Optimal Control
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Lecture - 10
Problem and Solution Session

So, last class we have discussed that, if an unconstraint optimation problem is there, we can solve it by using the KKT condition. We have discussed the KKT necessary condition agree?

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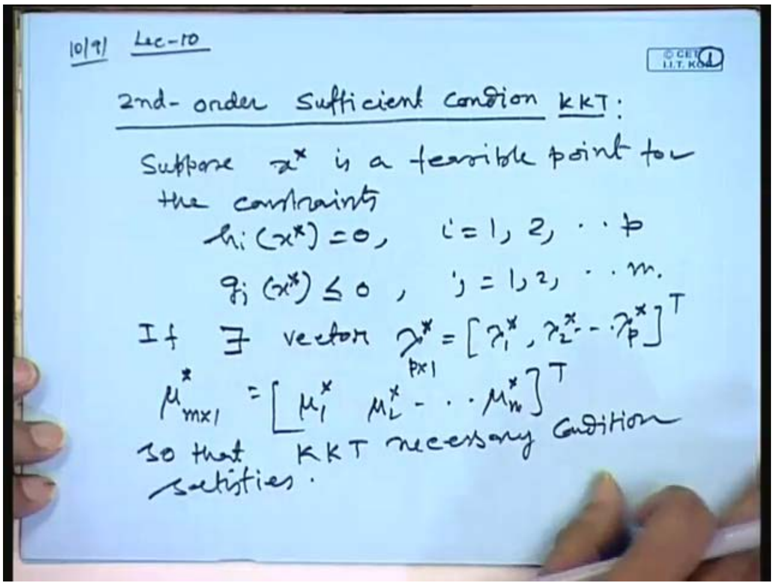

So, next is the sufficient condition. Whether the point, regular point is a minimum value of the function will give you or the maximum value of the function will give you that will be decided by using the sufficient condition. So, let us call that the second order sufficient conditions, KKT. So, suppose x star is the feasible point, feasible point means the point with lies in the, what is called feasible set. At that point, that equality constraints, inequality constraints must satisfy.

So, that is a feasible point. Suppose, x star is a feasible point, for the constraints, it must satisfy these constraints is equal to 0. We have a, there are p equality constraints, as well as it should satisfy the inequality constraints. Less than equal to 0, for j is equal to 1, 2 dot dot m. Now the question is if there is a, if there exist, this is called, there exists a vector. This symbol is there exists a vector. Lambda star is a, Lagrange multiplier, what is this one? Lambda 1 star, lambda 2 star dot dot lambda p star, because there are p equality constraints are there.

If there exists vector, whose dimension is p cross 1 and mu whose dimension is m cross 1 is equal to mu 1 star, mu 2 star dot dot mu m star, if there exist these and these vector. So that the KKT condition, so that KKT necessary conditions satisfied, condition satisfied. What is the necessary condition? Gradient of Lagrangian function must be equal to 0.

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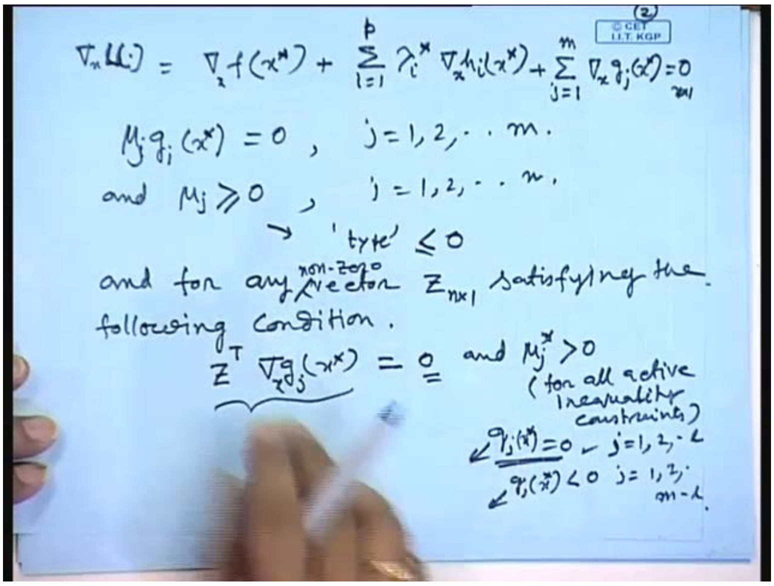

So, what is the gradient of Lagrangian function with respect to x? If u see that L, that gradient of L with respect to x is nothing but, a gradient of, f of x these plus summation of i s, that is we have shown last class. That is equal to lambda i star gradient of h i with respect to x this, plus summation of that j is equal to 1 to m gradient of this x g j x star, this must be equal to 0. Whose dimension is n cross 1, this dimension is n cross 1.

Again this dimension is n cross 1. So, and also there are, which is called switching function must satisfied, mu i, mu j, g j x star j must be equal to 0. For j is equal to1 to m. And mu is j, a Lagranian multiplayer associate with the inequality constraints, mu j must be greater than equal to 0. That we have shown in last class, 1 2 dot dot m. So, this is less than equal to, less than, greater than equal to 0, when the constant will be this type, this type of constant, constants are less than equal to 0. Then, this will be greater than equal to 0.

Again non-negative, mu is non-negative that we will see later in our discussions. So, this is the thing. Are satisfied and for any vector, for any use see this this conditions are satisfied. So, that the necessary conditions are satisfied, this conditions. And for any vector Z any non-0 vector, rather you write it non-0 vector. In any non-0 vector Z, whose dimension is n cross 1, satisfies, satisfying the following conditions. What is this following conditions? The gradient of g j of x star with respect to x, this multiplied by Z transpose way this must be equal to 0.

And mu j star equal to, greater 0, this greater than 0, for all active iniquity constants. For all active, for all active iniquity constants. Constants for all activity, constants means is, that if, x star is a your regular point, then at that point, that inequality constant, if it is a at that point g j of x is equal to 0, at x is equal to x star, where x is the regular point. Then it is called, the active constants. Again, because I mention it, we have a iniquity constant is small m, out of small m if at the regular point, if it is small l is the number of equality constant satisfied and remaining small m minus l is the, satisfies that inequality conditions. i is equal to 1 2, j is equal to 1 2 dot dot l and j of x star is less than equal to 0.

For j is equal to 1 2 dot dot m minus 1 constants, these are called active equality constants. These are called the inactive equality constants, this. So, this condition must be given. Naturally, when this is greater than 0, then g of j is your active constants. And gradient of that, at that point x star, gradient of that point with any vector into multiplied by these. Transfers of this, on this must be equal to 0, because that are, one is tangent and another is perpendicular to that one. So, this is a ((Refer Time: 09:21)) condition this must satisfy.

Next is Z transpose gradient of x g j of x star is less than equal to 0. When mu j and mu j star equal to 0, so this constant when mu j g j is in active constants, at that condition, this quantity will be less than equal to 0. Next is similarly, in equity constants, that with h Z transpose. Gradient of x with h i of x star is equal to 0, for all constants. This is for all constants h i x star is equal to 0, this is the all constants for active iniquity, means when g j of this, is 0.

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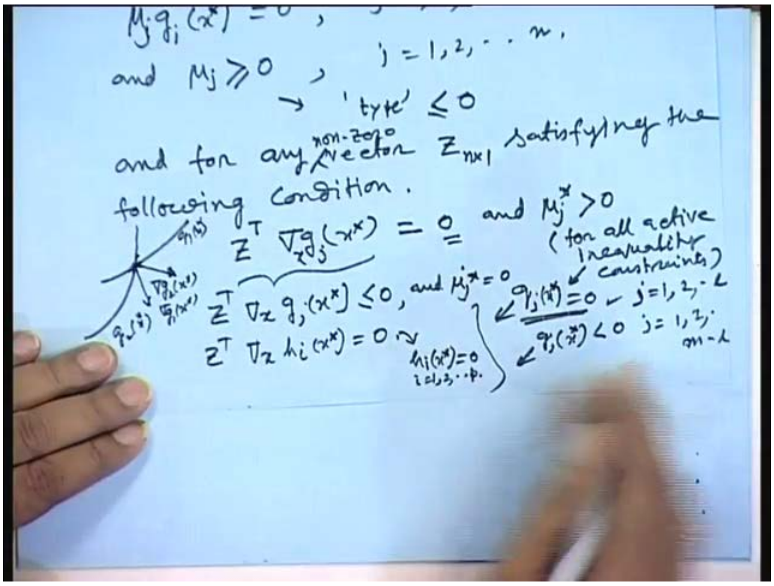

This condition, this is for inactive constants, this that means, if you have a let us call constant this is one constant is there, g 1 of x. This constant and another constant is like these way g 2 of x. At this point, the function below is 0. Now what you find? Of the gradient, of at this point up, because this equality constants are satisfied. At this point we will find out the gradient. That is you will draw the tangent corresponding to these, here and perpendicular to this one is that gradient of g 2 of x. And here, also draw the tangent corresponding to this curve and draw perpendicular to this one, is the gradient of g 1 of x star.

So, that direction Z tangent of this one, is Z transpose of this, must be this equal to 0. Similarly, h case also you can show it. So this is constant for i is equal to 1 2 dot dot p. So, this is a satisfied then, this point will, that point x star will be a then point, x star will be a what is called ((Refer Time: 12:01)) that if the following constants, because we are finding out the sufficient conditions, if the following constraints is true. That means Z transpose, that is Z is any vector is that second partial derivation of, derivative of, f of x at x is x x star, thes plus summation of i is equal to 1 to p, this is nothing but, a just like a un-constant optimation. We have done the second derivative of the function that is hessian matrix must be positive definite for minimum value of the function. hessian matrix must be negatively definite for maximum value of the function.

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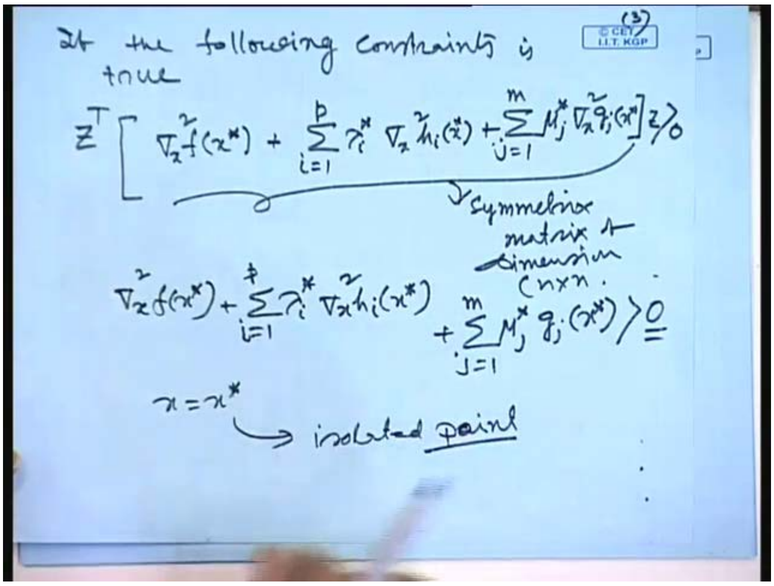

Similarly, these our, now the function is un constant it is a Lagrangian function. The Lagrangian function is similarly, second derivative this one with respect to x. What we are getting it is that lambda i star gradient of x h of i of x plus summation of j is equal to 1 to m mu j star gradient of x were g j of x star into Z, greater than 0. Greater than 0, means it should be a positive definite. This indicates this matrix, this is a matrix of dimension n cross n. This is symmetric matrix. Matrix of dimension n cross n and this quantity will be positive definite, provided this matrix is positive definite.

So, our condition is, this must be in our x is equal to x star, should be a minimum point, then this must be a positive definite matrix f of x star plus summation of j i is equal 1 to p lambda i star, gradient of this, not second derivative of this hessian matrix of h. So, this is star, this plus summation of j is equal to 1 to m mu j star g j of x star.

This matrix must be greater than 0, means it should be positive definite. Then we will call x is equal to x star, which will give you the minimum value of the function. And that point is a isolated point, that point is, a isolated point, optimum point. Isolated point means near, around this point there is no other minimum value of this functions. So, this is a isolated point means, there is no other, around this point will have a minimum value the function. So, this is a isolated point of this function.

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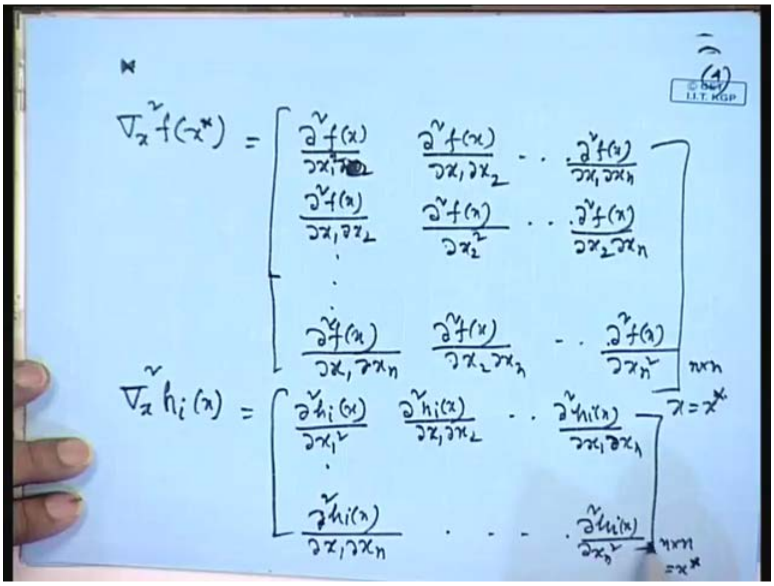

So, how to calculate this one you know this that is, if you write more clearly that. or Isolated point means, there is no other local minimum point in the neighborhood of extra. The isolated point once again, I mean isolated point is, means that there is no other local minimum point around x star, around that x star. So, let us say, how to calculate this one. x star that is, hessian matrix of f f of x is equal to x star. How to calculate? Is nothing but, if u recollect it is delta square of f of x, del x 1 del x 2. Sorry del x square f of f x then del x 1 del x 2 and since we have a n variables are there , we have a n variables are there. Then del square of f x del x 1 del x n.

Similarly, you can write it that there is del of x square, del x 1 del x 2 and del is f of x del x 2 square and in this way del of x n and del x square of del x of 2 del x n. If you continue like this way last term will be del f square del x 1 del x n then del square f and del x 2 del x n and last term is your del f square x del x square n. So, put this values at x is equal to x star. So, you have computed. and similarly, if you see this one del x batted Hessian matrix of h 1, it varies from i is equal to p will be h 1 you will again, you will get a matrix. You will get a matrix of dimension say, of dimension n cross n.

So, this dimension is n cross n. So, similarly, I am not writing this one del square f h i of x. You can get it in similar. Only you can say f will be replaced by h i this h will be replaced by h i function. Then you can write it, this is also n cross n. You have to put the value x x star again. So I just write it first for you, that h i of x del x 1 square h del i of x than del x 1 del x 2 and this way del square of 1 del x n and in this way. Last is del where i of x del x 1 del x n and if you do this one, last one del square i of x del x n square. So, again this is the second part of this, sufficient conditions we have written. All I now here we have to write it, if you see this one.

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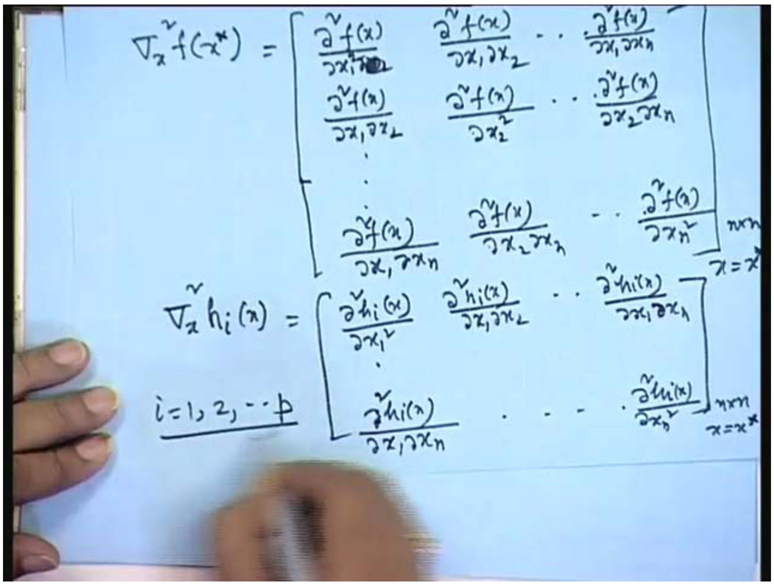

Here you have to write it i varies from 1 2 dot dot p.

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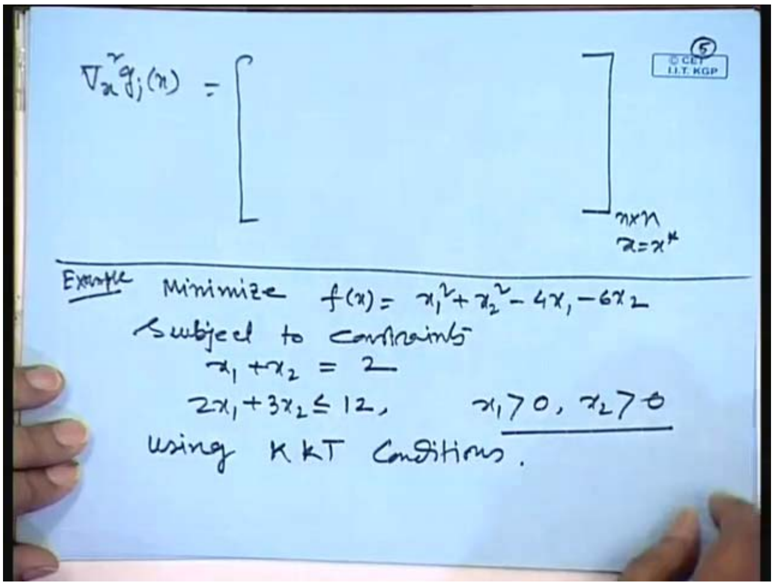

Similarly, can write it, delta x that g j of x. So, I will not repeat, only in place of a few replaced by g j. That is also n by n matrix, put the value x is equal to star and it is also symmetric matrix. Once you do this one, next is you have to check this matrix. You know this matrix, you know this matrix, you know now all put this together. You will resulted you will get another matrix. So, you will see whether that matrices positive definite or not. You know how to, how to test the positive definite matrix, not by Sylvester equality constraint, in equality conditions. You can find out if a matrix is positive definite matrix then you find out it is Eigen values are positive.

If all the Eigen values are positive then, it is a positive definite matrix. If all the Eigen values are negative then it is a negative definite matrix. Similarly, positive definite matrix to that matrix, the find of the Eigen values of this matrix. If some of the Eigen values are positive, some are negative what is 0, then it is positive semi definite matrix. Similarly, positive negative definite matrices, some of the Eigen values are negative and some of the Eigen values are 0 at least one will be 0. So, once you know all this one, then I can find of the second-order sufficient conditions.

This matrix positive definite, will call it is a, what is called, the function will give you the minimum value of the function. And x star is the isolated point. Isolated point means, this that the function does not have any other local minimum point around the x star. So, let us now solve a problem, then how to work out these problems, using the, our technique. That is what is called constant optimization problem, how to find out. Our problem is to minimize, example minimize f of x is equal to x 1 square plus x 2 square minus 4 x 1 minus 6 x 2 subject to, subject to constraints x 1 plus x 2 is equal to 2 and 2 x 1 plus 3 x 2 is less than or equal to 12.

We are assuming, that our site constant x is greater than 0 and x 2 is greater than 0. This so this problem you to solve it, by using KKT, KKT conditions. That is our problem. So what is our job? This is our equality constraints, they are an inequality constraints both are let us call both are inequality constraints and a this is our inequity constraint this. So, do not have any equity constraints. So, in that case h i of x is and 0 that lambda is associated with the equality constant. So Lagrange multiplier lambda i will not be there in our solution.

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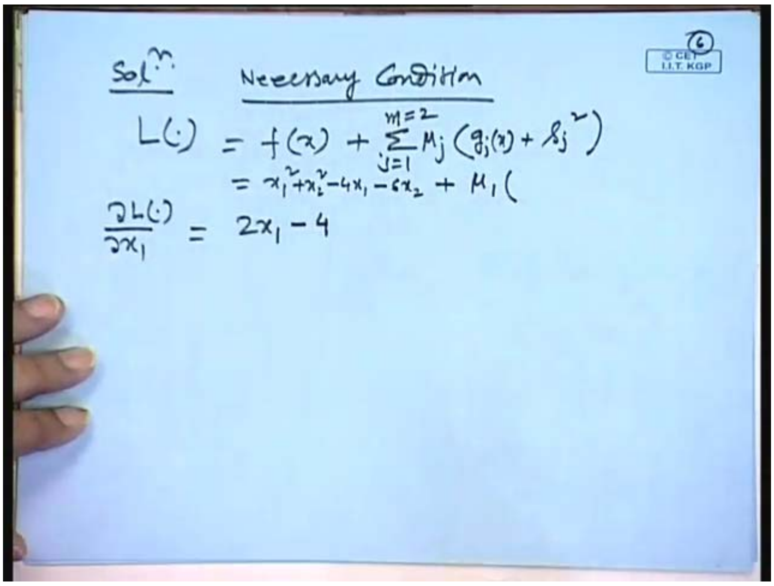

So that aside, first you write it, the solution of the problem. So, first let us write the necessary conditions. So, first write the Lagrangian function. That function f of x plus summation of there is no h equity constraint. So, that term will not come into the picture, there is an iniquity constraint j is equal to 1 2 m in our case is equal 2. There are two inequality constant if u see this. This then, mu j then our inequality constraint, we have converted into equality constraint.

If you see g j of x plus s j square. Now, I have is the necessary conditions for this one. So I will find out del 1 del x 1 is what is equal to 0, because so what is this one is a f of x of is that one. So, you differentiate with a x is x 1 sure to be 2 x 1 minus 4. We have a also if you see this one. Okay, I will write it. It will be clear instead of calling this from what is left of x, it is a x 1 square plus x star square minus 4 x 1 minus 6 x 2 plus mu 1 g 1 of x minus g 1 of x is 0 to what you say g 1 of x is equal to x 1 plus g 1 of x result to you see if you call this a g 1 of x x 1 minus x.

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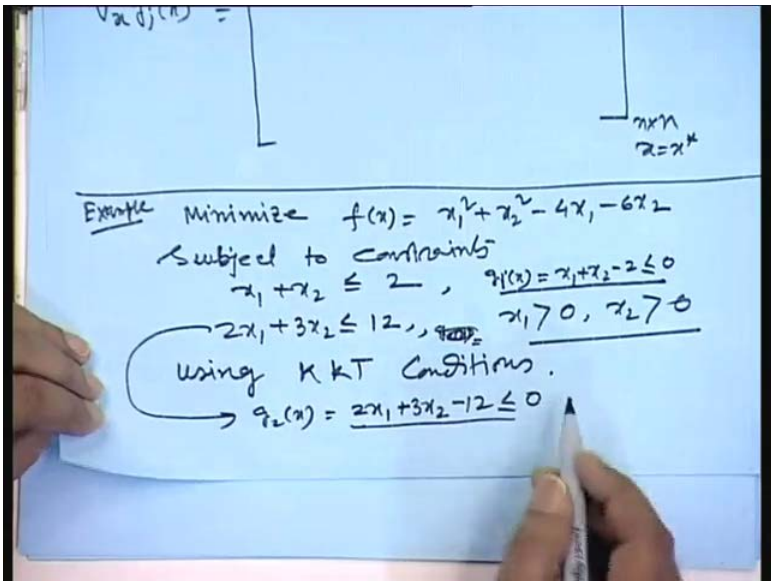

So I can write it this, if you see this one. I can write it this g 1 of x, g 1 of x is equal to x 1 plus x 2 minus 2 less then equal to 0. So this is our standard form, we have retained. Similarly, this is also you can write it g 2 of x. We can write it this one g 2 of x is equal to g 2 of x can write it is equal to 2 x 1 plus 3 x 2 minus 12 is less than 0. So, we have written our standard form type less than equal to 0.

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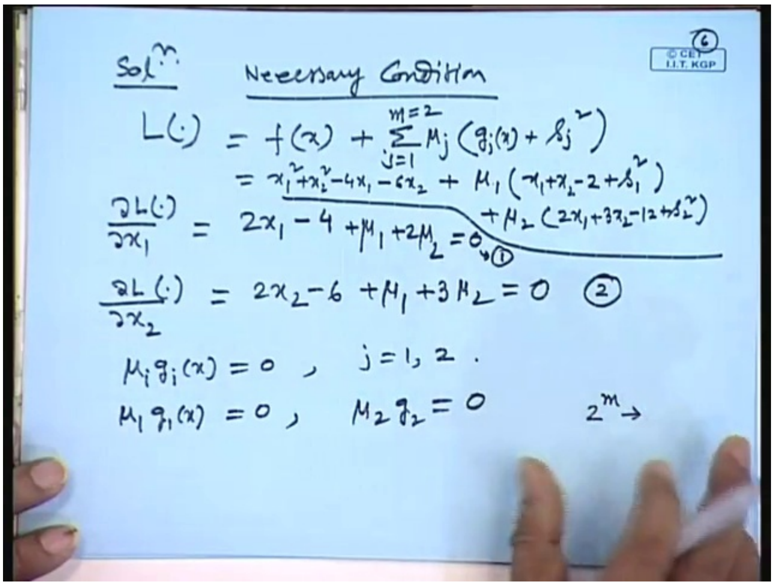

So, our g 2 1 g 1 of exigency x 1 plus x 2 minus 2 and we are adding with a some positive quantity. So, that it turns out to be at equity constant plus mu 2 then 2 x 1 that g

2, g 2 is your 2 x 1 plus 3 x 2 minus 12 plus s 2 square. Now I am differentiating this, with respect to x 1. So, it will be a 2 x 1 4 x then, here is x 1 is that mu 1 and no other x is here. Here again is there, is 2 mu 2. So, this equal to 0, so this one equation. Again you have to differentiate with respect to x 2, so if you differentiate with, so this is 2 x 2 minus 6 then it is a plus mu 1. Then plus 3 mu 2 to because I am differentiate it with h 2 again then this is there with respect to this. With so, that terms of these 0s, then this is the one equation, let us call, this is a equation number 1.

This is a equation number 1 and this is a equation number 2. We have a in fact, we have to differentiate with 1 with respect to there is no h. So, we need not to differentiate with respect to lambda. So, we have to differentiate with respect to our mu. Another saying one we have to differentiate with respect to s we have shown it that these two conditions can be combined together. Finally, we can write it that condition is mu i g j of x is equal to 0. In our case j is 1 and 2. So, what is this condition, will write it mu 1 g 1 of x must be 0. Another condition is that mu 2 g 2 is 0. So, we have, we have to solve these two equation.

How many equations? So, how many equations? So, how many unknown equations are there? x 1 x 2 mu 1 mu 2 and we have four unknowns. Two equation and from there we have to see, how to solve it. Look at this one, that there is this has a two possibilities, Satisfy this condition 1. Condition is mu 1, if mu 1 is 0 and what is called g 1 is not equal to 0, if g 1 is 0 but, mu 1 is not equal to 0. Again when mu 1 is equal to 0, g 1 is not equal to 0, that means it is inactive conditions when means g 1 is 0. But, mu 1 is not equal to 0 but, this condition is satisfied in that situation it is called active condition is satisfied.

Similarly, you have this, in general, if we have a small number of inequity constant. We have a 2 to the power of m possibilities are there to satisfy his equation. To deliver possibilities are, there when switching between switching 2 to the power of m, switching is possible to satisfy this equations. When that we have a j is equal to 1 to m that means m constants are there. So, in short if m constants are there, inequity constants are there then, we have a such types of equation. We have m equations are there, which in turn we have it 2 to the power of m. Switching conditions are there, to satisfy these equations. So, let us solve for this case.

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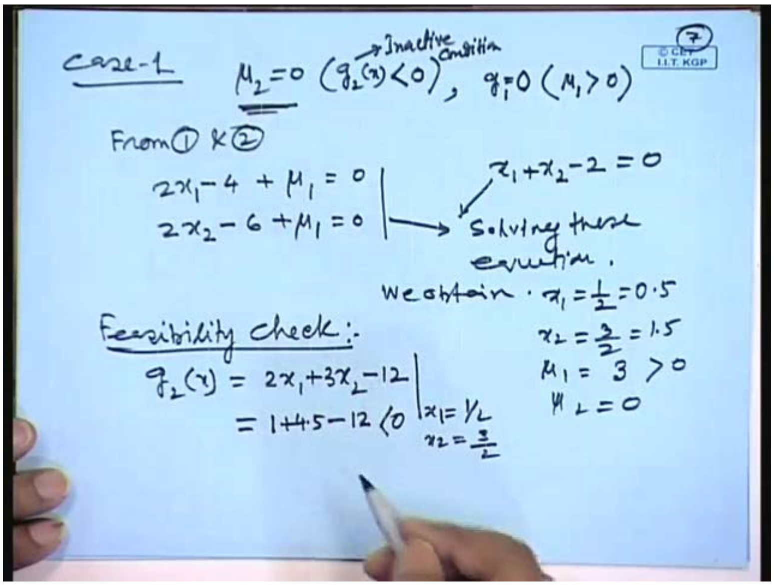

So, our first case we are considering that case one, mu 2 let us call 0 if mu 2 is 0, then g 2 cannot be 0. So, g 2 is our condition. Is g 2 is less then equal to 0? So, equal to cannot be of this. So, it is that inactive conditions. The condition is more relaxed, when you tell g 2 is a equal to 0. It is a active condition that means, it is a more tightened conditions, that means it should be on the curb of the on the line. If it is g x is a straight line, the register be on the line only, when g x is a active condition 0. So, you have this one condition mu is 0, then we can cut that g 1 is 0 g 1 affects g 1 is equal to 0. But, if g 1 is 0, then mu 1 is greater than 0, both cannot be 0. mu 1 g 1 mu 1 is equal to 0, it is enough to satisfy the conditions.

So, in this case what is the situation, we will see from 1 and 2. So, 1 and 2 equation I will import our say mu 2 is 0 mu 2 is 0. And there is a constraints of the g 1 is 0. So, we will put mu to 2 0 then, we will get 2 x 1 minus 4 plus mu 1 is equal to 0. Another equation 2 x 2 minus 6 plus mu 1 is equal to 0 and we have a g 1 is 0 g 1 is what? x 1 plus x 2 minus 2 is equal to 0. So, we have three unknowns there, so we have to solve 3 equations to solve.

By solving these equation, we get, we get we obtain x 1 is equal to half or point 5, x 2 is equal to 2 by 2 is 1 point 5 and mu 1 we got it 3 and mu 1 must be, I told when g 1 is active constant mu 1 must be positive non-negative number. If you get it by solving this negative, that means it does not give any solution for to become a what is called optimum value of the function. So, this must be this is 3 means greater than 0. And mu 2 is equal to 0, again this we got it.

Now, we have to see whatever this point is get, it must satisfy all feasible condition. All equity conditions, again an equality conditions, is satisfied that will call the point is in the inside the feasible space or feasible set of feasible region. So, let us say feasibility check. So, our g 2 we have to check it now. g 2 of x if you say g 2 of x is what twice x 1 plus 3 x 2 minus 12, if you see 2 of x and this we have to show it. It is our equation must satisfy your condition is less then equal to 0.

So, this condition, if report the value of x 1 is able to half value of x 2 is equal to 3 by 2, then what will get it, that one? We will get what is called, this, half means this is 1 and this is 3 by 2. 3 by 2 means, this is 9 by 2 means 4 point 5 minus 12. That is less than 0, 5 minus 12 is less than 0. So, this satisfied this condition. So this point may be one of the, what is called point, which will give, may give the minimum value of the functions. So, that we have to check it, makes. So, how to check it that one? So, this is dissatisfied all what is called constant equation at this one. That is x 1 is half and 3 by 2 will satisfy all constants.

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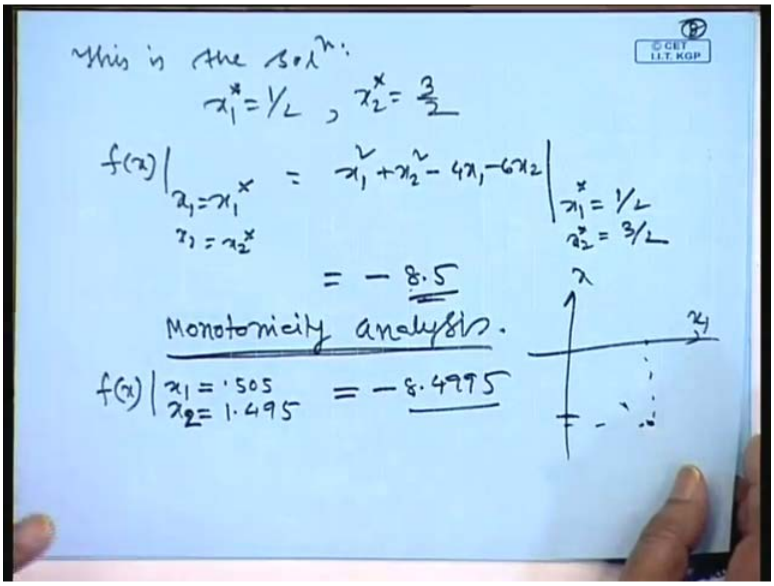

So, we can say this is the solution, the solution. That is x 1 is equal to x 2 star is equal to 3 by 2 is the solution. Here also, you can check it whether it is the minimum value? The function x m value function without checking the sufficient, sufficiency conditions and that condition will call that, how to check that one? Let us see, so let us solve a function that is x 1 is equal to x 1 star and x 2 is equal to x 2 star. This value is, I know x 1 plus x 2 minus 4 x 1 minus 6 x 2 put the value x 1 star is equal to half. And x 2 star is equal to 3 by 2 you will get this value minus 8 point 5.

So, this you got it, now how to say that, this is the value of the function at that point? But, how will you ensure that this will give you the minimum value of the function. One can check this thing by using, the monotonicity analysis. That is monotonicity analysis, what is this? I also, we have discussed you earlier that at this point, you give a small perturbations, if it is a minimum value. Give a perturbation around this point, the function value if it is a increase then it indicates, we have raised the minimum. We will again, so this called the mono policy analysis.

So, what is this? Let us call, we put the value of the function x is equal to this x 1 is equal to, let us call, we put the value function x is equal to this x 1 is equal to, let us form the value x 1 is 1 point 4 9 5. Instead of, what is called, that finding out the x 1 is not, this is x 2 is 1 point 1 instead of 1 point 5. I am writing 1 point 4 9 5 x 2, that means what is that the change I made? It point 0 0 5 and x 1 I made it, let us call point 5 0 5 point 0 5 added from 0 0 5 is part from this one. And if you put this value, at this one you will get the value is 4 point 8. Point 4 9 9 5, that is function is increased from minus 8 point 5 to minus 4 point 5.

It increased, if u see in the dimension x 1 and x 2 dimension eventually, if it is 8 point 5. Agree? This is corresponding to 8 point 5, agree? This is 8 point 4 9 5 agree? That means, this function value is increased from here to here. So, this indicates this is what is called, you get minimum value of the function. Also, we will check through what is called, the necessary condition or sufficient of condition of KKT condition.

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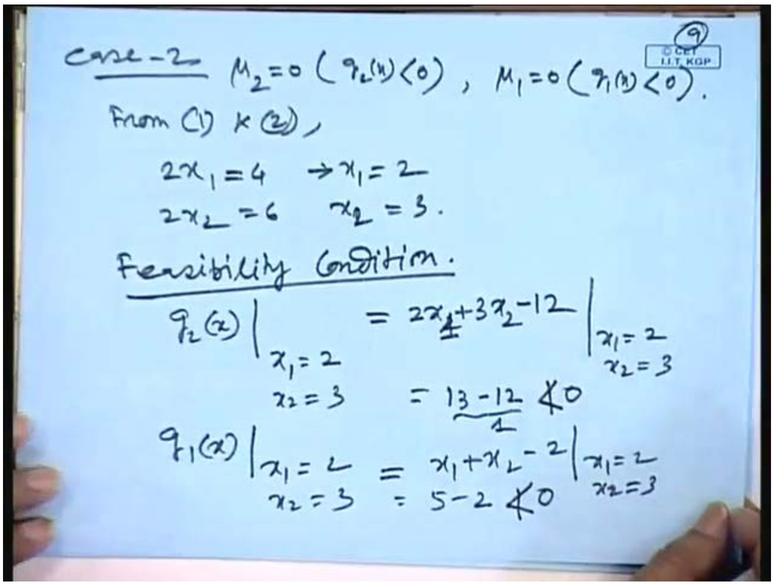

So case 2 maybe, case 2. Case 1 we have made it mu 2 is 0. I will make it mu 2 is 0. Now, there is another possibility is there, that mu 2 mu 1, is also 0. So, our case keeping mu 2 0. Same that is g 2 of x is less than 0. We are making mu 1 is 0, that means g 1 of x is less than 0 this. So, again from 1 and 2 we can write it twice x, I will put the value of mu 1 and mu 2 in that equation 1 and 2 equation. So, I will get 4. So, x 1 is equal 2, so twice x is equal to 6 and x 2 is equal to 3. So I have to check these value I got it. So, I have to check the feasibility conditions. We want mu 2 is 0, feasibility conditions and again I have to check the feasibility.

What is a feasibility condition? g 2 of x, have to find out g 2 of x. I have to find out the x 1 is equal to 2 and x 2 is equal to x 3, again let us see this one. What is g 2 of x? If you see this one x 2 minus x 2 plus 3 x, sorry x 2 x 1 plus 3 x 2 minus 12. So, this is our g 2 of x, put this value x 1 is equal to 2 x 2 is equal to 3 and you will get this value is 13 minus 12, which is not equal to, not equal to, which is less than, is not less than 0. This is means, this is equal to 1.

So, it does not satisfy this one. That means, we have a point 2 and 3, this is outside the feasible religion. Again feasible space. Similarly, 1 is enough to check that means, this is not the acceptable solution here. So, you can also check g 1 of x at x 1 is equal 2 x 2 is equal to 3 and in this g 1 of x you know, x 1 plus x 2 minus 2, put the value of x 1 is 2 x

2 is equal to 3. So, it is 5. So it is a 5 minus 2, which is not less than 0. Which is not less than 0. So these two condition, g 1.

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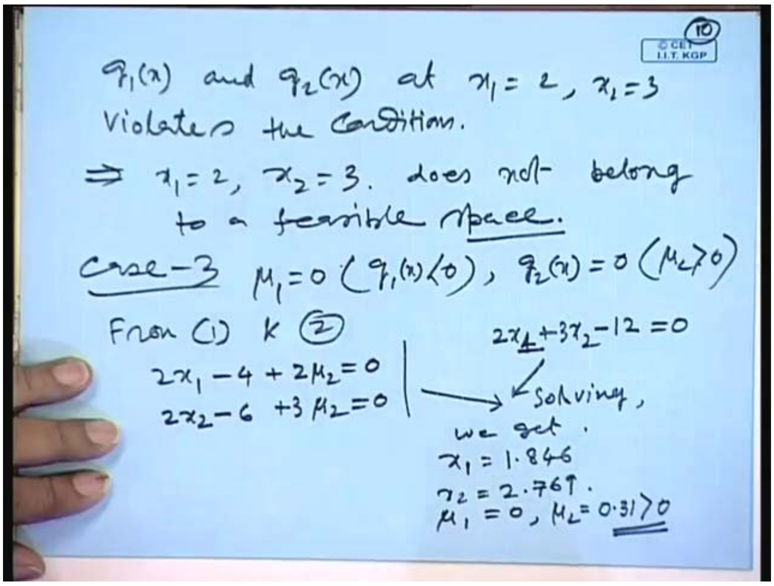

So, our conclusion is that g 1 and g 2 g 1 of x and g 2 of x at x 1 is equal 2 x 2, is equal to 3, violates that condition. So, our conclusion then, this is our, the condition that indicates x 1 is equal to 2 x 2 is equal to 3 x, is not, does not belong to feasible space. This does not belong to a feasible space or design space. So, this this case 2 will not be acceptable. Case 3 on another word it is not feasible space, there is no solution at that point. So, case 3, now I made mu 2 is 0, again. Then this another possibility, is you make it mu 1 is 0 if we make g 1 of x is less than 0, then you have a g 2 of x minus g 2 of x is equal to 0.

That means mu 2 greater than 0. So, this greater than equal to 0. So, let us see this one mu 1 is 0. So, form equation 1 and 2 from 1 and 2 I will get mu 1 is 0 in this event. So I will get it. Once again, if you see I will get twice x 1 minus 4 plus mu 1 is 0. To twice mu 2 is equal to 0, from equation 1 I will get this one. Putting mu 1 8 0 from equation 2 x 2, 2 x 2 minus 6 plus 3 mu 2 is equal to 0. I got it and we have a g 2 is 0 g 2 is 0 g 2 is, what if see, g 2 our 2 x 2 sorry 2 x 1 plus 3 x 2 minus 12 g 2 is 0. This one that make constant is on this. Solving we had a 1 2 3 unknowns are thee 3 equations are there, solving this.

Solving, we get we get x 1 is equal to x 1 is equal to 1 point 8 4 6, x 2 is equal to 2 point 7 6 9 and mu 1 is considered 0. And mu 2 is and mu 2 we are getting is point 3 1 which is greater than 0. Again, so this is satisfies all the conditions again. So, this maybe a one of the possible solution to get the minimum value of the function. But, first still have to check it, either what is called monotonicity analysis or using the, what is called by KKT, Second order conditions. So, let us check the feasibility conditions, our feasibility condition is this, condition must be satisfied g 1 is less than 0. This must be satisfied. So, our feasibility conditions is you see this one.

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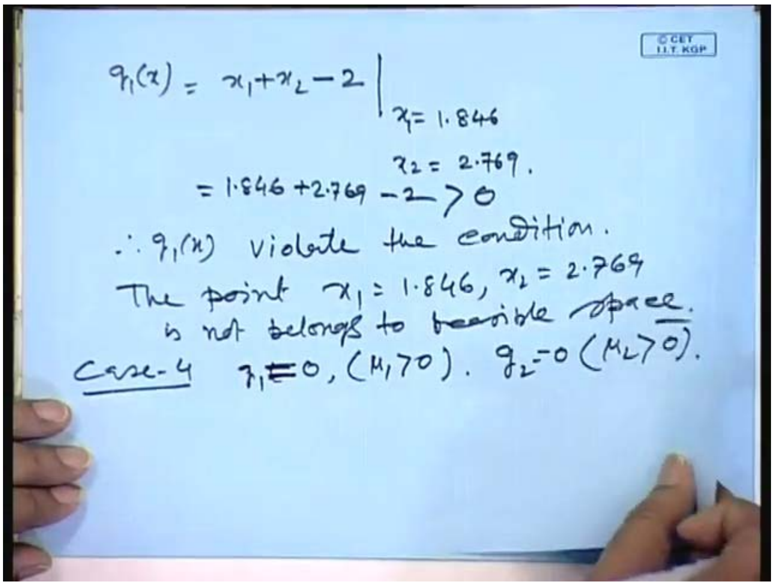

The feasibility condition g 1 of x, g 1 of x, what is g 1? x 2 minus g 1, x is minus 2. I think, so put the value x 1 is equal to that. We got it x 1 value is, how much we got it? x 1 we got is 1 point 8 4 6, x 2 is equal to 2 point 7 6 9. So, if you put this one g 1 is, you see does not satisfy. So, it is a 1 point 8 4 6 plus 2 point 7 6 9 minus 2, which is greater than 0. So, it does not satisfy the our monotonicity analysis. And, does not satisfy your constants. This must be less than 0. So, this cannot be a, once again this cannot be a g 1 of x, violates the condition.

So, the point, the point x 1 is equal to 1 point 8 4 6 and x 2 is equal to 2 point 7 6 9 cannot be a solution of our optimation problem, because it does not belong to any feasible region, feasible space. This point is not, does not belongs to feasible space. So, reject that solution. So, you say, this on our case 3, what we made it our case 3. Case 3 is that the done may get that g 1 is 0 and g 1 into g 2 must be 0. So, this will be less than 0.

So, this equation Z is coming from a equation number 2 and 3 putting mu 1 8 0, this then I have g 2 is 0. Solving this equation, I got this. So, it must satisfy this equation. When you are putting this equation, we got it, that one. It does not satisfy this one. Now case 4, what is the choices left? Now that g 1 is 0, which means mu 1 0 g 1 of is g 1 is equal to 0. Which means, mu 1 is greater than 0. Next choice is left, g 2 is 0, that means mu 2 is greater than 0.

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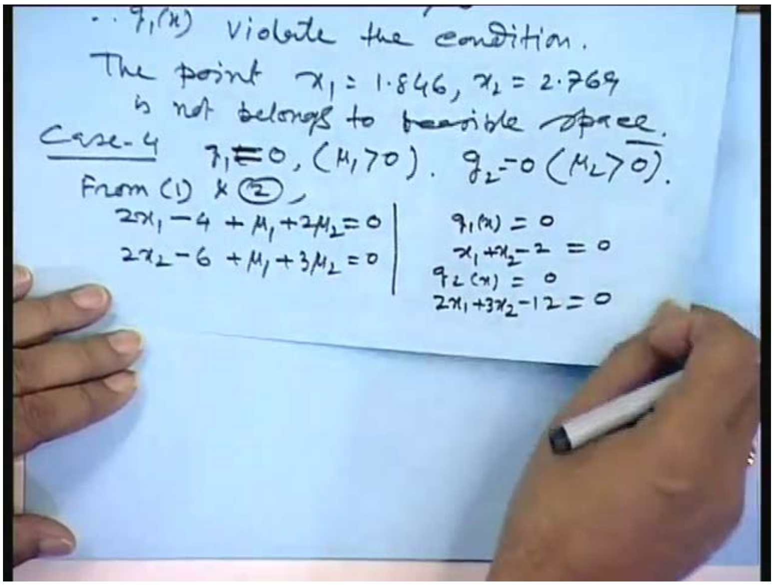

So, let us say using this equation in from 1 and 2. From 1 and 2, what you will get is 2 x 1 minus 4 plus mu 1 plus twice mu 2 is equal to 0. This I am getting from a equation 1. And then, form equation 2 I am getting is 2 x 2 minus 6 plus mu 1 plus 3 mu is equal to 0. So, this is your, getting from equation 2, this one. Now, so this will have a, how many variables are there? x 1 x 2 mu 1 mu 2. So, have a 4 variables are there. So, now use these conditions, because this is the way, who considers you only 0 g 1 of these 0 means g 1 is what? x 1 plus x 2 minus 2 is 0. So, this and g 2 is 0 means, that is 2 x 1 plus 3 x 2 minus 12 is equal to 0. So, we have 4 equations are there, 4 unknowns are there, mu 1 mu 2 mu 3 mu 4 x 1 x 2 mu 1 mu 2

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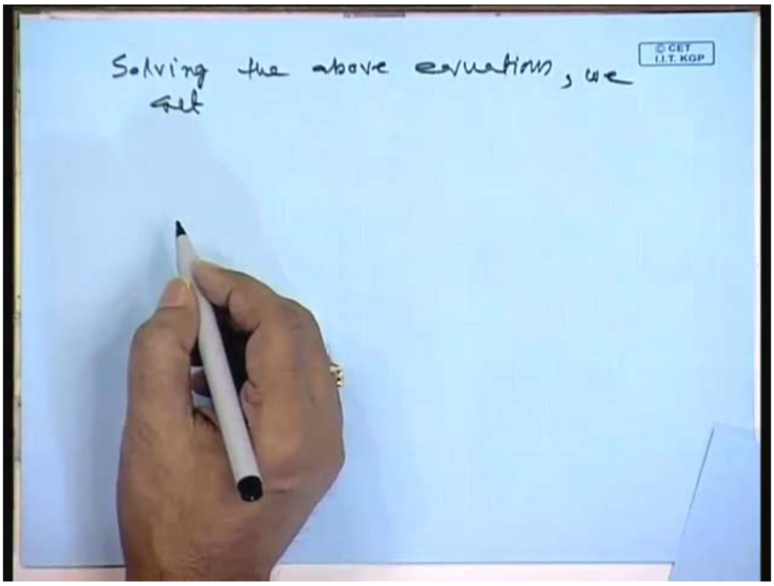

Solving these equations, we get, solving the above equation what we get? or you see this one, either this is the first, you solve this and this. This is a variable of 2 variables g 1 x, this you have 2 variables 2 equations.

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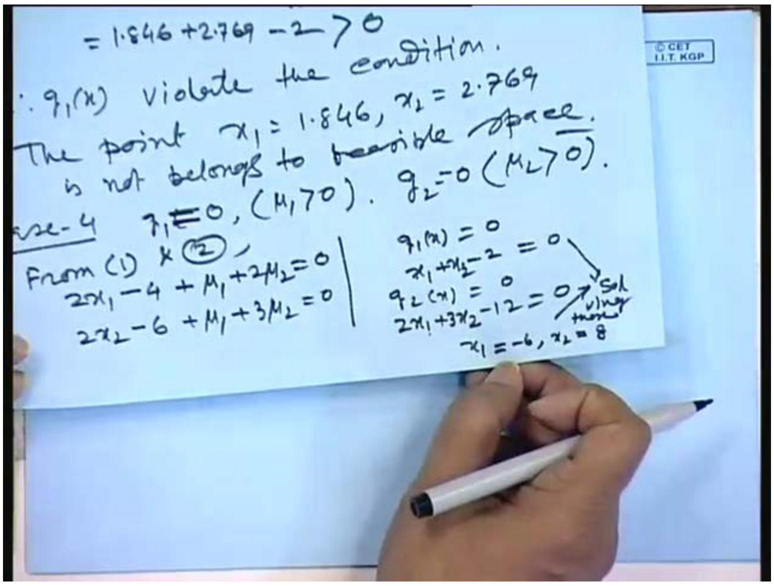

So, I can solve this or you can write it solving, we get, solving these equations, this equations we get x 1 is equal to minus 6 and x 2 is equal to 8. And it does not satisfy our, what is called our site constant we have put it x 1 is greater than equal to 0. x 2 is greater than 0.

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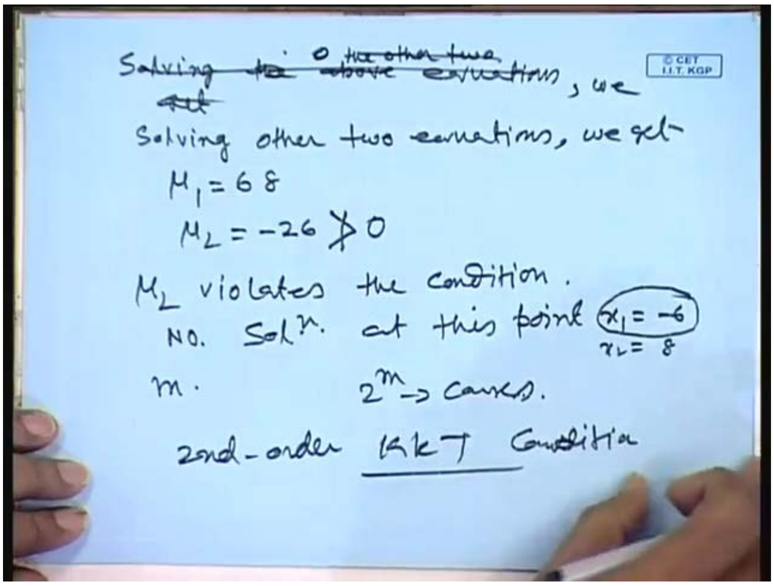

So that I will and solving the other 2 equations again, solving other 2 equations. Solving other two equations, we get mu 1 is equal to 68 and mu 2 is equal to minus 26 which is not greater than 0. So it also violates our what is called Lagrange multiplier conditions, that are site constants are violated. Our what is called the Lagrange multiplier for equity constants, that should be positive. But, it is become communicative, so mu 2 violates the conditions.

So no solution at this point. What is this point? x 1 is equal to minus 6 x 2 is equal to 8, not only this, you can stop it here, because this does not satisfy our site constants. Now if you say, out of the 4 cases now in short, if u have a, sorry if you have a n small and inequality constant you will hear to solve there, 2 to the power of m cases for switching the possibilities cases out of the you have to search, in which one will give you the feasible design space or feasible solution. At that point, whether it is a minimum or maximum one can check by monotonicity analysis or you can check by considering the second order KKT conditions. So, if you see the secondary and conditions.

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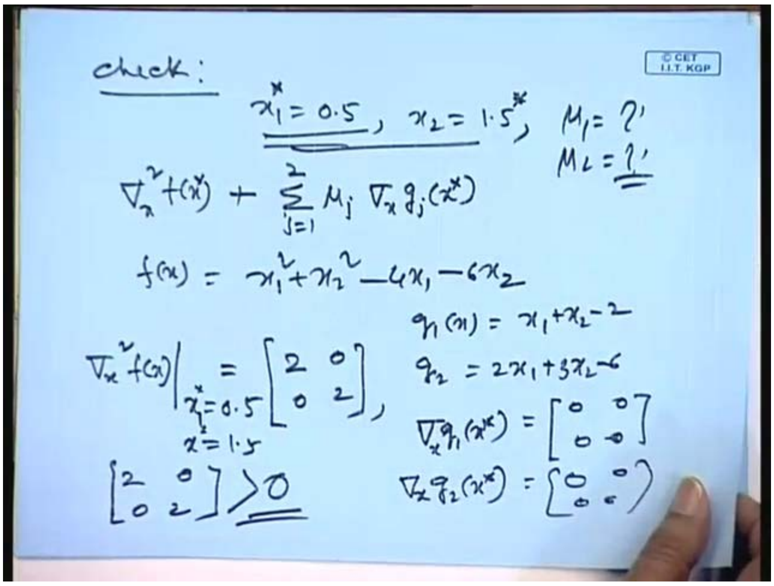

So, our acceptable or acceptable things is checked. Or acceptable x 1 is equal to point 5 and x 2 is equal to 1 point 5star. Again this, so you opt to check the KKT conditions, secondary KKT conditions. What is a secondary KKT conditions? f of x, this f of x plus summation of j is equal to 1 to 2 mu j mu j del x g j of x star, this star. So, this you have, because we do not have any, because equality constant. So, this two things, you have to find out.

So, let us see, what is f of x. Let us call our x 1 square, plus x star square minus 4 x minus 6 x 2 and what is g 1 of x is equal to x 1 plus 2 minus 2 g 2 of x is equal to 2 x 1 minus 2 x 3 x 2 minus 6. So, find out that, these of this at x 1 is equal to point 5. x 2 is equal to 1 point 5. And this will come, if you do this one, it will come 0 0 2 0. This second derivative of g 1 of g 1 of x star with respect to x, you will get a null matrix. Similarly, g x of g 2 of x you will get a null matrix. So, at this matrix is at this matrices is an ultimately get this one.

So, this is a positive definite matrix. So, 2 0 0 2 is a this matrix is positive definite matrix, by using Sylvester. So, our conclusion is, will get the optimum value means, minimum value of the function at this point. Corresponding mu, is what? The corresponding mu of that one is that mu you just saved. That mu got it positive quantity, if it recollects this one, mu 1 is equal to what? mu 2 is equal to what? We will see is a positive quantity. So, we will stop it here. Maybe we will discuss some of the issues related with the, what is call parameter variation, and objective functions, effect of the parameter variation and objective function.

Thank you.