Course name- Analog VLSI Design (108104193)
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Week- 2
Lecture- 5, Module-1

Welcome back, this is lecture 5. So, in the previous lecture, in lecture 4 we discussed mostly about how to take a non-linear circuit or rather how to take a non-linear element and do a linearization of that element right. Why do you want to do a linearization of the element? Because we saw that using the magic of Taylor series, we could break the incremental we could break the incremental voltages of the currents through the non-linear element into into linear and non-linear terms. And under the condition that the non-linear terms are are much much smaller than the linear term, we could neglect the higher order terms and we could essentially say that a whatever non-linear element we have to deal with under certain operating conditions we can linearize it. And what is the what is so holy about this method of linear linearization? It will the the beauty about the of this method is the fact that it will help us to do a linear circuit analysis. Because all our all our knowledge of handling circuits right, all our knowledge of handling circuits work efficiently as long as we can transform them into some sort of linear equations right.

So, and in the road to doing that what would we do? We took an example of an arbitrary non-linearity and we said that if the voltage across this is Vn the current through this is f of Vn right this is Vi this is RL. And what did we said? What were the steps? The step 1 was to find out the quiescent, the quiescent currents and voltages that is that is we do a numerical simulation or we do the numerical analysis or we do a graphical analysis. So, we find out the, find quiescent currents and voltages. And let us say after doing some numerical or graphical methods we could find that for a certain value of Vi the current through this network was I sub n and the voltage across this device was V sub n which means the voltage across RL is Vi minus V sub n which is nothing, but In times RL right.

So, we do a numerical analysis to to find these values right. So, do perform a numerical slash graphical analysis to find In and Vn ok. So, now that we have found what was the next thing? Next thing that we said was if, if my input was let us say Vi plus delta Vi, where delta Vi is a small increment right then I could in principle find out the relationship between the current that the incremental current and the incremental voltage let me use a different color here. So, let us say Vi plus delta Vi then obviously, there will be an incremental voltage across a non-linear element and the current that was In to start off will now change to In plus delta In and now the whole whole focus shifts to finding out is delta V finding out delta Vn and delta In given we have delta Vi right. So, now the focus shifts there and in order to in order to figure figure it out what should we do? Let me move this part here in order to do that in order to find out the relationship of the incremental quantities what we do? We replace we replace the non-linear element with a resistor what is the value of the resistor?

The value of the resistor is, 1 / del f / del V around the operating point Vn what happens to the resistor RL? This RL remains RL what happens to the input? Note that since we are since we are concerned about concerned about the delta Vi the input effectively becomes delta Vi correct.

So, this current that we will have after solving solving for this network will be delta In and this resist then the voltage drop that I will get across the linearized version of the non-linear element will be delta Vn right. So, this essentially was the crux of the entire lecture 4 right ok. So, now now that we have delta Vn and delta In right. So, let me write down this step also. So, this step was the second step the incremental equivalent right.

So, this network here is the incremental network right. So, now that we have got now that we have gotten the quiescent and the incremental. So, what is the final step? The final step is to find out the total final step is to find out the total In V right not only the increment. So, essentially what are we after? We are after if the input is Vi plus delta Vi and the and the network consists of an actual non-linear element right right. This is Vn plus delta Vn right and this current is In plus delta In right.

So, what is the what is the final solution? The final solution will be to find total voltages and currents for input Vi plus delta Vi we need to add up the responses from the quiescent and the incremental networks right. So, essentially we have found Vn and In right from operating conditions and we have found small delta Vn or rather delta Vn and delta In from the incremental conditions right. We have found capital Vn capital In from from this network right. We have found capital Vn capital In from this network and we have found delta Vn and delta In from this network right. So, once we have found these two what we need to do in order to find the total we simply need to add them up right.

I did not I missed a step here I did not evaluate delta In what is delta In? So, I know that delta In is delta Vi by RL plus RL plus this incremental resistance right. What is that incremental resistance? That is 1/ del f/ del V evaluated at Vn right.

$$\Delta I_N = \frac{\Delta V_i}{R_L + \dfrac{1}{\left(\partial f / \partial v_N\right)_{V_N}}}$$

So, once I know that this is del, delta In what is my total current? Total current is equal to In plus delta Vi/ RL plus 1/del f/ del Vn evaluated at Vn ok.

$$\text{Total current} = I_N + \frac{\Delta V_i}{R_L + \frac{1}{\left(\partial f / \partial v_n\right)_{V_N}}}$$

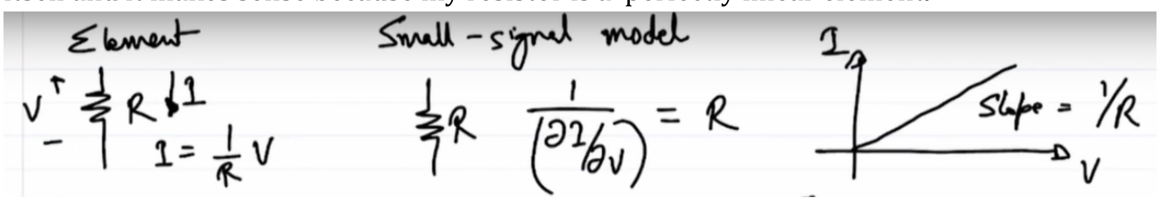
Fine fine ok great. So, again at the cost of repeating myself what did we do? What did we do in order to figure out the solution of this network? This becomes a 3 step process.

One find out the quiescent currents and voltages or the operating currents and the voltages using graphical or numerical methods. Step 2 linearize the circuit right sketch the incremental network. What is the incremental network? The incremental network is nothing, but the linearized version of the non-linear is a same network where the non-linear element has been has been replaced with their linearized version and and all the sources have been replaced with their incremental equivalent and then you then you find the the incremental currents and the voltages from the incremental network right and it is easy to find because this incremental network is fully linear. So, essentially now you are now you end up having 2 quantities one is the quiescent or the operating currents and voltages another is the incremental current and voltages in order to find out the total you just simply add them up and what is so great about this? The great thing about it this is that now if Vi if delta Vi changes right if delta Vi changes if delta Vi is let us say 1 millivolt I just simply replace 1 millivolt instead of delta Vi if delta Vi becomes 10 millivolts right I replace 10 millivolts instead of delta Vi if delta Vi is 100 millivolts I replace 100 millivolts in place of delta Vi and I do the incremental analysis again. So, you have to do the quiescent analysis once and at the cost of doing the cost quiescent analysis once you are able to do all the incremental analysis in a linearized fashion right.

So, that is the power and the strength of this of this method but note that you have to be slightly careful you will not you cannot do this for any values of delta Vi why? Because this incremental this linearized version of incremental equivalent is only valid when you can neglect the higher order terms and if delta Vi keeps on increasing the higher order terms for the Taylor series will be non negligible. So, in a sense there is a there is a maximum limit of delta Vi right beyond which this incremental linearized models are valid right and that and in this course will will not reach that limit, but it is, it is it is always instructive to keep that at the back of your mind and there is one more one more jargon that is used to that is used to refer to this incremental linearized model right. So, that is called a small signal model right and what by definition what is small signal. So, when, when the higher order terms of a Taylor series or not a of the Taylor series of the non-linear element can be neglected with respect to the linear term right. We call the linearized model a small signal equivalent or small signal model right.

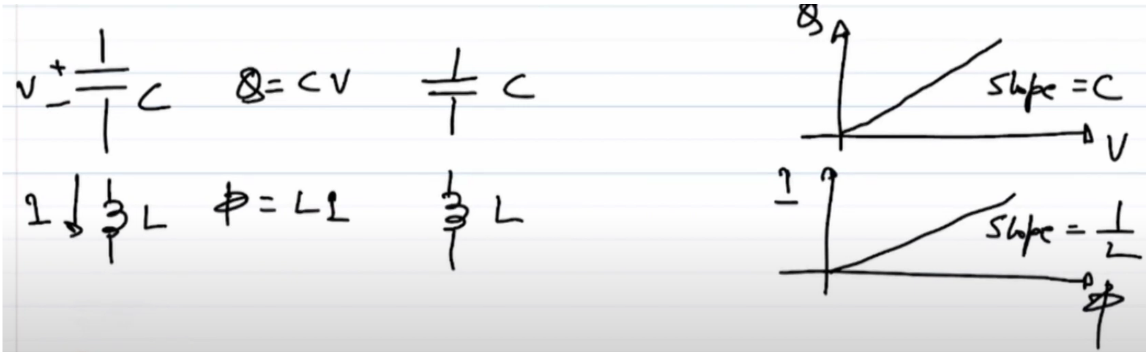
So, from here on in we will use this term small signal linear or linearized incremental model interchangeably ok. So, if you if you open up textbooks you will see that this term of small signal model is used ubiquitously. So, we will also refer to this as a small signal model. Now, before moving forward it is makes sense to understand what is the small signal equivalent of all the elements that we know right. For example, for example a resistor right say we have this resistor.

So, what is the small signal equivalent of a resistor? So, let me call write this as this is the element and this is the small signal model right. So, what is the small signal model of a resistor? How will you how will you get the small signal model of a resistor? Note that how did we get the small signal model in the first place? We needed to characterize the element. So, how do you characterize the resistor? A resistor is characterized by if I have a voltage V across this element and you have a current I that is flowing in the small signal the relationship between I and V right the relationship between I and V is I is equal to 1 over R times V. So, what will be the small signal equivalent of this? So, note that the I V characteristics of a resistor is a straight line and what is the small signal equivalent? The small signal equivalent whatever this element might be this is equal to 1/ del I/ del V right which is nothing, but R correct. So, the small signal equivalent of a resistor is the resistor itself and it makes sense because my resistor is a perfectly linear element.



What about a capacitor? What about a capacitor? So, when you see a circuit element that is a capacitor what is the governing relationship between of this capacitor? Primary governing relationship is between the charge stored across the in the plates of the capacitor and the voltage across the capacitor right. So, the governing relationship is Q equal to C times V right. So, like the resistor the governing relationship for a capacitor is is again a straight line right. So, what is the slope of this straight line? So, this slope is equal to 1 over R correct. What is the slope of this straight line? The slope of this straight line is C right.

So, the slope is C anywhere on this curve. So, essentially this capacitor gets replaced by a capacitor with a value C. Similarly, if you have an inductor right, we have the current I going through it. What is the governing relationship of an inductor? The governing relationship is phi is equal to L i where phi is the flux, i is the current that is going through it. So, essentially if I sketch i here and phi flux here again I get a straight line what is the slope? Slope is 1 over L right.



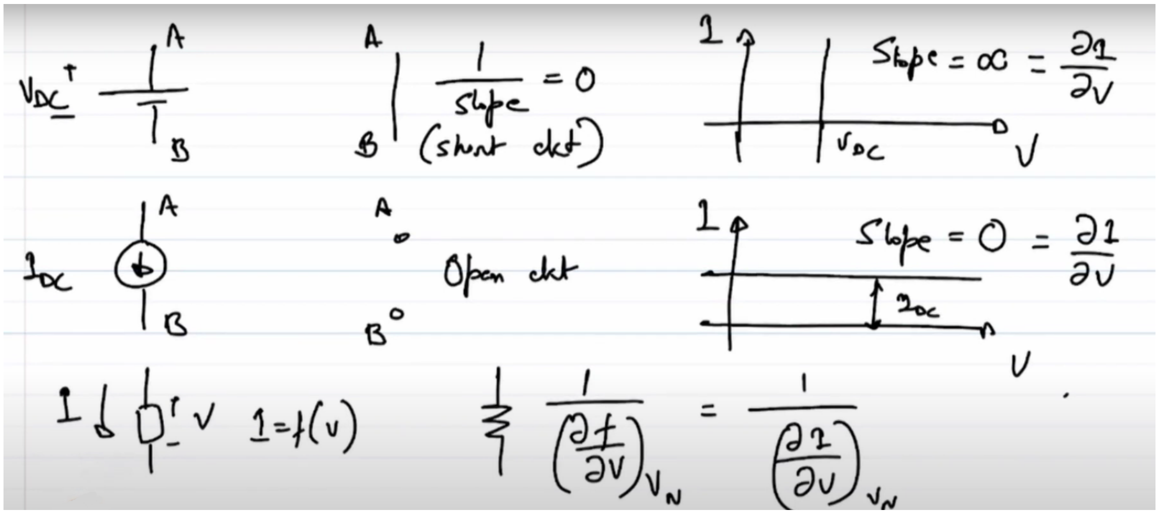
So, again this is a linear relationship. So, inductor remains an inductor no change correct. What about a voltage source? What about a constant voltage source? What is its linearized equivalent? What is this in small signal equivalent? So, again let us go back to our I V characteristics right. What is the let me call V DC. So, what is the I V characteristics of a voltage source? So, voltage remains constant regardless of the current.

So, this is a straight line perpendicular to the x axis. So, what is the slope? Slope equal to infinity. So, if I have to replace this with this incremental equivalent, what do I do? I replace it with an element which is 1 over the slope. So, what is this element? What is this element? If slope is infinity this is equal to 0 which means the impedance given the impedance given by this element is 0 which means this is a this is a short circuit right. So, this is a short circuit correct.

What is it if I have a constant current source? It is a I DC. So, again let us go back to our I V characteristics correct. So, this is V DC here. We go back to our I V characteristics. What is the I V characteristics of a of a constant current source? Again this is a flat line parallel to the V axis.

What is the slope? Slope equal to 0 which means 1 over slope is equal to infinity. So, you have to replace this I DC with an element right whose del I/ del V is what are the del V /del I? Yes, infinity right. So, let me a slope. So, this is nothing, but del I/ del V right.

So, this is again del I/del V. So, since del since del I/ del V is 0 in other words del V/ del I is infinity which means incremental resistance is infinity. So, you replace it with the open circuit right. So, this is this a this will be becomes a this will be see this is open circuit. And in general, in general if we have a non-linear element which has a current voltage relationship of I equal to f of V right I equal to f of V. You have to replace this non-linear element which is incremental equivalent which I mean if all the terms are real right and all the derivatives are real.



Then we will replace this non-linear equivalent with a register of value 1/ del f /del V V right around some operating point Vn or I can simply write in the form of I del I/ del V around some operating point Vn ok. Does it make sense?