Electrical Engineering Optimal Control
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Lecture - 13
Quadratic Optimization Problem Using Linear Programming

So, last class we have discussed about the convex set and convex function. And we have discussed the convex function, a set is said to be convex function, if any line segments formed between any two points. And that it belongs to that line segment belongs to that set, then it will be a called a convex set, then discuss the convex function.

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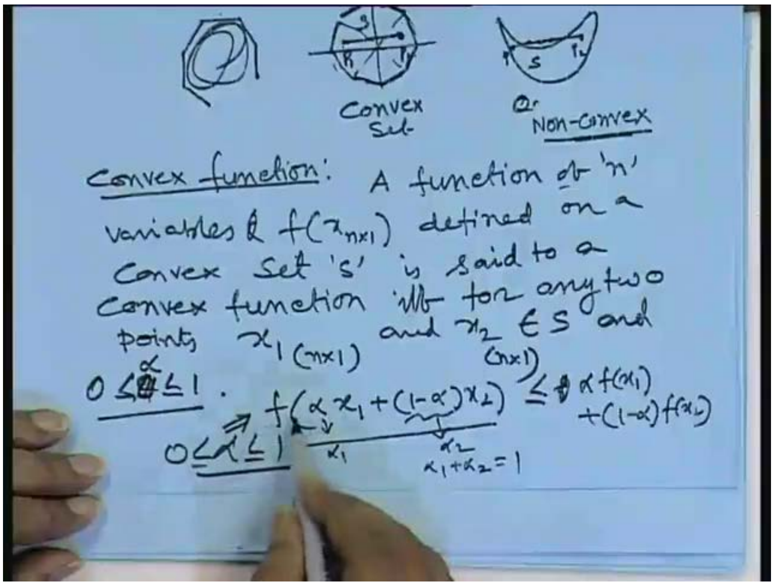

If a function is said to be convex function, that of n dimensional set in a convex set S is said to be convex function, if and only if this condition is satisfied. That f of alpha x 1, 1 minus alpha x 2 the function value is always less than equal to alpha into x 1 plus 1 minus alpha into x 2.

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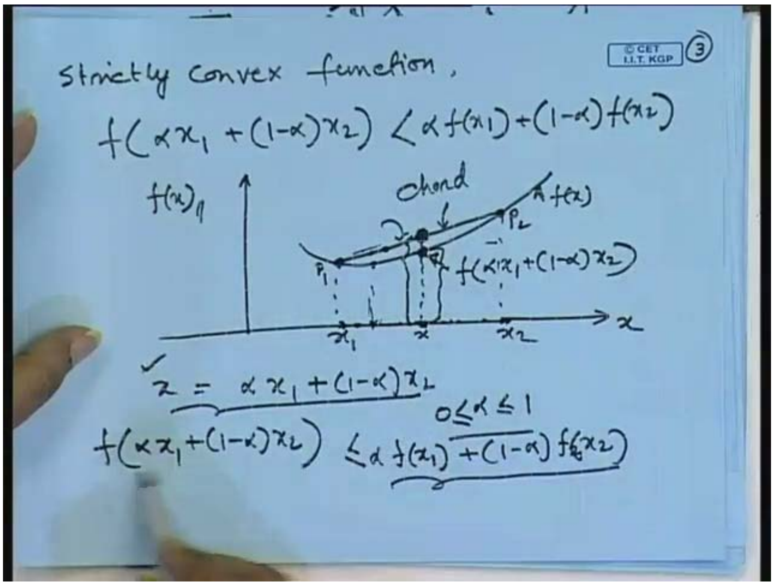

And this alpha varies from 1 to 0 to 1 any values of this one, this physical interpretation of this one, this function, if you see this one it is nothing but a, that one, this let us call f of x, x is a two variables x 1, x 2, then this x 1 and x 2 belongs to a convex set and. Then we can write it that any value between x and x 1 and x 2, I can write it alpha into x 1 plus 1 minus alpha into x 2 alpha is any value from 0 to 1. Then this function value at any point on this interval that x 1 to x 2 any value, this function value is always less than equal to the cord form from point 1 to point p 1, p 2 what is that cord is form. And that on the line, any point on this cord, this will be greater than function value. So, this is, if it is satisfied this condition it is called the convex function.

So, then we have discuss some of the properties of the convex function. So, a function is said to be convex function, if and only if that hessian matrix of this function is positive semi definite or positive definite, over the convex set that function, that whatever function is given. If the hessian matrix of this function is positive definite or positive semi definite over the convex set then it will be called the function is the convex function this is a test for convex function.

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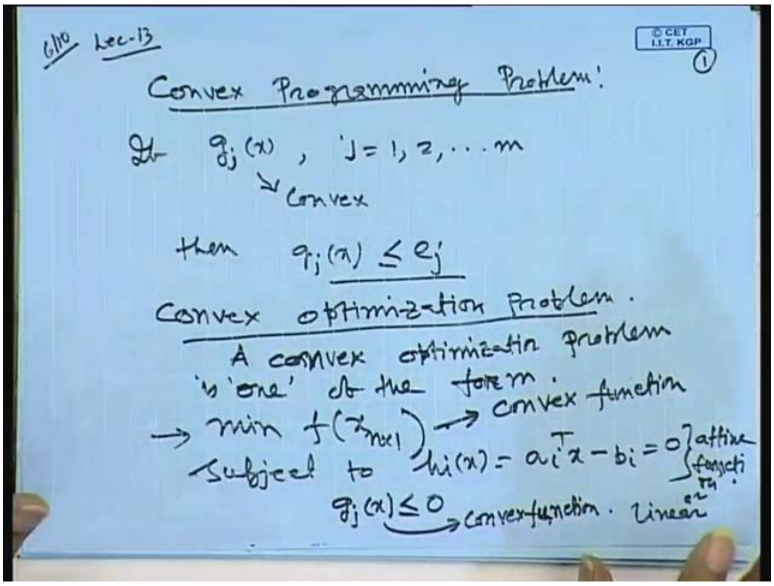

Then let us say what is the convex programming problem? Now, we will discuss convex programming problem. So, if you have a g j of x is a function which is defined, which is having a, this type of function we have a, let us call a m functions are there and each is convex, these are convex for j is equal to 1 to m. Then if this function are convex then g j of x is less than equal to e j, is also is a convex function, that intersection of this convex set is called individual, is defined by individual constant set or individual this. So, in other words you can say if g of x, g j of x is a convex function then g j acts less than equal to e j, also convex set, and their intersection is also a convex set, is also convex function g j.

So, next is your convex optimization problem, convex optimization problems. So, any optimization problem we know, we have a objective function as well as the constants. So, a convex function, a convex optimization problem is one, a convex optimization problem, a convex optimization problem is one which have the following form, is one of the form, is following form .What is it? The objective function is minimize f of x, x is that n variables, x is a n dimensional variables this equal to, this function will be a convex function and convex function, how you will test it? You find out the hessian matrix of this function, if it is a positive semi definite or positive definite then its function is convex function.

So, the function which you are supposed to minimize, that function must be convex and also subject to, subject to equality constant h i of x is equal to a i transpose x minus b i is equal to 0. So, this is our equality constant and that equation is affine function means, linear function is called, affine function means or it and linear in x linear function in x. So, if function is said to be convex optimizational function then objective function or cost function must be, cost function must be convex function, equality constant is affine function means, linear function. And equality, inequality constraints x j of x is less than equal to 0, that function must be also convex function.

Then we will call it is a convex optimization problems that means, in short we can say the objective function or cost function. If it is a convex function and inequality constant also a con, convex function and equality constant is affine function then minimization problem will call a convex optimization problem, is your that our j, j how many functions are there?

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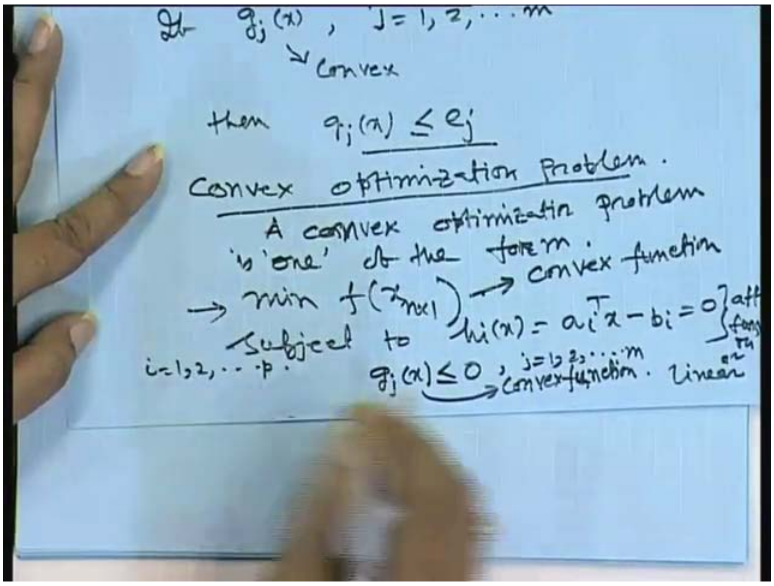

In general we have also mentioned earlier that, this we have a, that equal to m and the, i varies, that how many equality constants are there? i varies you can write it that, i varies from 1, 2 dot dot p.

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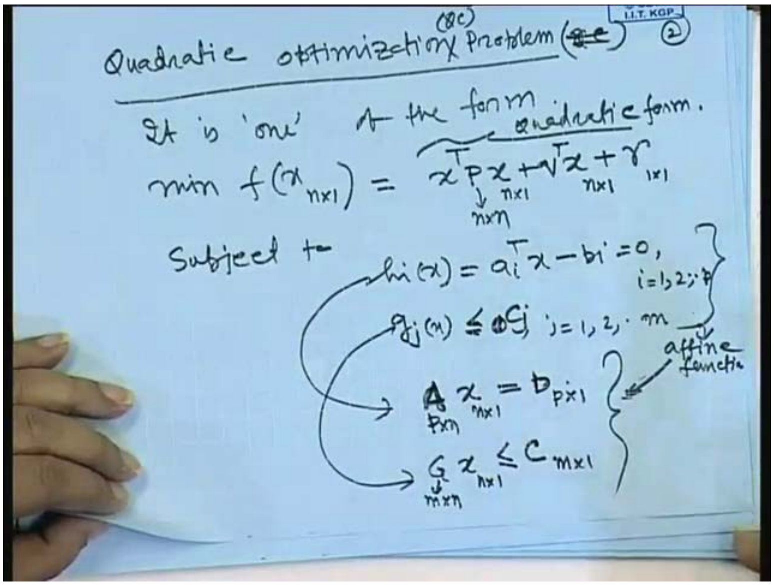

So, next is your quadratic optimization problem. So, in shot it is a QC problem, QC problem, quadratic optimization problems. So, you write it here bracket QC problem. So, in QC problem only the, you see the cost function or the objective function, convex function. But it should be quadratic form, convex function must be in quadratic function, convex function may be in any form, but it is a special case that convex function must be in quadratic form. And equality and inequality constant are affine functions of x then we will call it is a quadratic optimization problems, quadratic optimization problem.

So, according to our definition that quadric optimization problem is one of the form, that minimize f of x, x is a n variables of their and that must be I told you convex function, that convex function is a special form, which is quadratic form. So, in quadratic form already we have discuss earlier x transpose p x plus j quadratic form is like this, you transpose x plus some constant r. And it since our dimensional of x n cross 1 immediately to know, what is the dimension of q bar because this objective function is a scalar one. And this dimension is 1 cross 1 and this is since it is n cross 1, p must be n cross 1.

So, this will be a, what is call a convex function, but it is inquadratic form. Now this, function minimize this and what is in quadratic form, this whole thing is quadratic form, general quadratic form. And this quadratic form function must be convex function, we have already defined this quadratic form function will be convex function, provided the hessian matrix of this function is a positive semi definite or positive definite matrix.

Then, if it a quadratic form and convex function then subject to, subject to we have equality constant x h i of x is equal to a i transpose of x minus b i, if I considered this is equal to 0, i is equal to 1, 2 dot dot p. And g j of x is equal to, you can write, if you can write into this form is g j of these is less than equal to 0, j equal to 1, 2 dot dot m, m equations are there. So, if you write it in matrix form with this one, I can write it this is nothing but a your this quantity, I can write it as if it is a g is a matrix form into x is equal to b. And b is your dimension, if you write it this is metrics form copying all this, i is equal to 1 to p, this dimension is p cross 1.

And since n is this one, g dimension you know that it will be, p cross n. Similarly, I can write it that one, g of that one I can, let us call since it is A, better you write it A, since it is A, write it A. So, this I can write it G of x, G of x k x is equal to less than equal to you some constant equal instead of less than 0, you can put it some constant, let us call it is a C i C j, some constant also can put it. So, this equal to C A and that dimension is your m cross 1, this is n cross 1, this will be, this dimension will be m cross n.

So, it is writing in a matrix vector from this. So, this both this function or this function equal with this, this function which is, equality function is we have a p equality constant, we have a what is called m equality constant, inequality constant, this all the function must be affine function. These should be affine function of x means, linear this is also affine these and these are equal and from. So, these constants are affine function of, what is x.

Now you see, if you take that C affine means linear functions, if you take the hessian matrix of this affine function, linear function that hessian metrics will be 0, because if you take the partial derivative of this function twice then it, function will be 0, this one. So, this is quadratic optimization problem and in quadratic optimization problem, we minimize the quarter, a convex function which is in quadratic in nature, over a convex set.

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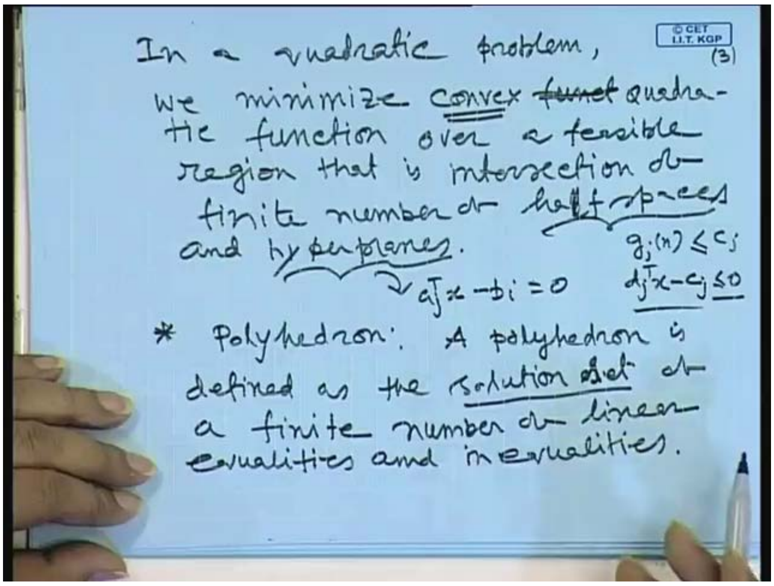

So, our practically in a quadratic, in a quadratic, that problem, we minimize a convex function, convex quadratic function, convex quadratic function over the, over a feasible region formed by, feasible region that is intersection of, intersection of finite number of, finite number of half of space, spaces and hyper planes. So, hyper plane is, is nothing but a, that constraints, that is equality constants of there, that is your h that is a i j x a i j into x minus b i is equal to 0, this is the hyper plane. Suppose, three-dimensional case x 1, x 2 so it you can write a one into x 1 plus 2 into x 2 to plus what is call a 3 into x 3 is equal to let us call b 1.

So, it is a hyper three-dimensional hyper plane and half spaces formed from the inequality constant, when the either this, that have space, either it is less than equal to 0 or than in one side of this surface will be there, if it is greater than this then it will be another side of the surface. So, it is generally it is expressed that is what is called g i j, g j of x is less than equal to c j in more specifically I can write it this one, g j of that d j transpose of x minus d j of x the d j of x this minus c j. If you take bring this that side, c j is equal to less equal to 0, that it can write it. So, this is inequality constant either it lies one of this surface or other half of the surface, depending upon the less than equal to 0 or greater than equal to 0. And it will be on the surface when equality sign is there.

So, the quarterly problem is nothing but a minimization of a quadratic function, quadratic, convex quadratic function over the region, which region? Intersection of this equality and on an equally constant, equality constant we call hyper plane and inequality constant is a high space. So, or you can say combination of this one is called the polyhedron, over a range of over a polyhedron. So, what is polyhedron?

A polyhedron is same this one, polyhedron is defined as the solution, set, the solution set, as the solution set of a finite number of equalities and inequalities constant, a finite number of, a finite number of linear equalities and inequalities. So, I told you that linear equalities, linear equalities constant is the, your hyper plane and linear inequality constants is a hyperspace.

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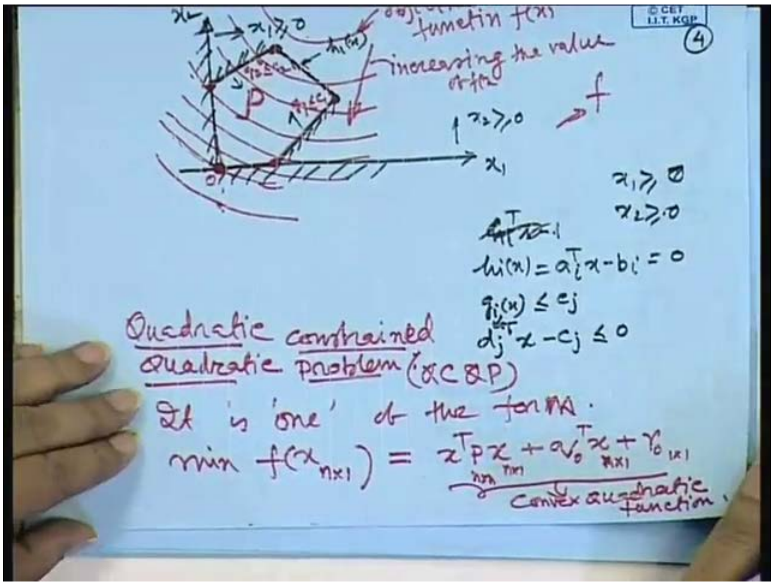

So, let us see an example, suppose we have a equality constraints. So, in a let us call is a , equality constant and inequality constant is a function of two variables x 1 and x 2. When I put this one, constant where x 1 is greater than equal to 0 and x 2 is greater than equal to 0, this indicates x 1 greater than 0 this indicates this space. And when x 2 is greater than equal to 0, this indicates this space that x 2 is greater than equal to 0, this one is x 1 greater than equal to 0. In addition to that, if we have a constant, equality constant just like h i transpose of x minus you see what I am writing this is not that constant I have written a i x that means, h i of x of, when we have written is a a i transpose of x minus b i is equal to 0.

This is equality constant and inequality constant and g j of x is less than equal to c j which I am writing in this one is, that symbol we have used the d j this one is, d j transpose into x minus c j is less than equal to 0. Suppose, if you plot it i told you it is a function of two variables x 1 and x 2, if you plot this one, this constraints. Let us call we have a some equality constant or inequality constant is there, let us call this is our equality constant h 1 of x, one equal to x that means it is on the line. This indicates is on the line and this is the, these are the inequality, two iniquity constants are there, let us call this into, this satisfy g i is less than equal to, g i is less than, g i minus c i is less than equal to in the, this place.

Similarly, another equality constant it indicates, let us call this area. So, our feasible region of the optimization problem is that one. So, it is a intersection of equality constant and inequality constant, intersection of this one. So, it is a polyhedron and intersection of this point is called vertices of the polyhedron. So, this is call a region of our, feasible region of our optimization problems that means, if your problem is like their minimize function of a, subject to let us call to one equality constant, which is shows on the line itself. And another two, let us call g 1 is this one, this is g 2 is that one and two inequality constant, whose feasible are less than equal to this. That g 1 is equal to, less than equal to c 1 and g 2 less than c 2 then represents this area, this portion and this represent this portion.

So, this is the, our feasible region and it, it forms with the in intersection of equality and inequality constant. You see, this indicates the two hops either, when it is g 1 is less than equal to c 1, we can assume this is that, that portion when g 1 minus g 1 minus c 1 is less than equal to 0, this, this portion. When g 1 minus g 2 is better than 0 this portion so that divides into two halts. In n dimensional case, it is divided into a n half spaces. So, an equality constants, it is on the, what is called in this case on the line, if it is more than two definition variables, it is on the hyper planes, that is.

So, each vertices are the vertices of the polyhedron and will show this letter after few lecture. We will show it the function, the function which are going to optimize, the solution of that optimize below the function will be any one of the optimization solution. We will get the vertices of this one, any vertices of this one, not any particular vertices of this one, but this point, this point, this point all are feasible solution, out of these five vertices 1, 2, 3, 4, 5, one vertices will give you the optimum value of the function, either maximum or minimum, that is we will show it there.

Now, if you see this one suppose, let us call the, our, that objective function is like this way because this is a convex function is, let us call it is like this way and this is our object functions f of x. And we are assuming that function value is increasing in this direction, increasing, this indicates that increasing the value of function, increasing the value of function f x. Now, you see clearly if you move this function towards these direction so if a pass to this vertices, this is one of the solution, feasible solution, if this pass through this one, is a feasible solution is there.

Again pass through this one again it is go on in this direction, the function value is increasing or if our problem is maximization of this function, I have to move this function in the right direction like this way. And up to this you see this point also, solution of the, our problem that means, at this point over point we will give with the maximum value of the function. Beyond that, function value no doubt more than this point value, but the solution is not feasible. In other words, the solution of this beyond that, the curve, this curve is not what is call, if you see this one this is not a feasible solution means, it does not satisfy at least one of the equality or inequality constant, may be more than one also.

So, our conclusion is if you, if you just do the what is call convex optimization problem, that function must be convex function sorry, quadratic optimization problem if you are doing, the function must be quadratic form, convex quadratic form. And the equality and inequality constants are your affine functions then you will get a polyhedron and each vertices of the polyhedron is a feasible solution of the problem, either maximization problem or minimization problems. Then out of this vertices, one vertices will give you the optimum value of the function whether, you want maximization problem or minimization problems. So, this is called a polyhedron.

Next is we have just discussed your, what is called quadratic optimization problem. Next is our quadratic constant optimization problems, quadratic constant, constraint, quadratic problem. So, previous problem is we have quadratic optimization problems, where that our objective function is a convex function, but it is a quadratic. That function is a object function is quadratic convex function and the, our constants are affine function whether, equality constant and inequality constant, both are in affine function. Next is, our problem is quadratic constant and quadratic problem, quadratic constant quadratic problems. In short it is QC quadratic constant quadratic problem QCQP.

So, it is it is one of the form, it is one of the form, the form is now minimize function x, f of x which dimension is n cross 1, this you have to minimize and that function is a quadratic form, only quadratic from that must function must be convex function. So, this function is a quadratic form and it is a convex function, that x transpose, let us call I am considering p x plus q 0 transpose of x plus r 0.

So, this dimension this since function is a scalar quantity n our x, I have considered n cross 1, correspondingly q dimension you will know immediately. And this since x is n cross 1, this will be n cross n based and this dimension 1 cross 1. So, this function quadratic constant quadratic problem though this function will be quadratic convex function. So, this function is convex quadratic form, convex quadratic function.

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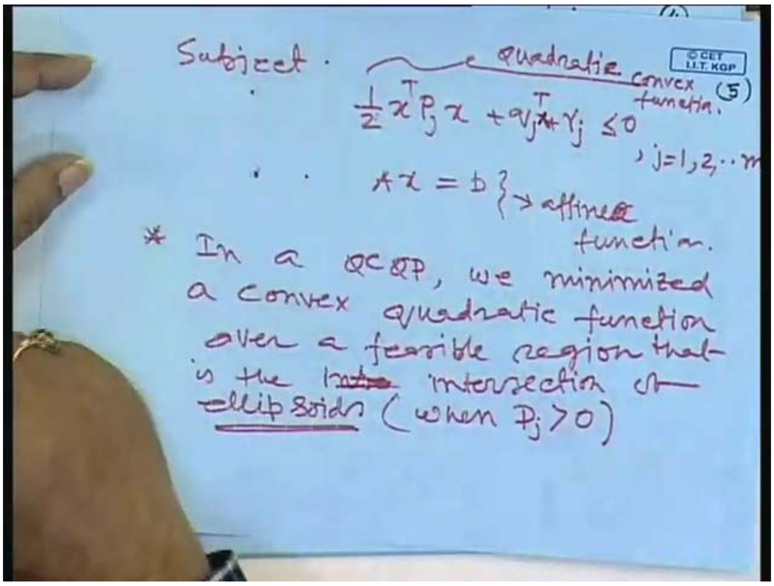

So, our cost function is convex quantity function, same as our quadratic optimization problems. In addition to that, our, what is call subject to equality constraints, an inequality constraints. So, our x transpose p j x plus q j this transpose plus r j is less than equal to 0, j is equal to 1, 2 dot dot m, there are inequality constants are there, which is a quadratic from non-linear, non-linear function. But that function is a quadratic form, function of this one and how many function, we have a m functions are there. And this function, it is a quadratic form or not to this one, that we do know, that you find out the hessian matrix of this one, quadratic convex function or not.

If the, that hessian metrics of this function is positive semi definite or positive definite, then function is a quadratic convex function, this one. And in addition to that we have a equality constant, which is a function of this is the affine function, this is the affine function means, linear function, linear function in x. So, that, that is, is quadratic, quadratic convex function, convex function. So, our objective function is quadratic converse function then our inequality constant is quadratic convex function, but we have a m inequality constant each is quadratic convex function. And our equality constant is affine, affine function in x then we will call this problem is called that quadratically constant, quadratic optimization problems, quadratic constant quadratic optimization problems QCQP problem.

So, in short we can write it in a, now you see our constant is what? Previously our constantly a equality constant and inequality constant was linear affine functions, intersection of that function we called is a polyhedron. That means, inequality constant forms the hyper half space and whereas inequality constant form is a hyper plane, the intersection of hyper space and hyper plane all these things, we get the polyhedron. And the polyhedron intersection points are called vertices of the polyhedron. So, here is no more this is affine function, this is a quadratic convex function and the just form is the ellipsoid form.

So, the intersection of ellipsoid and the linear equation, this form a ellipsoid in a quadratic constant quadratic problem, we minimize a convex quadratic function over a feasible region. How this feasible region is formed? The intersection of ellipsoids, this is from over a, over a feasible region that is the intersection of, intersection of, intersection of ellipsoid that means, when p j is greater than, greater than 0. So, this, in previous case all equality and linear equality constant, in equality constant was affine function and there is a form is over a region of polyhedron, but here over a region of what is called, ellipsoid, because this form is the ellipsoid, elliptical form of just this one.

So, remarks just see remarks, when this is p equal to 0, this quadratic not this one, that the function quadratic optimization function is that one. In quadratic optimization problems, if you see quadrant optimization problems, if p is equal 0 then what is this form? This term will not be there, this is there and this is the linear equation, affine equation. So, our objective function is linear inequality and equality constants is linear. So, quadratic optimization problems transform into a linear optimization problem, when the p is equal to p matrix is equal to null matrix, this term will not be there. So, linear, quadratic optimization problem special case, when p is equal to 0, this quadratic optimization problem boils down to a, what is called linear programming, linear optimization problems.

In quadratic constant optimization problem you see, our objective function is convex quadratic form, as well as inequality constant is quadratic, what is called convex function. In quadratic convex function, inequality constant if we assign, if you see this one, if we assign this quantity, if we assign this p j for all p j j is equal to 1 to m, this matrix is null matrix, this term is not there in equality constant, only linear term is there. Here I have missed a term x please note, x is missed here. So, if p j is equal to 0 this term will not be there, j is equal to 1 to m all cases that p j is 0. So, this is now becoming a linear affine function, but our more objective function is a quadratic convex function and when p j is 0, this tool equality and inequality constants are becoming affine function.

So, the QCQP problem turns out to be quadratic optimization problem, when p j equal to 0. So, it is special case of this one. Now, next our problem is that how to solve such type of optimization problems, that is next question. We use our KKT condition, what we have seen than we have to solve that one, that is one way of doing. Another way of doing is this, by linear programming, quadratic optimization problems we can solve by using the linear programming for this one.

First is considering the, what is called the KKT condition and then after the KKT conditions, if you do it necessary condition then you will solve it by using the linear programming. So, must know what is the linear programming. So, next topics will be the linear programming, how of solve a linear optimization problem using that numerical techniques.

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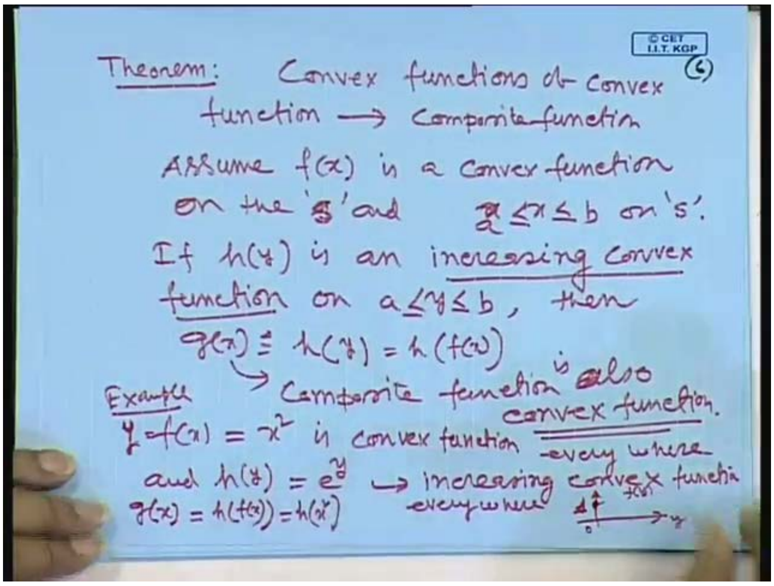

Before that, just one thing and just discuss about the, what is called that your, theorem one, convex function, convex function of, convex function, convex function of, convex function is called that, this is called composite function. You can say function of a function, function of a, function which each and each function is a convex function, but there is some different properties are there. So, assume the theorem telling, assume that f of x is a convex function, is a convex function on the set, on the convex set S is always, when you write S is the convex set, on the set convex S means, when you will get on the set it is a convex set. And over the range you can say, over the range f of x sorry, x is less than equal to b is less than equal to a.

And this on convex set S, if h of y, if h of y is an increasing convex set sorry, increasing f of x is a convex function, over the range on y is less than equal to f of y is less than equal to b. Then g of x is equal to h of, define h of y, y is a function of, y is a function of, y is a function of x, y is a or you can say y, which is a function of x. So, f is a you see, f is a convex function and h of y is also, h of f x is also convex function, but increasing convex function, over the range of a to b.

So, then it is called, this function is called composite function, which is also a convex function, this is also, this is a composite function is low, composite function also convex function. It is nothing but a convex of function, function, a function is a convex function and other function, which a function of that function, which is increasing nature in the same interval then it is called composite function over the function, of the composite function is also, is also convex function. Let us take one example that our f of x example, f of x is equal to x square is convex function and this convex function if you see, if just bought a x square versus f of x means y let us call, it is a parable type. And it is a convex for over the wall range of x, minus infinite to plus infinite, convex function everywhere, you can write everywhere.

So, f of x is a convex function and h of y is another function, which is the e the power of y and it is e to the power of y and this is, this is, that is h of y is increasing function and it is a convex function, is exponentially increasing function. If you see, if you plot this one is exponentially increasing and it is a convex function, by definition of convex function you see any point on this curve, join two points on the curve, if we join together you get a cord. And these two points belongs to a convex set any, any point on the convex set function value is always less than, what is the value of the point on the curve from the value a square.

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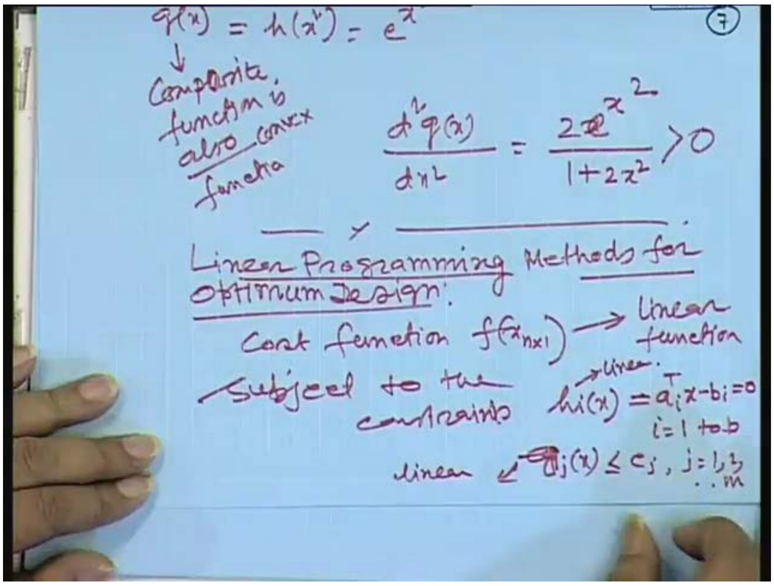

So, this is and this is able to y let us call, y is to f x is equal to this and this is, is increasing convex function, increasing convex function everywhere. If you plot it this one e to the power this, this is y and this is 0 from one and plotting, what is I am plotting? f of y, f of y I am plotting, if you see form to start from 1 and go on increasing this one. And this point is a convex function and increasing value of this function, this then g of x, that g of x is equal to h of, that is h of y, y is what? f of x that is right away, is equal to f of x is what? Is x square.

And what is h of y is a function of is nothing but a g of x, now I can write it, it is nothing but a h of x square and is nothing but a e to the power of x square. And this composite function, composite function is also, is also convex function. Now, how you check this is convex function? You just take the twice, twice differentiation of this function with respect to x then we will get this function value is always greater than 0, for any value of x. Let us say, if you take the second derivative of this, if you have more than one variable, I told you that function you have to differentiate means, that hessian matrix will get twice, that is function difference twice, that intern you will get hessian matrix, if the variable more than 1.

Then, that matrix must be positive definite or positive ((Refer time: 47:06)). And this case it is a, since it is a single variable, you just do it this one, you will get, if you do this one you will get it in e to power of x square plus this, into 2 x square, which is greater than equal to 0, for any value of x in the range. Because, you have seen that f of a function given example, f of x what we have consider x square, it is a convex function everywhere in the domain, that means minus infinity to plus infinity.

Similarly, that we have a function of each of y, which is h to the power of y, it is our y that is also the function of y, everywhere. And it is increasing convex function, that g of x is the function of y which in turn we can write x square, is this way. And this function is a convex function, the composite function, function of a function both is a convex resultant function is a convex function. So, this is our conclusion for this one.

Now I just mention it then how to solve the quadratic optimization problem, by using that what is call the linear programming. So, we will start for now, what is linear programming. So, next topic is your linear programming methods for optimizing, optimum design. So, we know first is we define, already we define that what is linear programming, that objective function, that means cost function, that means f of x, if it is a n variable this, this function is linear, linear function. And subject to the constraints, to the constraints and we have a constraint, equality constraint and inequality constraint, that h i of x is equal to 0, that h i of x I can write it into this form, a i transposed x minus b i is equal to 0 form and i is way to 1 to b.

This must be linear and also g, also g j of x is also that less than equal to 0 or less than equal to c i some constant, less than equal to c i or some constant, that is also our c j and j is equal to 1, 2 dot dot m. This is also linear, this also linear then this problem is called linear optimization problems. That means, you find out the value of x so that, it satisfies this constraint as, satisfy the both the constant as well as the function value will be minimum or maximum, cost function, this cost function let us call minimize this cost function or maximize.

So, this problem when all cost function, as well as inequality and equality constraints are linear, that of the optimization problem is called linear programming problems. So, to solve such type of problems, either analytically or numerically, so first we had to convert into a standard LP problems.

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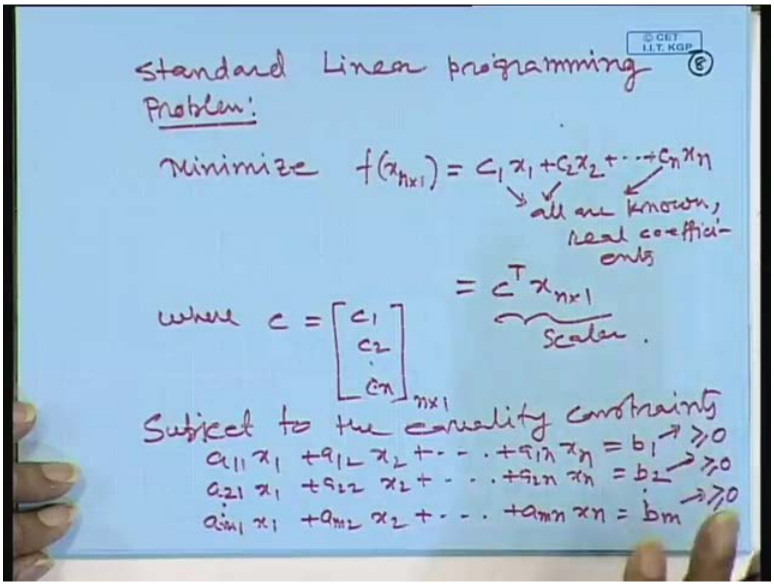

So, how will convert into a standard linear programming problem? The first will divide what is the standard linear programming problem. So, our problem is minimize, minimize the objective function, that objective function is f function of that x, which is function of n variables. Let us write they are digital expression and that expression, this function is linear. So, x 1 c 1 into x 1, c 2 into x 2 and we have a n variables. So, it is a c n into x m. So, you have to minimize this function and this function you can write it that, c 1 c 2 dot, dot c n all are known, all are known. And this function value is real and known, may be positive, negative, zero all function value is known is real function, known and real coefficients.

This I can write in terms of vector form, where c is equal to, you write it that c 1 c 2 dot, dot c n is a column vector and c transpose is row vector multiplied by column vector, this will be a scalar quantity. So, our objective is this one, subject to equality constraint and inequality constraint is not. So, our standard problem is there, our standard LP problem is minimize this one, subject to the standard problem, subject to that, subject to the equality constraint, subject to equality constraint, only. Suppose, if you have any inequality constraint, I can always convert into equality constraint.

So, that much standard LP problem is telling, minimize this linear function like this way and subject to this equality constraint x 1 plus a 1 x 2 plus dot, dot you want n x n. Because, we have a n variables of there, is equal to let us call it is a b 1. Similarly, second equation a 2 1 x 1, a 2 2 x 2 plus dot, dot a 2 n x n is equal to b 2. And in this way, we have a m equality constraints are there, a m 1 x 1, a m 2 x 2 plus dot, dot a m n x n dot, dot this is the b m.

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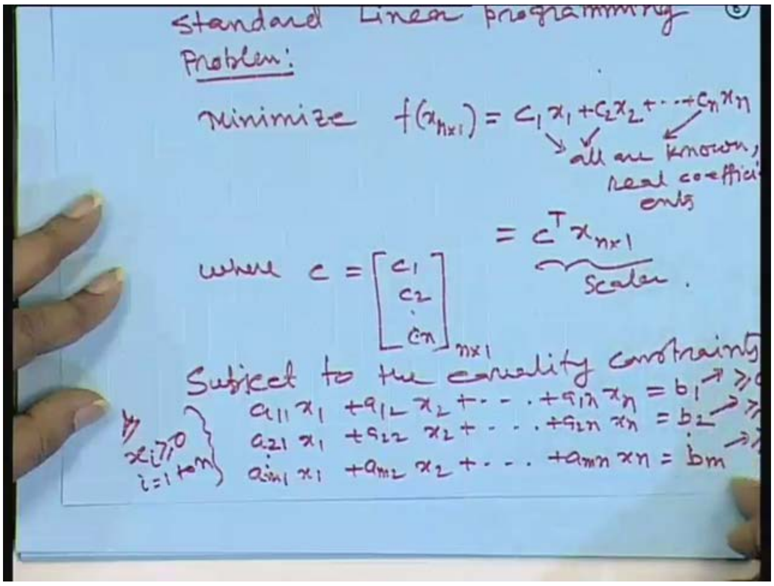

So, our standard LP problem, please remember minimize the linear function subject to our equality constraints and this inequality constraint b 1, b 2, b 3 dot, dot b m all are greater than equal to 0. This is the, their portrait like this way, all are greater than all audited 0. And at x i is better than equal to 0, for i is equal to 1 to n, this also must put constraint like this way.

So, minimize this function subject to equality constant like this way, right-hand side of this one which is a constant term, this a constant non-negative number, non-negative numbers, this. And x i is greater than a 0, for all values of x 1, x 2 and so on. So, if your problem is, if you having a inequality constraint, convert into equal to form, keeping the constraint term in the right hand side positive. Then it is a standard LP problem.

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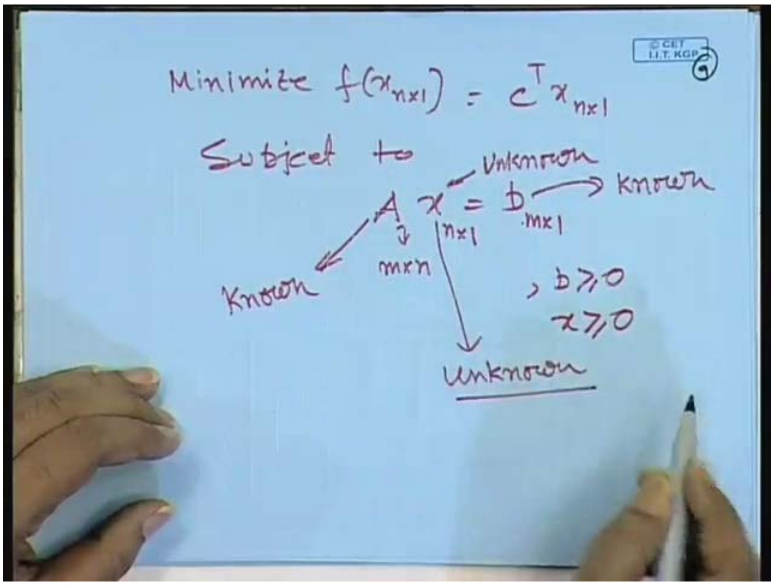

If I write it metrics and vector notation form, we can rewrite this optimization problem, minimize f of x is equal to C transpose x, this one subject to A into x is equal to b. And b dimension is m cross 1, n dimension is x dimension n. We have a m equations are there so that, the dimension is m cross n or b is, you can write greater than equal to 0 and x is greater equal to 0, x is vector that we have a each component of this x 1, x 2 dot dot x n is non-negative number at the greater than equal to 0. Similarly, b is a vector of m cross n that each is, each element of b is a non-negative number that means, right hand side of the matrix equality that, a x is equal to b that constant matrix b, must be, what is called positive.

Now, our problem is which is unknown? This x is unknown. So, our problem is minimize this function, subject of this constraint, equality constraint. In another words, you solve a x is equal to b in such a way, that function value will be giving minimum value. You are looking for some value of x 1, x 2 dot dot x n so that, this function value will be minimum as well as, this constant all satisfied. So, basically it is nothing but a solution of algebraic equation a x is equal to b this is known, this is known from the description of the optimization problem, this is known and this is unknown.

So, I will in general that what is call any problems are there, equality constraint, inequality constraint, if it is a linear form, I can always convert into a x is equal to b form. Keeping the right hand side term is positive quantity, that we can form, any problem, any linear programming problem can be convert into standard LP problem. So, I will stop it here today, next class I will continue this one.