## Course name- Analog VLSI Design (108104193)

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**Week- 2**
**Lecture- 4**

Welcome back, this is lecture 4. So, up until now we have discussed the necessity of using non-linearity in our design in order to get power amplification right. And we in the last lecture we saw that it is not only sufficient to have an extra power source in order to get power amplification, we also need we also need some sort of non-linearity ok. So, now when you get to dealing with non-linear devices right, we need to first develop a framework in order to how to handle non-linearity. Because I mean we are all used to handling sets of linear equations, we are quite confident in handling sets of linear equations from our high school algebra right. So, we would like to see what tools we need or what tools do we have in our mathematical database in order to handle non-linear equations.

And if at all is it necessary to use all the tools or can we do some mathematical jugglery in order to make our life simpler right. So, let us begin. So, let us assume that we have we have some non-linear element, let us say we assume we have some non-linear element and the characteristics of this non-linear element is I equal to some F of V,

$$I_n = f(V_n)$$

where I is the current through the non-linear element and V is the voltage across it right. So, let me be a bit more specific and I call this In, let me call this Vn, this is In, this is Vn ok.

And let us say we have used this non-linear element in our circuit right. Let us say I have a voltage source, let me call this Vin and I have used this non-linear element in our circuit and let us assume this is my resistance RL and this is the non-linear element in question. Ok. So, what tools do we have in our repository in order to solve in order to solve for currents and voltages in a network which has a non-linear element right. So, what we need? We need in order to figure out what will be the current in this loop, what will be the current I, what we need? We need essentially two things right.

We need to know how to apply KCL or KVL right. We need to go around the loop and impose the constraints of KCL or KVL and but is that sufficient? That is not sufficient because we need one more thing, we need I V characteristics, we need I V characteristics of each element in the loop right. So, essentially if I tell you the I V characteristics of each element in the loop then you can use that to write your KCL KVL equations and find out the find out the current right ok. So, what is the I V characteristics of a resistance? So, let us say let us start with the resistance RL. So, what will be the I V characteristics of R L? Let us call this voltage VL, let us call this current IL.

When we are trying to plot an I V characteristics it is customary to plot the voltage in the in the in the X axis and the current in the Y axis right. So, in case in case of a resistance we know it is simple Ohm's law. So, it will be a straight line going through the origin. What will be the slope of the origin? Rather what will be the slope of this line? The slope of this line will be constant and will be equal to 1 over R L right. So, the slope is the resistance of the line.

What will be the I V characteristics of the voltage source? For since the source typically pushes current out let us mark the I in this direction. So, this is Vin this is Iin. What will be the what will be the I V cap? Note that a voltage source by definition does not have any does not allow any change in voltage regardless of whatever current that flows through it which means the voltage will be constant which means your I V cap for the voltage source will be a straight line a vertical line ok fine. So, what else what else we need? We have another element we have another element another non-linear element. And we are calling the, calling this element Vin right and calling this current Iin.

So, we need the I V cap of this device in order to write my KCL or KVL equations right. So, this needs to be given. So, I have not given you anything in this in this particular problem. So, let us assume let us assume some some sort of curve. So, let us assume the curve is something like this ok.

So, now that we now that we let us say we have this we have this network. Now, that we have this network how do you how do you think we should go about solving this with the first thing that we should do it is we should write the we should write the loop equation and let us it is a single loop right. So, we can write the KVL. So, what will be the KVL? KVL will be V in will be equal to V in plus I R L correct. And how will you go about finding out finding out this current I? Now note that note that I have I have the I V characteristics of the non-linear element right and since everything is in series all the elements are in series the currents in the loop or currents through all the elements are identical correct.

So, essentially this I is equal to I n which means that this I n is equal to V in minus V n by R L right.  So, let me split it up and write it in this format V in by R L minus V n by R L ok.  So, what is this equation telling us?

$$V_{in} = V_n + I R_L$$

$$I_n = \frac{V_{in}}{R_L} - \frac{V_n}{R_L}$$

If we stare at this equation and also stare at the I V characteristics of of the device of the non-linear device what do I see?  I see that this device is a plot of I n and V n and this equation is also a plot is also  expressing a relation between I n and V n when the device is being used in the network  right.  So, when the device is being used in the network the governing equation is I n equal to V in  over R L minus V n by R L.  So, in order to figure out what will be the what will be the I what will be the actual  values of I n and V n right under the conditions of the network what should we do?  We should simply try to equate the currents in the device we should simply try to equate  the current through the non-linear device to try to equate this current I n right with  with the current that is flowing through the flowing through the resistor correct.

What is the current that is flowing through the resistor?  The current that is flowing through the resistor is V in minus V n by R L and there are several  ways of going about doing it right right there are several ways of going about doing this  matching right.  One of the popular ways is graphical and what do we do?  We simply we simply sketch this equation right we simply superimpose this equation on the  You can start off with let us say 1 volt and then it can go to 1.1 volt, it can go to 2  volt, it can go to 0.5 volt and so on and so forth.  So, does that mean does that mean we will have to go and reevaluate this plot again  and again and again over and over again every time I change the value of the input.

What I am essentially saying is that what if so this plot that we have seen right.  So, this plot that we have seen here is for a certain value of Vin correct.  So, because this curve is (Vin -Vn)/RL right.  So, if Vin changes right if Vin increases let us say Vin increases what will this where  will this plot go if Vin increases this plot will move slightly depending upon how much  it has increased this plot will move upwards right and we will have a new solution a different  Vin will be slightly higher let us call it Vin + delta Vin. Similarly, we will have a slightly different Iin and let us call this Iin + delta Iin ok.

$$\Delta I_N = \frac{\Delta V_{in}}{R} - \frac{\Delta V_N}{R}$$

$$\Delta V_{in} = \Delta V_N + \Delta I_N R$$

So, does that mean that like does that mean that for this new solution we will have to redo the entire work again or is there a smarter way of going about this right. As it turns out there is one and that is what we will be concentrating next. So, what will we do we simply recognize we simply recognize the fact that we simply recognize the fact that this current I in correct this current I in is equal to V in by R minus V in by R ok. And let us say let us say I have increased V in to some value of V in plus delta V in correct. So, let me let me mark it here this is for the case when V in is increased to V in plus delta V in and if I increase if I increase this voltage right what will happen what will happen to the current the current will increase and since current increases Vin will also increase right.

So, because that is how the Iin - Vin plot is. So, this let me mark let me also call this voltage in the new voltage of new voltage across the non-linear device Vin plus delta Vin right. So, this will be the new plot correct. So, in other words what I am saying is if I have increased this voltage from Vin -> Vin + delta Vin, this current will increase from iin -> Iin + delta, Iin similarly this voltage drop across this device will increase from V in to V in plus delta V in right that is what being is being expressed graphical. Similarly, if I express the same thing similarly if we write our new equations what will we see we will see I in plus delta I in will be equal to V in plus delta V in over R minus V in plus delta V in over R, correct.

Now, if we subtract the top equation from the bottom equation right why are we interested in subtracting we are interested in subtracting because you see that there are lots of common terms between the top equation and the bottom equation and we are essentially trying to figure out what we are essentially trying to figure out what will be this delta I in and delta V in R simply because if we can get that information of delta I in and delta V in we will be able to know the new solution right. So, if we subtract the first from equation 1 from 2 so this is 1 is 2, so 2 minus 1 what will what will we get if we do that we will get delta I in will be equal to delta V in over R minus delta V in over R which effectively we can write it as delta V in is equal to delta V in plus delta I in times R ok. So, let us run out of page let us go to the next page. So, again what are we saying we are saying the new equation our new equation relating the incremental change is nothing, but the change in input that is delta V in is related to is related to the changes delta V in that is the volt that the change of voltage across a non-linear network delta I in times R or R L right R L ok. So, now what do you think is the is the next step, now you might turn around and say that hey it looks like in order to figure this out right in order to solve this in order to solve this equation I still need to know what is the relationship between delta V in and delta I in right.

We still need the relation between delta V in and delta I in correct. So, are not we back to the same problem we were having these problems earlier we had to resort to graphical solutions simply because we did not have a linear relationship between between I and V of the non-linear device right. It was because we did not have a non-linear relationship between I and V of the non-linear device we had to resort to a graphical solution. Had this had this new device if it had a linear equation if it had a linear I V characteristics You can start off with let us say 1 volt and then it can go to 1.1 volt, it can go to 2 volt, it can go to 0.

5 volt and so on and so forth. So, does that mean does that mean we will have to go and reevaluate this plot again and again and again over and over again every time I change the value of the input. What I am essentially saying is that what if so this plot that we have seen right. So, this plot that we have seen here is for a certain value of Vin correct. So, because this curve is (Vin -Vn)/RL right.

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And let us say let us say I have increased Vin to some value of Vin plus delta Vin correct. So, let me let me mark it here this is for the case when Vin is increased to Vin plus delta Vin and if I increase if I increase this voltage right what will happen what will happen to the current the current will increase and since current increases Vin will also increase right. So, because that is how the Iin - Vin plot is. So, this let me mark let me also call this voltage in the new voltage of new voltage across the non-linear device Vin plus delta Vin right. So, this will be the new plot correct.

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So, again what are we saying we are saying the new equation our new equation relating the incremental change is nothing, but the change in input that is delta Vin is related to is related to the changes delta Vin that is the volt that the change of voltage across a non-linear network delta I in times R or R L right R L ok. So, now what do you think is the is the next step, now you might turn around and say that hey it looks like in order to figure this out right in order to solve this in order to solve this equation I still need to know what is the relationship between delta Vin and delta I in right. We still need the relation between delta Vin and delta I in correct. So, are not we back to the same problem we were having these problems earlier we had to resort to graphical solutions simply because we did not have a linear relationship between between I and V of the non-linear device right. It was because we did not have a non-linear relationship between I and V of the non-linear device we had to resort to a graphical solution.

Had this had this new device if it had a linear equation if it had a linear I V characteristics right then we could have as well written two separate linear equations and found a solution and we are good at that we have been doing it from high school right to to system of linear equations we know how to solve. But looks like here after doing all this increment subtraction and all we have ended up with the same problem where we still need the relationship between delta V in and delta I in right. While it might seem while it might seem that we are back to square one let me show you why we are not right. So, here comes the magic of mathematics. So, we know that we know that this voltage Vn right this voltage Vn and the current I n through this device is non-linear and let us say it is some non-linear function f and we express current we express the current In equal to f of Vn where f of Vn is some non-linear function where f is a some non-linear function right.

Now given that given that I know we know for sure we have found the solution for one particular case of In and Vn we know that In is equal to f of Vn this solution has been found iteratively or graphically or using numerical analysis. In whatever ways we could we found some definitive solution for a particular point of the I-V characteristics and we have found In equal to f of Vn right this is a known value this is a number right fine. So, now what we are saying is if Vn changes if Vn changes for some reason if Vn changes to Vn plus delta Vn and In changes to In plus delta In is there a relationship that I can exploit. So, if this is the case what is my new relationship my new relationship is In plus delta In is equal to f of Vn plus delta Vn correct. So, now you see that what is this term telling us, what is this telling us this is essentially telling us we again plot the I-V characteristics over non-linear device this plot is essentially telling us or rather this expression is telling us if we know a solution for a particular In Vn can we know can we predict a solution in the neighborhood of this existing solution.

So, essentially we are saying if we increase Vn by some delta Vn we know that the current will increase we know that this current will increase by delta In right. So, is it possible to predict is it possible to predict that change right and as it turns out it is possible and who is who will help us Taylor series will help us right. So, how what how should I expand how should I expand this function we already know the function the value of the function exactly at In comma Vn. So, let us expand it around that function In comma V n. So, this will be f of Vn plus the slope of the curve plus the slope of the curve around the around this of around this solution In Vn right.

What is the slope the slope of the curve is L f del V n multiplied by the by the change multiplied by the change that is del V n correct plus I am running out of space. So, let me move it to the left plus del Vn squared by factorial 2 del 2 f del V n squared plus dot dot dot ok. So, let us let us go to the next page and write it in a cleaner fashion. So, what are we saying we are essentially saying that my new In that is In plus delta In is f of Vn plus del Vn f del Vn at around that what is the slope the slope is around the operating point Vn plus higher order terms. Let me write at least one of the higher order terms and then approximate ok.

$$\Delta I_N = \Delta V_N \left.\frac{\partial f}{\partial V_n}\right|_{V_N} + \text{higher order terms.}$$

And what we what we know we already know that In is equal to f of V n. So, again subtracting the bottom equation from the top what we get we get delta In is equal to delta Vn times del f sorry I think I messed up the notations of it this should be small and did I mess it up here yes I messed it up here small because that the variable right plus dot dot dot that is higher derivative right ok. So, what is this telling us this is this is telling us it looks like it looks like if we know the slope of the curve or if we know that derivatives of the curve around the point at which I have already I already have the solution right I have already have the solution at Vn P capital N comma I capital N. On top of that if I also know the derivatives right if I know all the derivatives of the plot around that point then it is possible to find definitively a value of delta In if I know delta Vn right this is what this this equation is telling us, but this we can as engineers we can further take it one step forward right and the first step that will take is if delta V n is small enough such that we can neglect the higher order terms right this is a very important important assumption if delta V n is small enough such that we can neglect the higher order terms then what we get we get delta In is equal to delta Vn times delta del F del Vn around the operating point Vn ok. So this is crucial so if this is true if this is true let us go back and revisit our equation let us go back and revisit this equation how does this equation how does it help us to rewrite this equation what is it what is this telling us this is essentially telling us so let us let us go to the next page and write both the equations together.

So what is it what is it telling us is telling us that this plot this circuit that I had was related by the incremental quantities were related by this equation that is delta Vn was equal to delta Vn plus delta In times R L right, but the new equation that we got the new expression that we got is In delta In is equal to delta Vn times del F del Vn around the operating point Vn

$$\Delta V_{in} = \Delta V_N + \Delta I_n R_L$$

$$\Delta I_N = \Delta V_N \left.\frac{\partial f}{\partial v_n}\right|_{V_N}$$

$$\Rightarrow \quad \Delta V_N = \Delta I_N \Big/ \left(\partial f/\partial v_N\right)\Big|_{V_N}$$

$$\Delta V_{in} = \Delta I_N \left[ \frac{1}{\left(\partial f/\partial v_N\right)\big|_{V_N}} + R_L \right]$$

So what is so great about this the great thing about this is now we have got a linear relationship between the incremental quantities delta In and delta Vn Note that the linear relationship is between the incremental quantities provided we are able to neglect the higher order derivatives right again note that this is not the relationship between that exact In and exact Vn this is the relationship between delta In and delta Vn Now replace this delta Vn right so in other words I can express this delta Vn as delta In over del F del Vn around the operating point Vn So what do we get we get delta Vin is equal to delta In times 1 over del F del V n around the operating point Vn plus. So what is this what does this equation remind you of this equation is telling us that there is a linear relationship between the current right the current that is flowing through the loop right note that delta In is equal to delta I this is this is by default because it is the same loop what is this telling us this is telling us that the relationship between the incremental quantity right the relationship between the incremental quantity delta In and the incremental input delta Vin is linear right or in other words this is telling us delta Vin over delta In is nothing but 1 by this derivative plus R L ok. So what does this equation remind you of this is nothing but the equation on the right or the expression on the right is the summation of two quantities one is an explicit resistance and what is the other one the other one is another quantity which is not a physical resistance but what will be its unit it will also be a of a resistance right. So what is again what is this what is the pictorial description what is the pictorial description of this equation of this equation what is a pictorial description of this equation a pictorial description of this equation is nothing but I have a let us assume if I have a incremental voltage source delta Vin this is looks like this delta Vin if I apply an incremental voltage delta Vin the current that delta In will go through right is proportional to resistance RL and another resistance and the value of the resistance is nothing but 1 by del f/del Vn right. So the reciprocal of so the beauty of this beauty of this incremental analysis is a fact that you are able to replace you are able to replace the nonlinear element with an arbitrarily arbitrary function arbitrary I V characteristics with a resistor in the incremental domain right.

Again when can you do this when can you do this replacement you can do this replacement when and only when the incremental quantities are small enough so that you can neglect the higher order derivatives right. So this is so this is vital so let me write this down. So if the increment in voltages or currents through a nonlinear device is small enough such that the higher derivatives can be neglected with respect to the first derivative we can replace the nonlinear element with an element with a linear IV care or rather delta I delta V care correct. And in this case right in this case your what is the relation between delta IV care so the relationship between IV is delta I is del f / del V around the operating point times L V or in other words in this case we have been able to transform a nonlinear element into a incrementally linearized equivalent which follows incrementally linearized element which follows Ohm s law in the incremental sense right. So this follows Ohm s law in the incremental sense ok.

So quickly what did we do we essentially said that if I have a nonlinear element any arbitrary nonlinear element having an IV care some I equal to f of V and I have some increment I have some input Vin and I want to find out what is the current through this loop right through this loop for any arbitrary value of Vin what should I do step 1. Step 1 is find 1 In Vn numerically somehow let me just say somehow this somehow might be your favorite numerical approximation your favorite numerical method if every graphical method your something that you have got from simulator or something ok. What will be the next step next step will be to find any incremental change In current or voltage due to incremental change In Vin what should we do we should replace we should replace the nonlinear element with its incremental equivalent and what is the incremental equivalent so essentially In the incremental sense we are replacing the nonlinear equivalent which is with resistance and the value of the resistance is 1 over del f /del Vn. This resistance RL remains RL and the incremental sense is voltage gets replaced with the incremental voltage right ok. So, then the incremental current delta In this I call it In this is delta In the incremental current delta In will get will have a linear dependence on the incremental voltage delta V ok.

So, step 1 is somehow find that the In and the Vn somehow find it using numerical graphical methods once you have that you can find any increment In the any incremental effects on the network by simply replacing the nonlinear element that you have with their incremental equivalent and what is the incremental equivalent the incremental equivalent is nothing but 1 over the slope of the IV characteristics of the device right 1 over the slope of the IV characteristic of the device around the point at which you have evaluated the IV characteristics In the first place right. So, In other words if the device that we had had a characteristics like this and your initial In and Vn were here right what would have been the slope the slope would have been something like this right. So, this slope would have been this slope is 1 by del f /del Vn. However, if you had chosen your values In such a way that the solution would have been here correct. So, In the first case maybe the solution was here maybe In the first case the solution was here but let us say you chose your device In such a way that the solution was here correct.

So, then what would that slope be what would the new slope be the new slope would have been something like this. So, this would have been your new register of value 1 over del f/ del Vn around the new operating point let us say this is let us say VN1, IN1, VN2 right. So, In a sense this register that we use to replace the nonlinear element the value of the register depends upon where the evaluation of the original currents and the voltages were made right because ultimately we should be cognizant of the fact that the this is nothing but use of Taylor series right. We have used Taylor series In order to figure out the changes and by our knowledge of Taylor series we know that any incremental change right is also a function of from where the change has occurred ok. And that is why it is important to note from where the change has occurred and this pivot point from where the change is occurring we call it the operating points right.

So, this Vn, In are called operating point of the non-linear or non-linear element ok. This is also called quiescent point often abbreviated as Q point right. So, In the literature you will see that these terminologies are used interchangeably and we will also use these terminologies whenever the need might arise, sometimes we will call it the operating point, sometimes we might call it the Q point, but at the back of your mind you should understand that they all essentially mean the In the same thing ok. So, before we end today's lecture what is the take home point? The take home point is essentially the fact that it is possible to linearize any non-linear element under certain condition and what is that condition? The condition is that you can take the IV characteristics of the device and you can see what is the rate of change of the current with respect to the rate of change of voltage In the IV care. And if you see that if that rate of change is such that I can only approximate that change with and with a linearized model right, we can neglect the higher order terms of the Taylor series then it is possible to express the incremental change In current with the, it is possible to express relationship between the incremental changes of currents and voltages with the proportionally constant right, essentially making the non-linear device a linear element.

And now that we have a linear element we should be able to replace the non-linear device or the non-linear element with this linearized equivalent and once we have that we have come to our, we have essentially come back to a very linear circuit and we know we have all the tools to evaluate a set of linear equations that might arise from any linear network. .