some elementary operations, what is the operation you have to do it? If you see this one that 2 x 4 sorry this is x 2 x 3 that is 2 x 4, 2 x 4 now if you do will intimate you add equation two to equation three then this variable is eliminated. So, you can write it at 3 at 3 equation 3 with 2 equation 3, equation 2 is equation 2 is added with equation 3 add 2 with equation 3, right?

Then if you add what is this if you do this one than you say this one that will coming equation number this is 2 add 3 sorry it is a 3 add 3 equation number two just minute with the this add 2 with equation number three that know this equation you add that one if at this one you will get x 1 plus x 2 plus 3 x 3 is equal to 9. Let us call this equation number four, so what I did it this equation added with this one so ultimately it has come x 1 plus x 3 plus 3 x 3 has come to 9 and this equation I re-write here, this is the equation number this is two.

And this equation I rewrite here if I rewrite here are what you will get it x 1 plus 2 x 3 plus 2 x 4 is equal to 6, let us call this is equation number five. So, still you say it is not converted into canonical form because x 1 coefficient x 2 coefficient one, but x 4 in x 2 is not in this equation x 4 is not in this vision, but its coefficient is 2. So, I have to divide over both side by 2.

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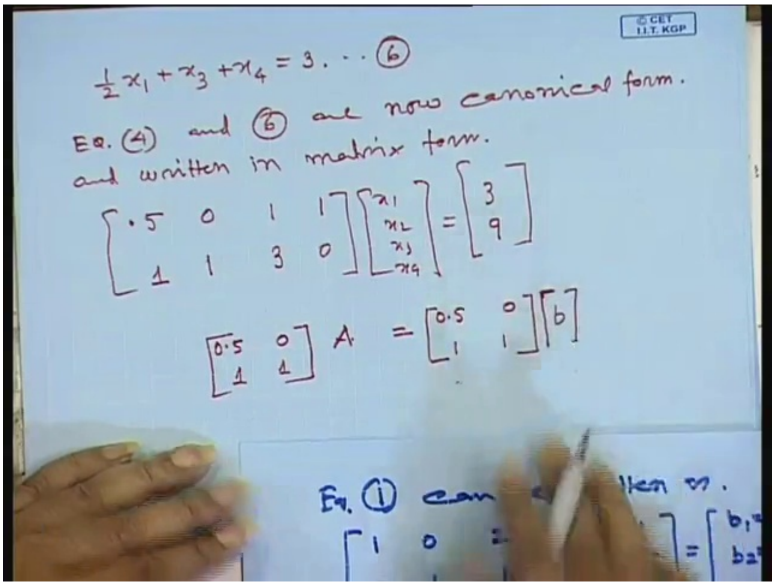

If I divide both side by 2 than equation will come half x 1 plus x 3 plus x 4 is equal to this is be divided by 2. That means, 3 so let us call this equation number 3 now equation 3 and 4 if you see equation not that three is four and six equation, equation four and six are now canonical form. So, this now you see if this equation four and six are in canonical form, that coefficient of x 4 is 1 and coefficient of x 2 is also 1.

Now, if you assign the what is our non-basic variables a state where you will get it that x 4 value and your x 2 values, but we want to do in matrix form that that one. So, our if you see now our equation that equation now canonical form and written as, written in matrix form. So, if you write matrix form this will be a 0.5011 then it is a 1 equation four you see equation 1 I am writing 1 1 3 0 1 1 3 0 this and this is x 1 x 2 x 3 and x 4 is equal to you are getting 3 and 9.

Now, you see this and this it is a canonical form this, so this we can get directly from the original matrix A, this expression we can get directly from original matrix A by using a row operation, what row operation is that now telling in terms of matrix. So, just see this one that this equation that what we got from this equation this, this equation and this equation, what it did it here?

We did elementary row operation basically elementary row operation in order to get equation number four and six see the operation you got this is nothing but matrix 1 2 2 0 1 1 a what it did it here I add equation number this equation number six the equation number that two. I just divided by immediately I divided by if you see the equation number two, that this one equation number two which is written here like this I divided by 2. So, that is a first you divided by 2 means 0.5 whole equation I divided by 5 ultimately I am doing it here that a matrix I multiplied by first row you divided by 0.5.

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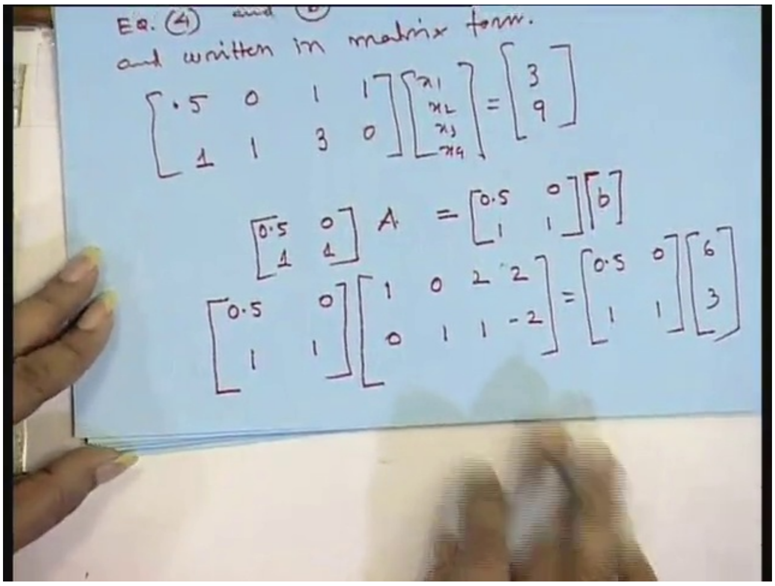

Means, 0.5, 0 and other elements are remaining same and second element what we did it second element operation I did it this and this I added the equation number two, and equation number three I added. So, the value this added one and one so this a is equal to the both side you have to multiply of that equation A is equal to 0.5 0 1 1 and this is equal to your B.

Now, I will write our a matrix 0.5 0 1 1 our A matrix is what see our a matrix is If you see our a matrix 1 0 2 2 0 1 1 minus 2 is equal to that our B matrix is 6 3. If you see our B matrix that is our 6 3, mind it what did it here the elementary row operation that is translated into matrix that operation. First word is this equation I divided by 2 this equation I divided by 2, which is coming that one is divided by 2 and then I add this equation with that equation. Now, if you see this equation multiplied by this and this so this into this so this element is multiplied by 0.5, this into this 0 this into this 0.5 multiplied by this into this 0.5 it is multiplied.

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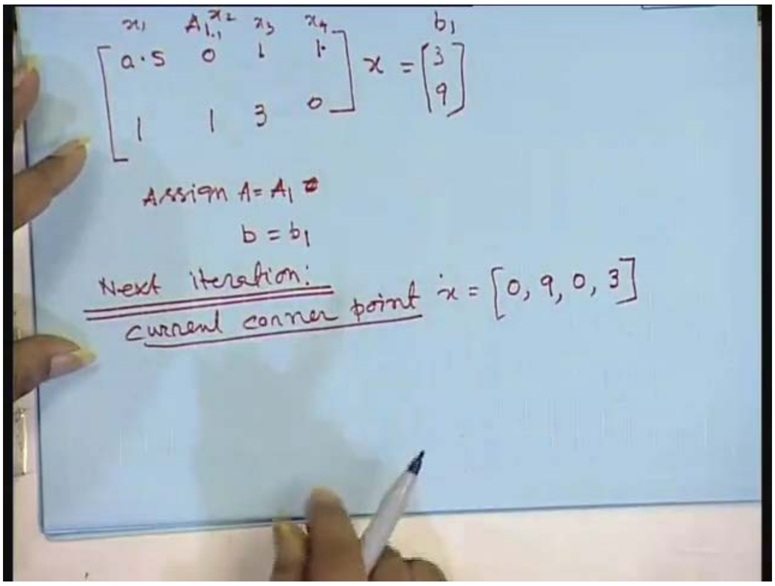

So, ultimately we are getting this multiplication this matrix is coming 0.5 0 1 1 than this and this part I completed, this into this there are I am adding you see this row added with this one this into this plus his into this I am writing. So, this is one next is this into this plus this into this that means, this two rows are adding this one than this operation means, this matrix multiplication by this row means, this two row are adding by that both are in 1 1. So, this is this and this 3 that this is 3 and this and this is 0, and this is what this I multiplied by this whole thing here x is here that I missed it x is here.

So, ultimately this is coming that one and this case you see this into this 3, so this is coming into 3. So, this into this, this are the added means 9 and exactly you see what I got it here that is I can do matrix operation now I can do it matrix operation now in this way. So, after doing this one as if this is our now matrix is changed A 1 and B 1 now I assign, assign A 1 is equal to A 1 is equal to A and B 1 is equal to B 1 as if this is the equation constants are given you minimize our objective functions.

So, now what to do next, you know at this point this is our x 1 x 2 and this column x 3 and this column x 4. So, x 1 x 2 and your x 4 are the used basic variables x 1 and x 4 are the non basic variables. Now, I will change similar to our earlier method I will change one of the non basic variable will live as a basic variables and one of the basic will live as a non basic variables, will proceed in the similar manner.

So, the way I explained this one the same thing you can do it next iteration, next iteration. So, current corner point is what current vortex or current corner point, our non basic variable is x 1 and x 3. So, it will be x is equal to 0 than we got x 2 value is what just now you have calculated value after we have seen that value you got it, what is the x 2 value we got it 9, if you see than x 3 is non basic variable 0, x 4 value we got it 3. This is our current corner point, now we will see check it this one.

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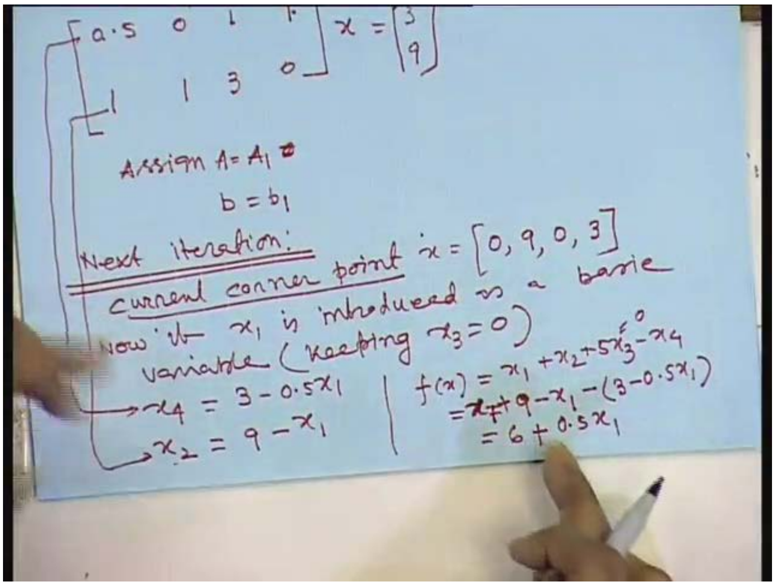

Now, if x one is introduced as a basic variable is introduced as a basic variables keeping x 3 is 0, then from this equation what will get it? This is our this equation a in this equation we get it that way x 4 is equal to this is x 3 0 this into this 0.5 x 1 this is 0 this is

0. So, x 4 is equal to you get 3 minus 0.5 x 1. Similarly, second equation from this equation we are getting that one will get this one this will be x 2 is your 9 minus x 1.

Now, see our objective function f of x our objective function this is f of x objective function is what x 1 plus x 2 plus 5 x 3 minus x 4, our basic variables you seen it we have changed it. Now, this is x 1 this is a non basic variable, we have to now change into basic variable x 3 value is 0. So, I will put it x 4 value x 4 value is x 1 value is what x 1 value is 0, x 1 value is 0 than x 2 value is your 9. So, you will write x 2 value in terms of this so 9 minus x 1 than your x 4 value is x 4 value you will write as 3 minus 0.5 x 1. So, ultimately it is coming 6 minus 6 minus minus x 1.

Now, see x 1 is now is used a basic variables that means that value previously x 1 is 0. Now, we are increasing this value if you increase this value our reduction cost reduction value is negative coefficient is negative. So, x 1 value is if you increase it, that value will what this what just see this what let's see this is x 1 value and this is x 1 value is what this is this x 1 is missed it, this x 1 is that x 1 because x 1 value is now not equal to 0. Now, it is a basic variable other than 0 so x 1, x 2 value is 9 minus x 1 x 3 value is 0 x 4 value is that agreed. Now, you see this one since x 1 value is from 0 x 1 is positive if we that if use that x 4 as a, if you use that x 1 as a basic variables than this will increase the function below. So, x 1 cannot be the x 1 cannot be the basic variable so what choice is left x 4 sorry x 3 with x 3, if you try with x 3 similar logic.

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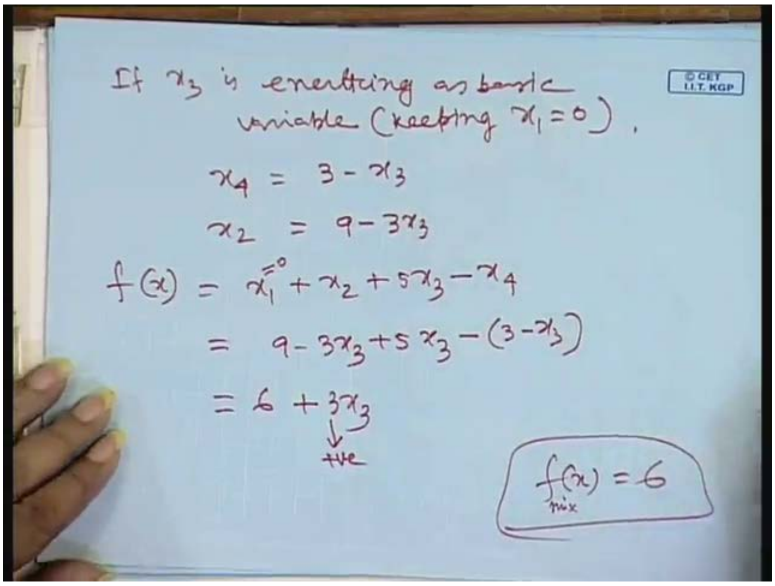

If x 3 is entering as basic variable keeping x 1 is equal to 0, than what will get it x 4 from this equation x 4 is equal to 3 minus x 3 and x 2 is equal to 9 minus 3 x 3. And what is our corresponding objective is x 1 plus x 2 plus 5 x 3 minus 5 x 4 this is now entering as a basic variable is not equal to 0 and x 1 value is now 0 because x 1 is keeping that value x 2 is 9 minus 3 x 3 plus 5 x 3 minus 3 minus x 2 value I am writing x 3. So, if you simplify 6 that 5 this is like a 6 minus 3 the plus x 3. So, this coefficient is positive reduction co efficient is positive, so exact value from non basic variable value 0 to some positive value you go it the functional value is increasing.

So, what is your conclusion we cannot change, whatever the non basic variables are there, agreed? And whatever the basic variables are there, these are the previous equation these are the our actual solution of the optimal point and it will give the minimum value of the function. For this problem we are getting minimum value of the function is f minim is 6 ((Refer Time: 54:46)). So, next class we will show the how use the matrix, the matrix form all these things, so will stop it here today.