Optimal Control

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Lecture - 14
Matrix form of the simplex Method

Last class we have discussed what is convex optimization problem. In convex optimization problem, that objective function is a convex function, and your in ability constant is a convex function. Our equity constantly is a fine function then have discussed that what do we mean by the quadratic convex optimization problem in that situation that our objective function is a quadratic convex function. Then our inequality and equality constants are a fine function means linear function in it convex set. Then we have discussed that what is what it is quadratic constraint quadratic opposition problems. In that situation objective function is a quadratic convex function. The equality constant also quadratic convex function and equality constant is a fine function then problem is how to solve these problems.

Then one way of solving is that you just apply the KKT necessary condition. Then after that one can solve it by different techniques one of the technique I am discussing here is the linear programming technique. So, in linear programming problems that if you see this one, that our a problem is linear programming problem, when the our objective function is linear and as well as our all constants. The equality constant and inequality constants are also linear in order to solve this problem by simplex method. You to convert for first simplest method means innovative method innovative methods, you to first convert the that linear programming into standard L P problem.

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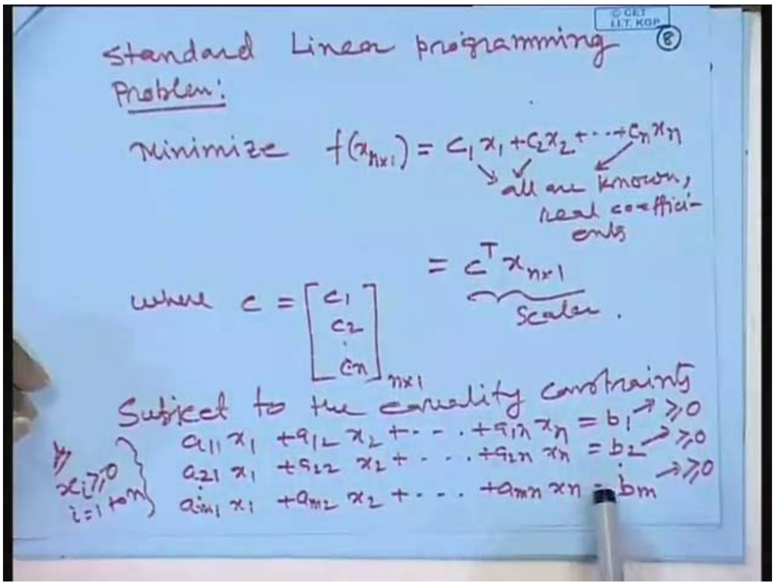

What is standard L P problem? Is there minimize the function, which is a linear function, which is written c transpose x subject to a all are equality constants? So, if you have a in the problem actual problem, if we have a inequality constants are there one can convert into one has to convert into equality constants. If you put into this form equality constant and the right hand side is the constant. That constants are greater than equal to 0 means non-negative number and our all design variables, x 1, x 2 dot x n are also greater than equal to 0 the non negative number.

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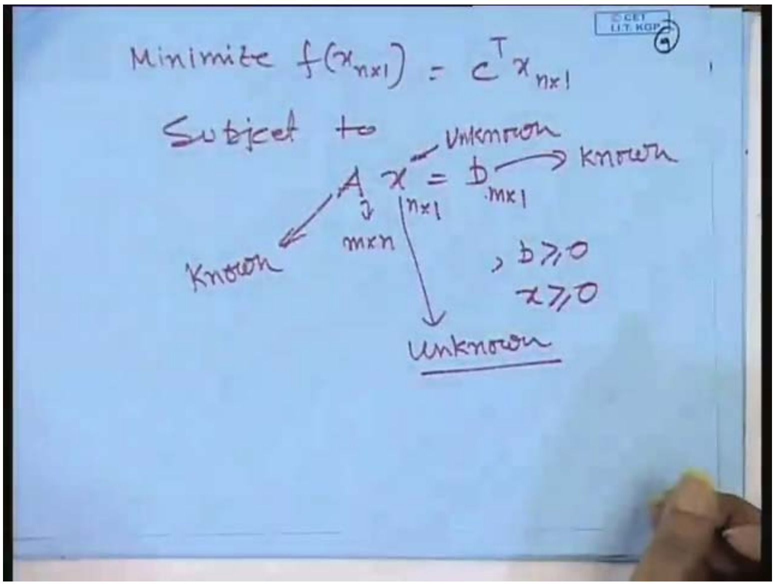

So, this is our any or linear operation problems, if you have to do it first you to convert it into a standard L P problems. What is standard L P problem, we just mention it here. Now, see how to convert the standard optimization problem a linear programming problem to convert first interest under optimization problems.

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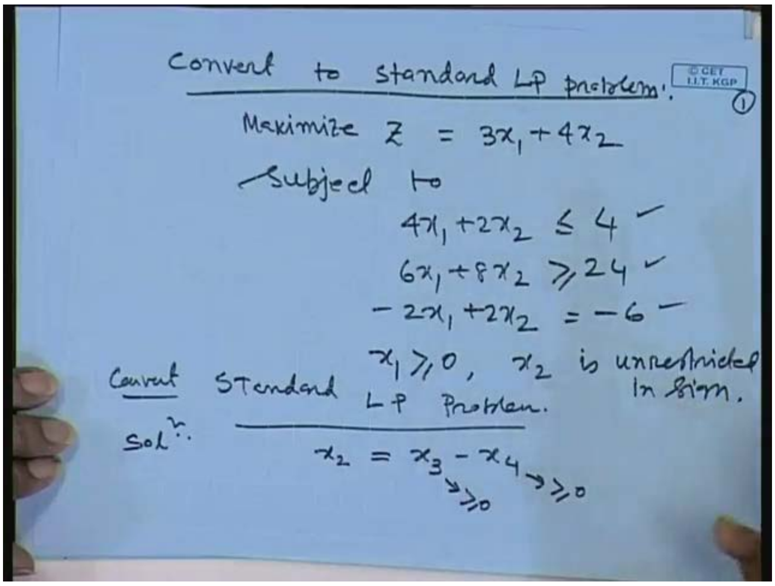

So, let us take some example and mention convert to standard l p problem. So, our problem is let us call maximize Z is equal to 3 x 1 plus 4 x 2 an subject to 4 x 1 plus 2 x 2 is less than or equal to 4 and 6 x 1 plus 8 x 2 is greater than equal to 24. We have eight minus 2 x 1 plus 2 x 2 is equal to minus 6. We have given x 1 is greater than zero and x 2 is unrestricted in sign unrestricted in sign, this is the situation. That means that x 2 value can be positive negative and zero, whereas at x 1 is greater than equal to 0. So, we have a different types of constants of their, that is less than equal to type greater than equal type equal to type is there.

So, first is if want to solve this problem by simplex method means innovative method. First you have to convert into a standard L P problems. Let us see how to convert the standard L P problems standard. So, firstly this is the unrestricted in sign, so the x 2 because all variables here must be greater than equal to 0 for standard L P problem. All inequality constants we have to convert into a equality constants. The right inside of the equality constant must be positive. So, this step we have to do it here. So, let us see I can always write that our we can just solution not the solution of the problem, how to convert into standard L P problems. So, our x 2 I can always write it x 3 minus x 4 there is no other variable other then x 1 and x 2.

So, I written x 2 is x 3 minus x 4, where x 3 is greater than equal to 4 x 4 is greater than equal 0. So, which turn the value of x 2 depending upon the value of x 3 and x 4. It can be a positive it can be negative, it can be zero. So, we introduce in place of x 2 in this problem by x 3 and x 4 that makes the minus x 4. So, this part is now becoming x 3 is greater than zero x 4 is greater than equal to 0. So, x 2 now will replace by two new additional variables. Next is to convert these equation into a equality constant, the right hand side of the of equality constant must be positive. So, how to convert that one? So, let us take the first equation.

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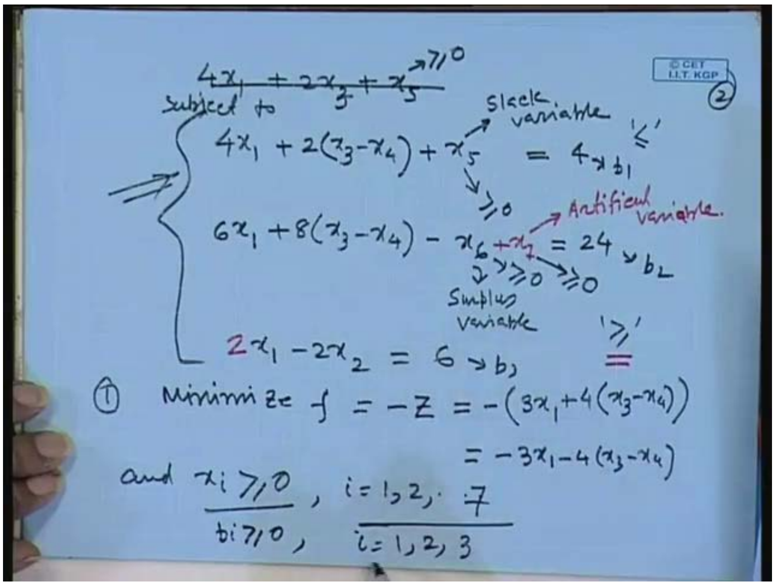

4 x 1 plus 2 x 2 if you see this one it is less than equal to 4, that made this a positive quantity again is the positive quantity. That means left hand side left hand side of this expression is less than 4. It indicates that if you add some positive quantity with the lead me in the in this expression of left hand side. Than I can make it this inequality constant as a equality constants. So, I can write it these plus a new available here we have x 3 x 4 is there. So, I have to write it that x 5 some positive quantity. So, this is our main values greater than equal to 0. I told you whenever you will get it x 2 that will replace by x 3 minus x 4. So, I can write it these, now if you write it x 3 minus I just write this one once again, because there are two x 2.

So, I will replace by x 2 buy x 3 minus x 4 and that is less than equal to 4, that means right inside is a positive quantity and left hand side is less than the right hand side. So, I have to add some positive quantity in the left hand side expressions. So, this is a x 5 x 3 is there x 4 the next is where ever the x 5. So, that variables are all called slack variables and that variables is greater than equal to 0. So, this equal to you 4, so first equation, which is given inequality we have converted into a equality constants equality constant. As well as right hand side of the section is a positive quantity. So, next equation if you see this is 6 x 1 plus you see 8 x 2.

So, x 2 will be replaced by x 3 and x 4 and it is greater than equal to 24 greater than equal to 4. That means it indicates that this quantity right hand side quantity is I have to subtract in the left hand side expression something. So, that it will become equal to 24. So, it is eight into to eight into x 3 minus x 4 I have to subtract some positive quantity in from this expression minus up to your gone up to a x 5. So, I will put it x 6. Again, I will write this is equal to 24, but there is some problem is here. If you write in this let us call we have a this equation is there. Two equations are there for the time can be highly asked you to solve this one for the time being. If you see there are six unknown variables and to there are two equations are there, how you solve it infinite number of solutions of there.

Suppose, if you consider done x 2 x 3 0 x 1 x 3 x 4 is 0. Then x 5 is equal to 4, but x 6 is equal to minus 24, but x 6 value is positive. So, when you have a such type of inequality constant is not enough just introducing, a minus in variable. So, that there will call surplus variable surplus variable there is not and have just the, if you add a surplus variable. Because, why has service that this quantity is greater than that this quantity is greater than 24. Something you to subtract means in surplus from this video. So, that is why it is called surplus variable. Just now I mention you if you ask to solve only these two equation. It is equally to equation really than I can say there six variable to equations of there.

So, we have a infinity number solution if you just consider that at x 1 at x 1 x 3 x 4 is 0. Then one case is x 5 is equal to 4 it is satisfied, because that x 5 our positive quantity. I just said positive quantity satisfied and x 6 also our greater than equal to 0 our positive quantity. Because, I am subtracting some positive quantity to get 24 when made it x 1 is 0 x 3 is 0 x 4 is 0, that x 6 value is coming minus 24 just contradicts. So, this is we need to add what is called another variable that variable after x 6 it is at 7. So, this variable is called artificial variable.

So, when you have a such type of equation some expression greater than equal to some quantity, this one to get. It then you have a two variables you can introduce one is surplus available another is artificial variable. So, this is the second now third equation you see we have a equality with the equality means this equation is there equality constants of their, but on the things, we have a right hand side is a minus. We have to make it plus. So, both sides you multiplied by minus, if you multiplied by both side minus it is a twice x 1 minus twice x 2 is equal to 6. Our artificially variable I forgot to mention in our artificial variable also nonnegative number greater than equal to 0 artificial variable remain.

So, you see this all this equality constants all these things we have converted, but what is this standard problem? It is given convert into stander, it is given mix maximization problem. So, our problem will be, now if you see this one problem is minimize F, F is nothing but I told maximization problems. If I know how to minimize a function I can use same technique here. So, it is nothing but a, what the function we have supposed to maximize disagreed with minus that function is minus Z. You know that is 3 x 1 plus 4 x 2 while x 2 I replace by x 3 and x 4. So, it is a minus I am writing at x 1, I mean the z value 3 x 1 plus 4 and x 2 I will replaced by x 3 minus x 4. So, in altogether it will be a minus 3 x 1 at minus 3 x 1, then your minus 4 x 3 and x 4.

So, you a repeat once again, so our that this problem we first, we have convert into standard L P problem maximization minimization of functions, same as minimization of this function, same as that minus z of these. You minimize subject to this constant, which is converted into a equality constant. We have introduced when there is if this type of symbols are there equality constants i. We need only ones slack variables, which is greater than equal to 0. When you a this type of a equality constant rather we need two variables. One is surplus variable another is artificial variable. Both are greater than equal to 0 and equality side right hand side unit is not a negative way positive quantity. We have to make by some means to make a positive.

So, problem is minimize subject to this constant and all right hand side constants are becoming greater than equal to 0. Now, same also all variables artificial variable than your surplus variable slack variable, our design variable all are also greater than equal to 0. So, the minimize this one subject to subject of this. You can write it and what is this x i is greater than equal to 0, where i is equal to 1, 2 dot in our case come up to 7. So, I am writing first is here minimize the subject to constants.

These constants which is converted into equality constants, with x i is greater than equal to 0 i is one to end. All these are right hand side that is b i is greater than equal to 0. Our i is how many equations are there? 1 2 and 3 i is equal to I can write this i is equal to 1, 2, 3. Suppose, if you consider this b 1 this is b 2 and this is b 3 than this I can write it. So, at this moment we know how to convert a every problem in the standard L P problem linear programming problem, how to convert into standard L P problem. Suppose, you may have a some constants x 1 see x on is greater than equal to 5 or less than equal to 45 greater than equal to 3.

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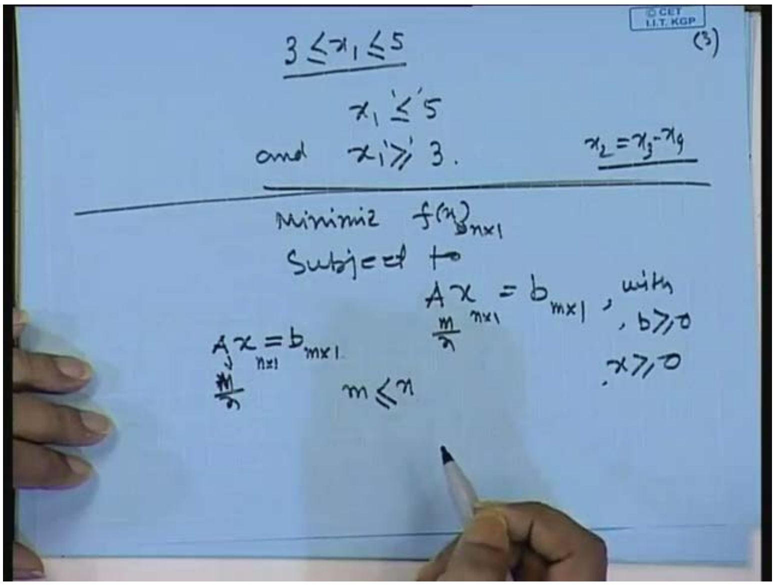

So, this I can easily convert into two equality constants. I can write it here one is less than equal to 5 and x 1 greater than equal to 3 this. You will know how to convert this into equality constant. Similar way you can write it, because this is a what is called this type of things, you can it is enough to introduce only one slack variables. So, here is this one greater than equal to x 1 some expression greater than equal to some constant quantity. So, that to introduce two variables, so that means so one can convert linear general linear programming to standard L P problems. So, and in our case if x 2 is I am restricted sign than x 2 is if you 2 is related to x 3 and x 4.

So, let us see this one next that, now basically if you see this one minimize f of x subject to a into x is equal to b this is n cross. If the dimension of the variable of this and if you have the m equal to equations are there. Then this is the basic equation with b is greater than equal to 0 and x is greater than 0 this vector greater than 0 means each element of this vector of dimension m cross n is greater than equal to 0. Similarly, x also each element of x greater than 0 means greater than equal to 0 means greater than equal to 0. Now, our problem is you see we are looking for solution of this eligible equation linear equation. We have to find out what is the value of x.

So, that these constants are satisfied this. This are satisfied L P form it will has to satisfy, but this constant also for any value of x, x value is not negative number solve this one. Simultaneously this value of x should meet the minimum value of the function. So, that is our job for this one, so it is nothing but a solution a set of equations. So, if you recollect your basic linear eligible, what is called solution of a set of equation. It is A x this a m this is m cross 1 this is a m this is a m cross n. Then we are assuming number of equation this m indicates the number of equation is less than equal to n. So, we get different solution of this one. One can write it this way, we form you see our this is solution we want to do it.

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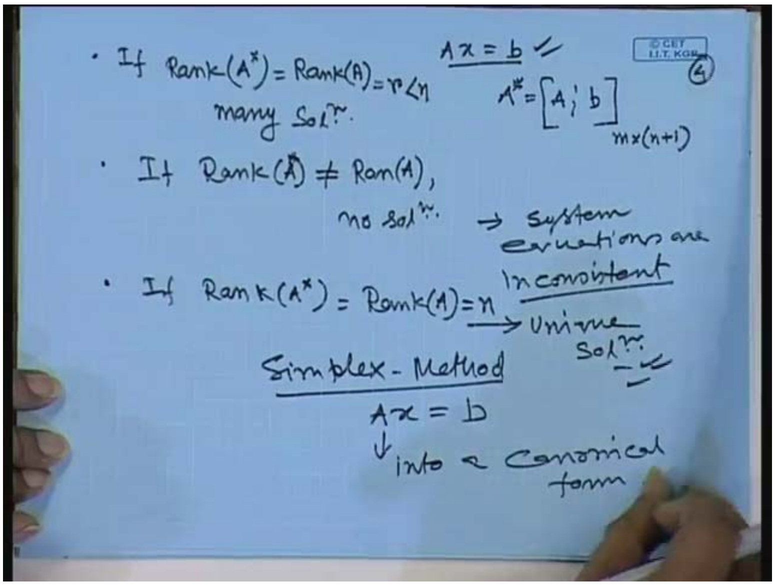

We form a metrics are the metrics system metrics and right hand side metrics. So, this dimension will be m cross n plus one this A. So, if you just recollect our basic results of solution of eligible equation, that is rank of a star is equal to rank A is equal to r both the ranks are this is the argument metrics is r than, which r is less than n. Then our conclusion is that we have many solutions multiple solutions true. If the rank a is not equal to rank a no solution exists. In other words you can say the system equations are in consistence. In other words the system equations are in consistent.

That is some equation is equal to let us hope b 1 and same equation is equal to b 2 same constants between the two equations. The third one if the rank of this a star if rank of a star is equal to rank of A. Than we have a unique solution that means in others words rank of this equal to this equal to n. This n is the number of columns of metrics. So, this will be when we when it is asymmetric metric, sorry not symmetric, that is square metrics. So, you have a square metrics and the rank is full of the square metrics, than you have a unique solution.

So, this miscellaneous solving that type of equation, so in order to solve this one if you recollect in a undergraduate class, that we are solving a set of algebraic linear equation by some numerical techniques. That means gross elimination method gross Jordan method always thins gross civil method risk. This type of seen. It is a gross elimination method is nothing but a row operations were are doing to eliminate some of the variables from the some equations.

So, you also will do the same thing here remain. So, first in simplex method simplex method is nothing but a simplex method to solve a set of algebraic equation simplex method solves a set of equation of algebraic equations and a by using the numerical techniques or the is called the iterative process. So, whatever to do first that you convert thus sustain equation system eligible equation into a canonical form. What is canonical form, that I will just discuss here. Suppose, we have is, suppose we have a system a x is equal to m cross n.

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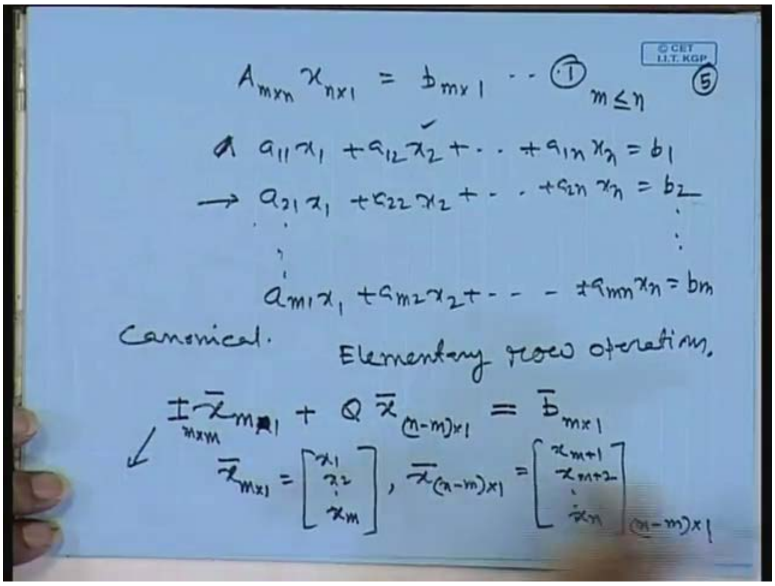

This is n cross 1 is equal to b m cross 1 and it is a equation number one and will m is number of equation is much less than the number of design variables as n. So, what I will do it, we have a n equation is there in first equation. We can consider that variable x 1 is their only act not in first equation variable x 1 x 2 x 3. Maybe, x n that have just writing this one. This is the standard equation is there a 1 1 x plus a 1 2 x 2 plus dot a 1 and x n is equal to b 1 then a to 1 x 1 plus a to 2 x 2 extra plus dot dot a to n, x n is equal to be two. If you continue we have m set of equation is there want m one x 1 plus A m two x 2 plus dot plus A m n x n is equal to b m.

So, you say in all equation in general x 1 x 2 all these things are there. So, what will do it in canonical form representation canonical representation of sets is a set of equation, will keep x 1 only in equation number one again and x 2. Only in equation number only in equation number one and will keep x 2 only in equation number two. No other equation extra will be there. So, in this since we have A m equations of there. We can make it m variables x 1 and two up to x, x m this variables are present only in one equation. Only x 1 will be involved in one equation and no other extra will be involved in this equation. Let us call two equation do not involve another equation. Similarly, x m involved in last equation and other equation action would not be there.

So, this I can do by elementary row operations, how to do it? I multiplied by this first history a 1 divided by a. You divide this equation by a 1 1 again then the multiplied by equation by 8 to 1 subtract from equation two. So, x 2 is go out similarly multiplied by first equation by a three, one subtract from third equation, that a f x 1 will removed from this one. So, in this way similarly, x 2 will be only in this equation, so x 1 is not there. So, you divide by this one what are the coefficient will divide by that one to convert into a only coefficient of this one coefficient associated with x 1 x 2 unto x n is one alteration.

So, you eliminate x 2 from equation number two three dot and so on, you continue up to variable x n. So, after doing this elementary after doing elementary row operations one can write this above equation like this way, x m the dimension of x is equal to m cross this dimension. I am writing this is x m I am writing the dimension is m components are there x 1 x dot m complain. So, I am writing even writing this one it indicates m complaint m cross one again. Then this remaining variables remaining variables will be your q is a metrics, that variable should be n minus m into 1. So, this is or you can just this equal to b bar and that is dimension the m cross 1. So, you say the first equation or you can say it is nothing but i multiplied by these i dimension is m cross m.

This you can write it also, mind it our this or you can use some other symbols of right our x m is our variables are x 1 x 2 dot x m. If you just put it call if you to put it just a differentiate x bar m. Then it is cross one is then these x bar were then these can you know x bar is what this one x bar n minus m cross 1. What is this 1 x of m plus 1 x of m plus 2 dot x n whose dimension is your n minus m into 1. So, this I can write it into this form.

So, this representation is call canonical form representation, once you get it this canonical form representation, you say we have a n variables are there. Whereas, wherever we have a n variables are there, but we have only m equations are there. So, you can get is infinte number of solutions. Let us see how we are telling the infinte number of solution of this. At the moment I can say, if you solve this equation one or equation, what or it frequently equation two.

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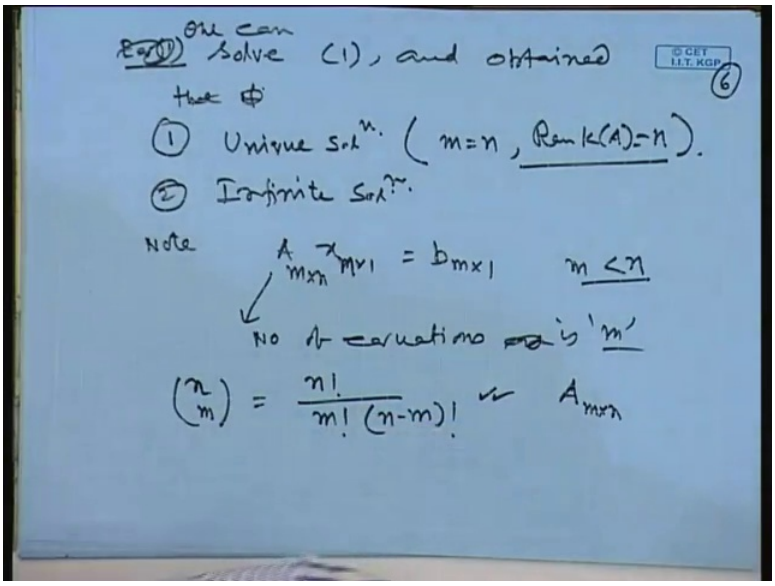

If you solve it again you have a either you solve equation one or two, which is a equation two is canonical form solve it. One can solve equation one and obtain the following solution and can obtain the following solution following solution. What is this solution one is I may get the unique solution. When I get it unique solution, when any linear equations, that when the ordered of this order or use a rank of the metrics is n and fuller. Then you will get unique solutions unique solution unique solution will get it, when m is equal to n rank of a is equal to n in two.

You have a in finite solution many solution in finite solution. I told you when you get this infinite solution of this one when rank of a star is equal to rank of a, a is equal to r, which is less than small n again. That is this conditions, if you see that condition many solution than you have a no solution. Then you have a unbounded solution.

There are four types of solution exists to the typo systems. Now, note that that our equation can just look at this expression A x m cross sorry n cross 1. This is m cross n is equal to be m cross 1, we are considering m is less than n particle to not equal to by the way. Then we have seen this one that number of equation number of equation, in this system is number of equations are is m a number of variables is n. Now, you can use in this one if these metrics is full rank, this is metrics a full rank. These than we can make it, what is called that n of m number of linearly independent combination. We can make it that means, this is an factorial n divided by factual m n minus m.

So, how many columns of there in a metrics, whose dimension is n, n columns are there out of m columns. There is m columns are linearly independent. If it is full rank of metrics n, then we have a n columns are there small n common columns are there out of the small n. I am picking up m linearly independent practice. So, we can get it such typo number of linearly independent sets of that one. So, it will now recollect that one, you see this one our canonical form. If you see this is our canonical form representation or canonical form representation of, I write it here

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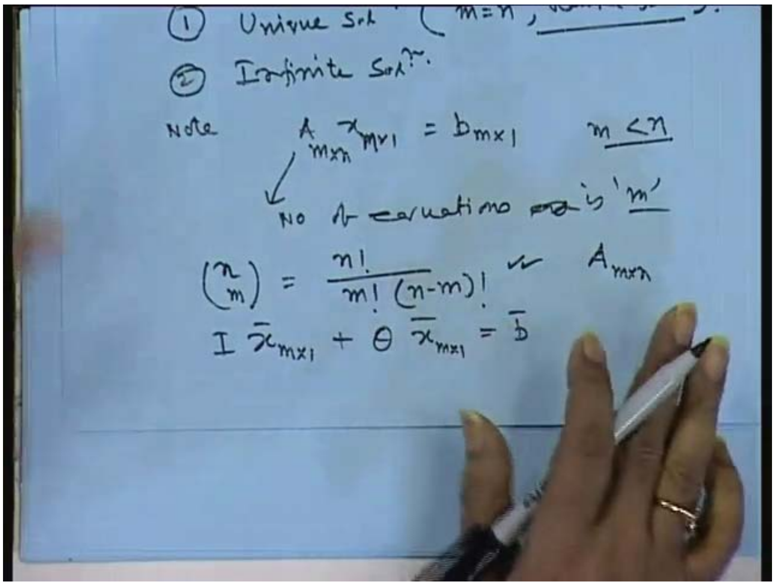

That is I into x bar of m cross 1 is q x bar of m cross 1 is equal to b bar, this earlier seen how to do it. So, from now know on words I am telling, that as if I have a system itself is a canonical form m. This is the m that is A m variables of their. This is why this dimension is n minus m cross 1 dimension.

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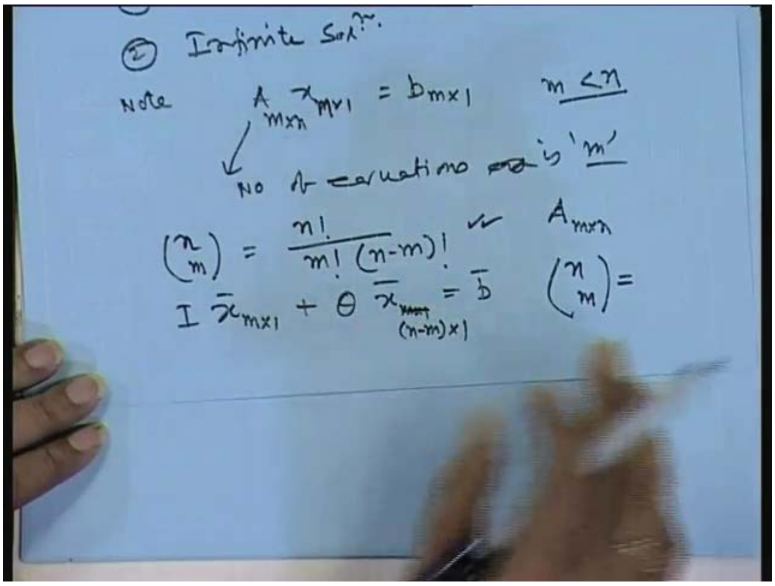

Now, you see if you assign this n minus m variable assign is 0. Then is values is nothing but A b. So, how many because you have n variables I can select n minus m variables in what combination ways that we have two variables of m n out of these. I am taking is m variables, which is not equal to 0 and remaining is 0. So, many solution you can get it I consider this is equal to 0. So, I will get it one solution I can consider another combination one variable from here it will come here and one variable is coming here.

So, then there will be another solution. So, if we assign that our now our variables can split up into two parts. One is basic variable another is the non basic variables. How many non basic variables of their there are n minus m variables are there. How many basic variables are there is a m basic variables are there. When will assign the non basic variability is zero the solution, what will get it is called the basic variable solution. So, if you see this one that the total number you can, let that the m be the rank of A, which is a full rank.

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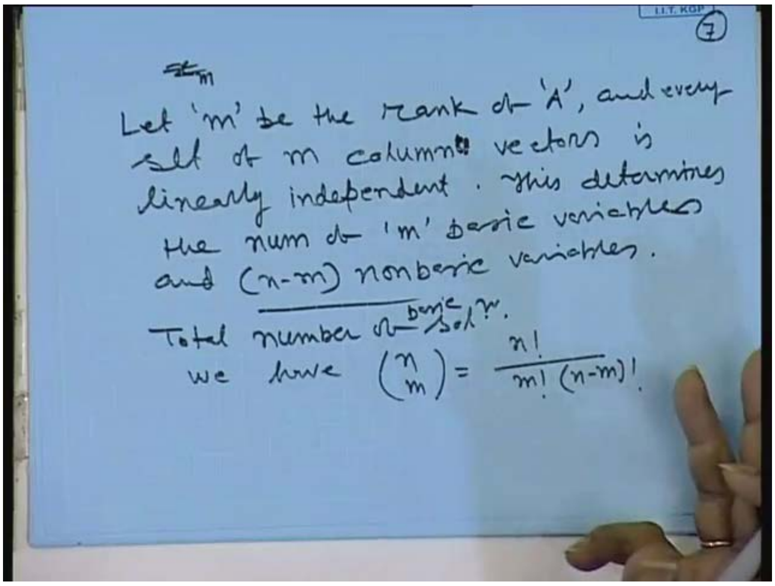

Every set of m columns m column vectors is linearly independent linearly independent. So, this tells us how many combinations of linearly independent vectors will get it. This determines the number of m basic variables we variables and n minus m non-basic variables. So, that means this if we assign the n minus 1 variable value. Something we will we can get the remaining m variables by solving m equations. Because, I told other and variables of there a m equations, that to you to assign n minus m variables value. You can which in turn you will get it what is call m equations.

Now, that n equation m variables are there what is m variable x 1 x 2 x n x m or any variables may be form x 1 to x 1, which is m reacts to m variables of x will be unknown. That can be obtained by solving the set of equations. So, when you assign thus this non basic variable value is 0, then solution is what is called basic variable solutions. So, total number of solution we have total number of solution, we have put the number of basic solution. We have n m is equal to factorial m n factorial m minus n.

So, let us see and this solution is this solution, m equations are there, n variables are there. Basically, we have as this number of solution under out of this solution, some maybe feasible solution, someone not be feasible solution according to our optimization problems. So, let us take one example would how to find out the solution of this one.

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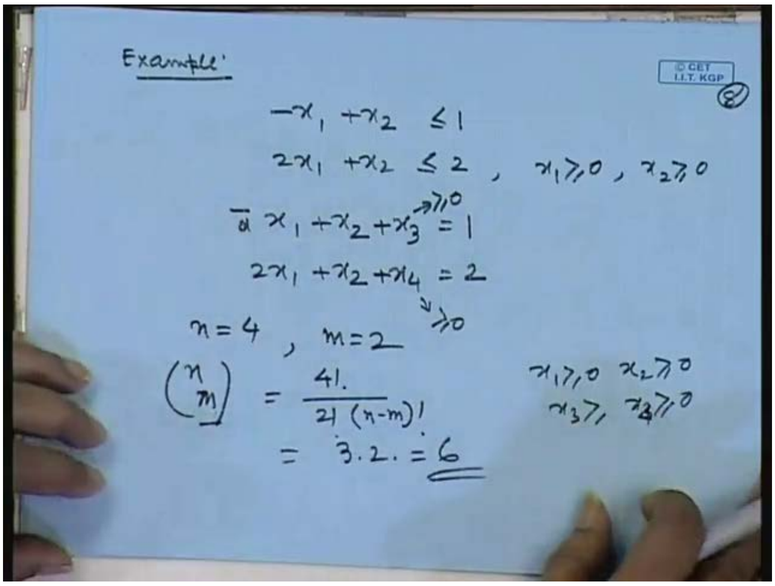

Minus x 1 plus x 2 is less than equal to 0 twice x 1 plus 2 is less than equal 2 x 1 is greater than equal to 2 x 2 is greater than equal to 0. So, let us go to this equation and this two equation I can convert it into a, what is call equality equation. That means minus x 1 minus x 1 plus x 2 and this quantity is less than one. That means I have to add some new variables. Let us say x 3 is equal to 1 and this is 2 x 1 plus x 2 this less than this one. When I to add some other variable let us say x 4 is equal 2. We have a given x 1 value x 2 value greater than 0.

Now, you solve it this equation and there is no restriction I put it in x 3 is of course, this x 3 is greater than 0, x 4 is greater than equal to 0. Because, this some positive quantity I am adding some positive quantity in turn, that all variables are greater than equal to 0. If it is violates this condition, you will get some solution woodwork, which violates this condition. Any one of these than it is not feasible solution and it is not as satisfying these constants, so let us call how to solve this one.

So, you have a four unknowns two equations of there. So, I can easily assign two equation values of that two variables value, what is call equal to 0. Let us assign x 1 is 0 x 2 is 0. So, can get it here what is call x 3 is 3 x 4 is 2 and this is one of the solution. Another value solution is what the x 1 is 0, I consider x 2 0 and x 3 0 all this things you consider. So, if we just consider our this is one choice of we got it zero benefit choices. If you do the choice like let us call x 1 is 0. Then you will get it this way and is even have

A x 1, if I put it 0 than our three unknowns are there anyone of this. You can put it 0 let us call your out it that x 3 is 0. Then you will get another solution.

So, in turn if you take the four variables of their two equations are there you assign two variables. Randomly anything any combination you assigned zero. Other variables will can get it. So, in our case, now n is equal to four number of variables are 4 n number of equation is two. So, these are is this what is the solution how many solutions are there you will get it for this one first is how many solution factorial 4, 2. This is n minus m factorial 2.

So, in total I will get this four that means is 3.2 is a six solution, but it is not necessary. That six solutions will give you the will satisfy this equation. Satisfy the constant also, but we will get the six solutions it. Try it here you will get six solution I can tell you what is a six solution. I took let us say x 1 is 0 x 2 is 0, than one solution than I take x 1 0 x 3 0. Then we are well x 2 and x 4 solve it you get another solution. If you have a x 3 x 2 is 0 x 3 is 0 and A x 1 and x 4 another solution. In this way that means any two variables, you assigned zero. Remain two variables you find out and in this way you can get number of solution n m, c m two variables. At the time you are as what is called if it is m variables at the time you are grouping to get the solution of this one form out of n variables. So, in this way they proceed is solution, I just cannot just mention it the solution of these to find all basic solution.

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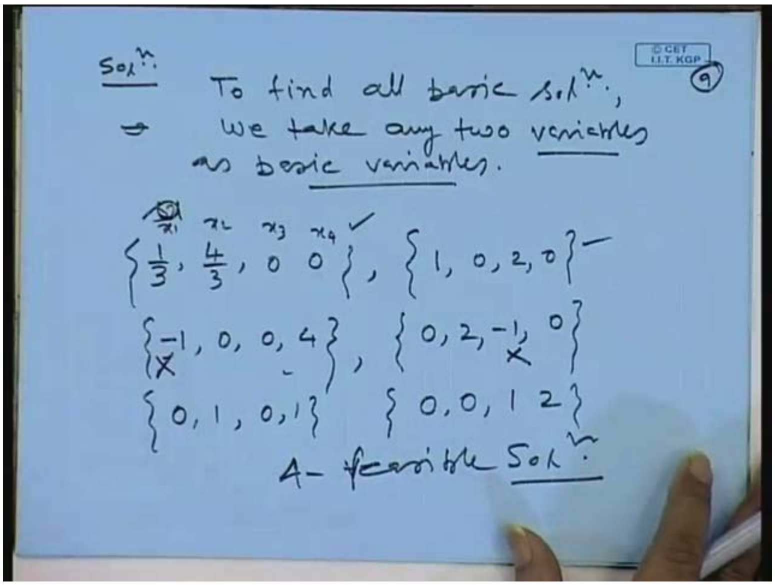

Please, remember the basic solution I will call of that equation once you convert into a what is call canonical form canonical form. Because, we have a in this case we have a two equations at their. So, if I convert let us call x 1 in presenting equation number one. It should not be present in equation number two. Similarly, x 2 present in equation number one. It should not be present in equation number two. So, our x 1 and x 2 are canonical form. You can consider these variables also variables only x 3 present only in equation number. This x 4 is visibility equation number four only it equation is number two, this also canonical initially. So, in this way I can make it six combination of this one.

Now, total to find the basic solution we take any two variables as basic variables any two variables with a basic variables, means that to variables in this to equation, that to variable present only in one equation. If it is a x 1 you taken as the basic variable x 1 will present in first equation, x 2 is you take another basic variable present in second equation. If you take x 2 and x 3 than x 2 present in first equation and x 3 present in only second equation. In this way we have a two variables at a time I am taking the basic variables. So, you will get a solution of find the basic solution we take two we get the basic variable set.

So, we have will have a solution one third 4, 3, 0, 0. This is x 1 you can say x 2 this is x 3 and x 4. This is one solution will can get it please you can check it also another solution 0 1 0 2 0. Another solution am getting is minus 1 0 0 4, another solution I am getting it 0 2 minus 1 0. Another, solution in 0 1 0 1 and another solution is 0 0 1 2. So, you say we have a four variables two variables and at a time two variables, at the time we assigned zero and remained two variables. We can find out by solving the two equations.

So, we got it this and out of the six basic variables as a six solution. You see that there are four feasible solutions is there. This is not valid, this is cross this we cannot accept, because our x 1 x 2 dot x 3 x 4 all are greater than 0, but it is showing x 1 negative. Similarly, this is not acceptable this is x 3 is negative, this is also not acceptable. So, we have a our basic solution we have a 6 out of 6. I told you in the beginning that when you are discussing that we have a basic solution. It does not mean all the basic solution will give you the feasible solution feasible solutions, means that the solution must satisfy our what is called equality constants.

Also, our that the constant that x i is greater than zero. It may not satisfy also and if it is a feasible solution than satisfied all the constants. The all site what is call constants also, out of this six basic variables. We got four is the feasible solution four, we have a feasible solution. So, what is the what is the basic variables? Basic variables is thus set of variables we have a set of variables out of this. Thus, if you get your way is we have a what is call that set of variables is there in the basic solution in the basic solution. The corresponding variable, which is non-zero the corresponding variable, which is non-zero they are called is basic variables.

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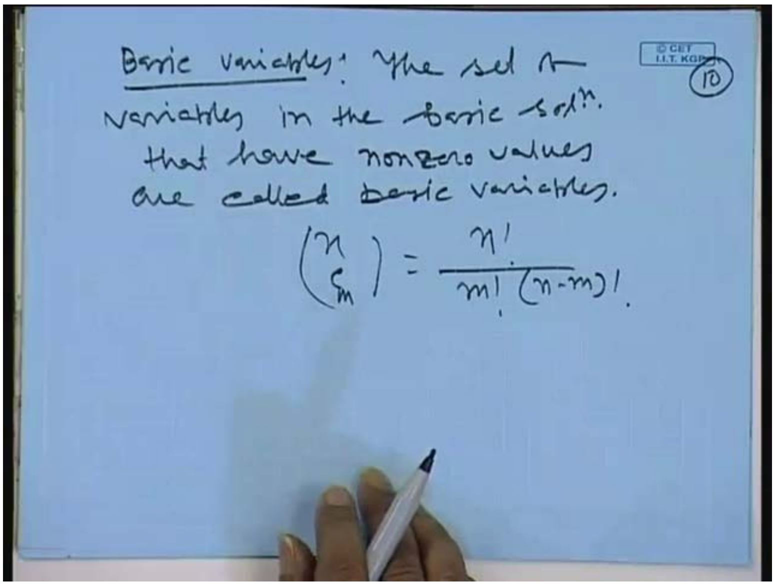

So, the set of variables in the basic solution that have non zero values, are called basic variables. So, the variables whose values are all zeros, then we will call that the variable that variables are called non-basic variables. So, I told you that if you have A m variables are there m equations are there we have A m basic variables. We assigned the non-basic variables value non-basic variables value is 0 and immediately will get the what is the basic variables solution. We have at that type of combination we have A n c m that is what we have seen factorial n factorial m minus m factorial.

So, what is the basic physical solution? We have a basic solution out of these who solution are feasible, then it is call basic feasible solution again. Next is optimal solution, you got a value what is a feasible solution out of this feasible solution who solution will give the optimum value of the function. It is called the feasible optimum feasible solution again and non-degenerative solution basic non-degenerative, basic variables solution non-degenerative basic variables solution. What is that mean? You got the feasible solution of this one again if the basic variable the basic variable values are all non-zero then it is call non-degenerative basic variable solution. So, next class will just discuss how to solve the element of what is called the linear metrics equation, in the metrics form. Then we will solving in tabular form.