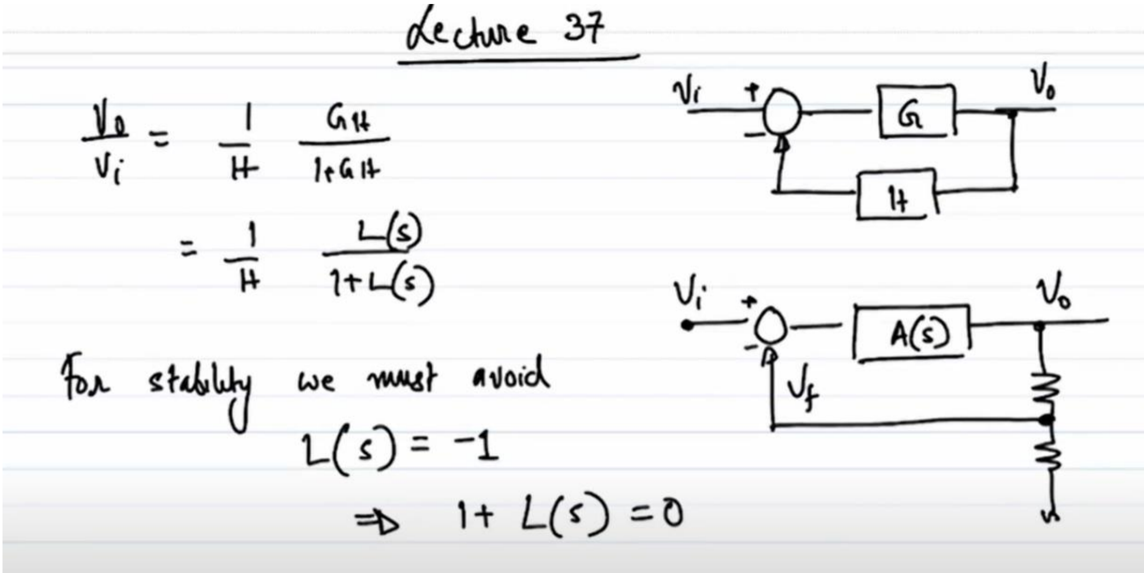
Course name- Analog VLSI Design (108104193)
Professor – Dr. Imon Mondal
Department – Electrical Engineering
Institute – Indian Institute of Technology Kanpur
Week- 12
Lecture- 37, module-01

Welcome back, this is lecture 37. So, in the previous lecture we were looking into how to ensure that our circuit is or negative feedback loop is always stable right. So, all the good things about negative feedbacks are it gives us it gives us precise gain, it gives us a gain which is invariant to ambient temperature, it gives us a gain which is also largely invariant to the load resistance RL all are good, but one problem one possible problem is it can lead to instability. And we saw an example of how it can lead to instability in the in the previous lecture and essentially your V0 over Vi is 1 over H times GH by 1 plus GH. If we are assuming the negative feedback loop is of this form right, this is also a critical assumption. If we assume that the negative feedback loop is of this form, then we can express then we can express V0 over Vi as 1 over h times GH by 1 plus GH and what is GH? GH is a loop gain.

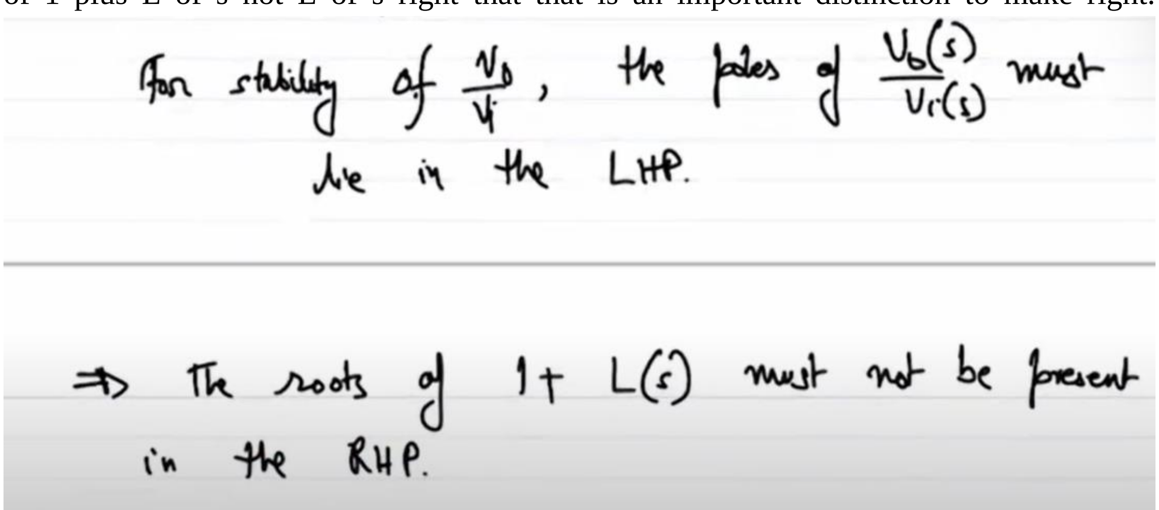
So, essentially this becomes L by 1 plus L. Now, having gone through the having gone through frequency dependent characteristics for so long, we know well that these L or loop gain are functions of S right, they are dependent on frequency and the examples that we saw where if we have a amplifier here, amplifier transfer function is obviously a function of frequency because for example, the common source amplifier, it has capacitance at various places and its frequency is and its gain is not same across frequency right. And more importantly and more importantly, we also saw that as the as the input changes right or rather as the frequency changes, there is some phase, some phase lag starts to build in between V0 and Vi right. At very low frequencies V0, the phase of V0 is almost equal to that of Vi, but as frequency increases the phase, the phase alignment between V0 and Vi also starts to differ and we saw in the previous lecture there is that can lead to a potential problem and what was the problem? If the feedback voltage Vf is right, if the phase of the feedback voltage Vf was exactly equal to 180 degree with respect to Vi and also if the feedback voltage was of equal amplitude of that of Vi, then we end up in a we end up building an oscillator right.

So instead of an amplifier, we end up building an oscillator, so that is not a good thing right. So, what was the condition that we wanted to avoid? We wanted to avoid, so for stability we must avoid L of j omega equal to minus 1 or in other words we need to avoid 1 plus L of j omega equal to 0 right. So, if we can avoid this or let us let me put in s domain because we know s equal to j omega we can put any time right. So, if we can avoid 1 plus L of s equal to 0, then we know that this guy will be this guy will be stable right, but we know one more thing right. We know one more thing from our control system basic control system understanding.



The basic control system understanding tells us that for stability we also must avoid for stability the rule the stability of the closed loop system of V0 over Vi right. For the stability of V0 over Vi, the poles of V0 over Vi must lie in the left half plane the left half s plane right ok. So, what does that imply? This implies that the roots of 1 plus L of s must be must not be present must not be present in the right half s plane right. These are the consequences of stability right. So, we know that poles have to be in the left half plane, but poles of what? Poles of 1 poles of V0 over Vi note that not poles of L of s right.

So, you have to ensure that the poles of V0 over Vi that is 1 that is the roots of 1 plus L of s have to be on the left half plane right. So, let me make a distinction here and say that roots of 1 plus L of s not L of s right that that is an important distinction to make right.



So, the roots of L of 1 plus L of s must be on the left half plane or must not be on the right half plane. So, why are we saying this? We are saying this because long long time back Professor Nyquist came up with a very neat way of figuring out if the roots are indeed in the right half plane or not right. So, one might say you might say that I can write out the polynomial L of s will be some polynomial higher order polynomial and then I can find out the roots.

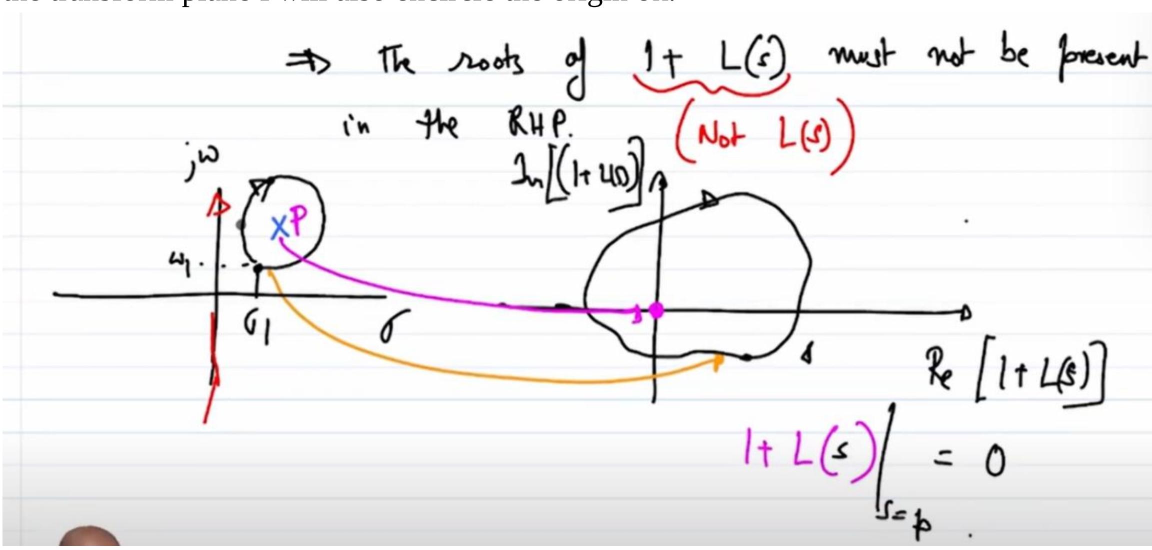
Yes in principle we can do that. However the problem is and the problem is if L of s becomes a polynomial of the order of 3 or higher right then you cannot have a closed form solution of the roots ok. So, since you cannot have a closed form solution of the roots you cannot develop an intuition as to as to what as to whether my circuit will be stable or not right. So, that is why we need to figure out we need to figure out whether the roots lie in the right half plane or not. We need not we are not particularly bothered about finding out where exactly the roots are we are trying to figure out whether the roots are on the right half plane or not right.

So, that is what that is what we will do we will we will let me take you through a quick refresher of Nyquist criteria from your undergraduate control system courses and it is essentially says the following. So, let us say I have this is the s plane that is sigma j omega right. So, what are we looking for? We are essentially looking for the location of the poles of L of s right. So, if let us say the poles of L of s exist in the right half plane right. So, if the poles of L of s exist in the right half plane let us say one pole exist right.

Then if I develop if I draw a contour if I draw a contour of or forget about the contour for the time being all it says is that. So, let us say we have a let us say a pole exist somewhere in the s plane and I evaluate the transfer function at for all values of s in s in a closed contour need not be a circle in a closed contour then and if I correspondingly plot if I correspondingly plot real of L of s or let us say real yeah. So, real of L of s and imaginary L of s right. If I plot this what will I get? So essentially what we are trying to say is that let us say I select some point here ok. I select some sigma 1 omega 1 point and I map this point on the L plane right on the L of s plane or let us say let us do one let us take a step back and say that we are trying to find out this is the real of 1 plus L of s this is imaginary of 1 plus L of s ok.

So, all we are saying is that if for every point in the in the s plane there will be a corresponding mapping point in the in the L plane right. So, essentially I will probably have a point somewhere here right. So, maybe so this maps to this point let us say this maps to this point let us say I take another point somewhere over here maybe this maps to this point and I map and take another point here maybe this maps to this point and so on and I keep on taking these points and I go in a in a counterclockwise direction right. So, what will end up happening? I will end up coming back to the to the same point right. I will I will essentially come back here right by moving in some contour in the 1 plus L plane I will move in some contour and I do not know what the contour is, but I know that it will be a closed loop I will come back.

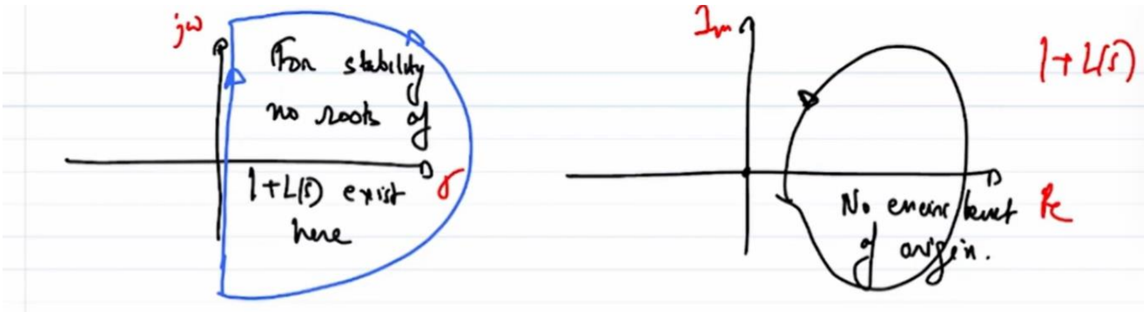
But what Nyquist showed that if in the j if in the in the s plane right if in the s plane the root of 1 plus L of s the root of that function right exist then what is going to happen then this closed contour this closed contour in the 1 plus L plane must encircle the origin. Or in other words what I what I am what should happen is that in this case the origin should be inside the contour right. The intuitive way of understanding this is what is the what is the what is let us say this is the this is the pole what is 1 plus L p what is the root of 1 plus L of s what is what is 1 plus L of s at s equal to p by definition by definition 1 plus L if p is supposed to be the root of 1 plus L of s then by definition at P1 plus L of s has to be 0 right. So, in other words this pole maps to this pole maps to the origin pole maps to the origin in the in the other plane ok in the transform plane ok. So, if I if I take a closed contour which in the s plane which which encircles which encircles the root in the s plane then in the in the transform plane I will also encircle the origin ok.



So, that is what that is what Nyquist told us right and then he further further extended this extended this finding and and said that in in stability cases in in while finding stability all we have to ensure that the roots of 1 plus L of s does not exist anywhere in this entire s plane right right. So, so we have to ensure that for stability for stability no roots of 1 plus L of s exist here right. We must ensure that this this actually happens. So, in order to ensure that what we need to ensure we need to ensure that if we take a closed contour in in the s plane right this is this is sigma this is j omega this is let us say 1 plus L of s. If we take this is real imaginary if we take a closed contour in the in the s plane right that like the way I have shown here then if s if a pole if if the roots of 1 plus L of s does not exist then I should not be encircling the origin correct.

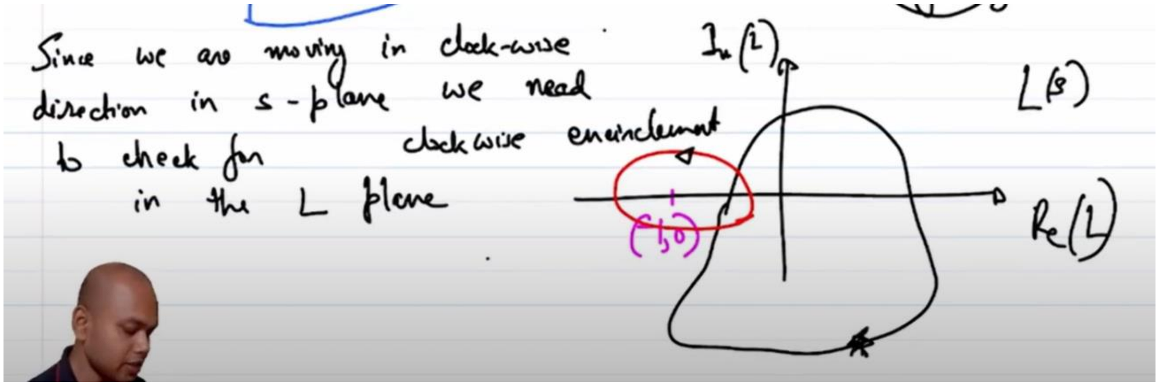
So, all it says that if so maybe I will be I will be doing something like this maybe I will be doing right. So it will do some some funny structure right we will have some funny contour. So, if no pole exists then no encirclement of origin ok. So this so by looking at the looking at the plots we should be able to figure out whether we actually have a we actually have a closed loop pole on the right hand side or not. If we have a closed loop pole on the right hand side then we are in we are in trouble right ok.

So, now then we further say that instead of plotting 1 plus L of s and trying to figure out whether we are encircling the origin or not let us plot L of s and try to figure out whether it



encircles minus 1 or not right. They are essentially the same thing. So, in other words what we are saying is instead of plotting 1 plus L of s let us plot L of s and say this is real of L imaginary of L and instead of focusing on the origin we will focus on the point (-1,0) right and we will do the same exercise and all we have to see is all we have to see ensure is that if the contour if the closed contour if the closed contour in the L domain does not encircle (-1,0) then we are then we are fine right that is all we are trying to ensure ok. So, it further says that if we are doing a further offshoot of this is to ensure that we are moving in the right direction by what I mean by that is if we move but since we are moving in anti-clockwise direction in s-plane we need to ensure we need to check for anti-clockwise encirclement in the L plane right. So, what I mean by that is that if let us say you find a contour which has a clockwise I think I made a mistake here yeah sorry this should be clockwise encirclement right.

So, this should be since we are moving in the clockwise direction in the s-plane right this is since we are moving in the clockwise we need to ensure that we need to check for clockwise encirclement in the L plane right. So, let us say for some transfer function you we moved in the anti-clockwise direction in the s-plane, but when we came back to L plane we found that it has a it has an encirclement, but in the anti-clockwise direction right. So, that is might we say is that that is not a problem right.



So, this is stable right this is still stable ok. So, we have to keep a track of whether the encirclement is a clockwise or anti-clockwise direction ok.

So, this might seem a bit confusing if we without the aid of an example. So, let us take an example and then try to figure out right ok. So, now when let us take an example of a first order system let us say L of s is equal to some K0 by 1 plus s by P1 ok. So, let us let us try to figure out what the Nyquist plot of this will be ok. So, when we are doing Nyquist plot what are you essentially doing? See the smart thing about Nyquist stability criteria is this.

So, when we are doing Nyquist plot we are starting from the origin and we are moving in the j omega direction right. We are starting from the origin we are moving in the j omega direction and we are going to all the way to infinity right we are going all the way to infinity. So, what happens to the transfer function in any realistic transfer function when we go omega to infinity because in any realistic transfer function the number of poles is always greater than number of 0s, but especially it is kind of a low pass transfer function eventually everything has to go to 0. So, if you go to infinite frequency the transfer function essentially goes to 0 right and then when you go to infinite frequency and for s equal to infinity essentially you do this encirclement business for this entire encirclement the transfer function remains 0 right and when you come back here and then you go from minus infinity to j to origin you essentially are travelling that 0 to j omega plot contour or 0 to j omega line in the reverse direction right. So, in a sense we all we have to do is to plot all we have to do is to plot the transfer function between 0 to omega equal to infinity right that is the cool thing about Nyquist criteria.

So, let us do that. So, if we do that what will happen. So, this is L of s this is L plane L of s plane ok and what is the point that we need to avoid we need to avoid this (-1,0) point this is real this is imaginary real of L imaginary of L ok. So, what is as we as we traverse the j omega axis so we are we have to put s equal to j omega right. So, your L of j omega is equal to a0 by 1 plus j omega by P1 right. So, mod of L is a0 by under root 1 plus omega square by P1 square and angle of L is minus tan inverse omega by P1 ok fine.

So, let us start. So, let us say we start from omega equal to 0 right if at omega equal to 0 what is what is mod of L mod of L is a0 if a0 is high enough it is completely real it will be somewhere somewhere in the right hand side in the L plane ok. And then if I increase omega a0 will decrease right. So, we have to it will probably come to the left hand side, but will it come to the left hand side this way or will it come this way will it will it go up or will it go it will nose dive. So, what from where can we get that information we can get that information from the angle right. So, what is the angle now angle is negative right.

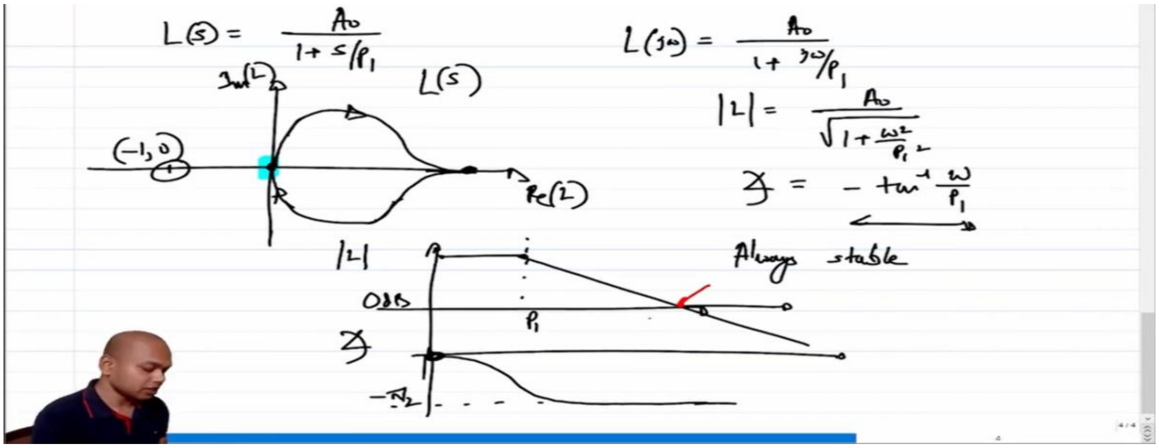
So, the angle is negative which means it is going it is going down right. So, so that is good. So, let us go down right. So, this will have this will go down and at omega tends to infinity what is going to happen at omega tends to infinity mod f l goes to 0 right. So, it will be it will go to the origin, but from what angle will it go to the origin right.

So, it can go to the origin from this side you can go to this origin from this side you can go to the origin from this side it can go to the origin from this side from which angle will it go. So, clearly if we put omega equal to infinity then the angle becomes minus 90 degree. So, which means that this plot will go will go to the origin using this contour. So, this is when right this is when we have moved from 0 to infinity right. So, when I move to infinity the transfer function is 0.

So, when I do this entire circlement right of the entire right hand side s plane what will the transfer function be it will still be 0. Then when I go from minus infinity to 0 along the j omega axis what will I see I will again see the same transfer function right, but it is mirror image right. Because now essentially for whatever omega we had we have we can put minus omega which see them same transfer function, but it is mirror image. So, the transfer function will something look like this right. So, regardless of the shape of the transfer function on my drawing scale all you have to appreciate from this is this guy is getting nowhere close to this critical (-1,0) point right.

So, since this is nowhere close to the (-1,0) point this is always stable right. So, this is always stable fine. We can understand this from the bode plot also right. So, bode plot of a first order system what will it look like mod of L. So, this is something like straight line till P1 and then it goes like this to minus infinity b and what is the angle of L? The angle of L will start from 0 and it will go to minus pi by 2.

Since this is never going close to minus pi right. So, we can as well say that this guy is always stable ok fine. So, by the way where is this (-1,0) point in the bode plot? So, when I say (-1,0) what do I essentially mean? It means a magnitude of loop gain is 1 and the angle of the loop gain is pi right. So, where is this where does the magnitude of the loop can go to 1? We have to if this is a 0 dB line.



So, the magnitude of the loop can goes to 1 where the loop can crosses the 0 dB line and that frequency at which it crosses is called unity gain frequency right or omega U G B right.

So, the frequency at which L of S or L of G omega mod of L of G omega goes to 1 is omega U G B right. So, or omega U. So, we will use omega U in this course ok fine. So, what we need to check? We need to check that whenever the gain becomes unity what is the phase? If the phase is pi then we have a problem pi or minus pi we have a problem if the when the gain is unity phase is not minus pi we do not have a problem right ok fine. So, clearly in a first order system that is not possible.

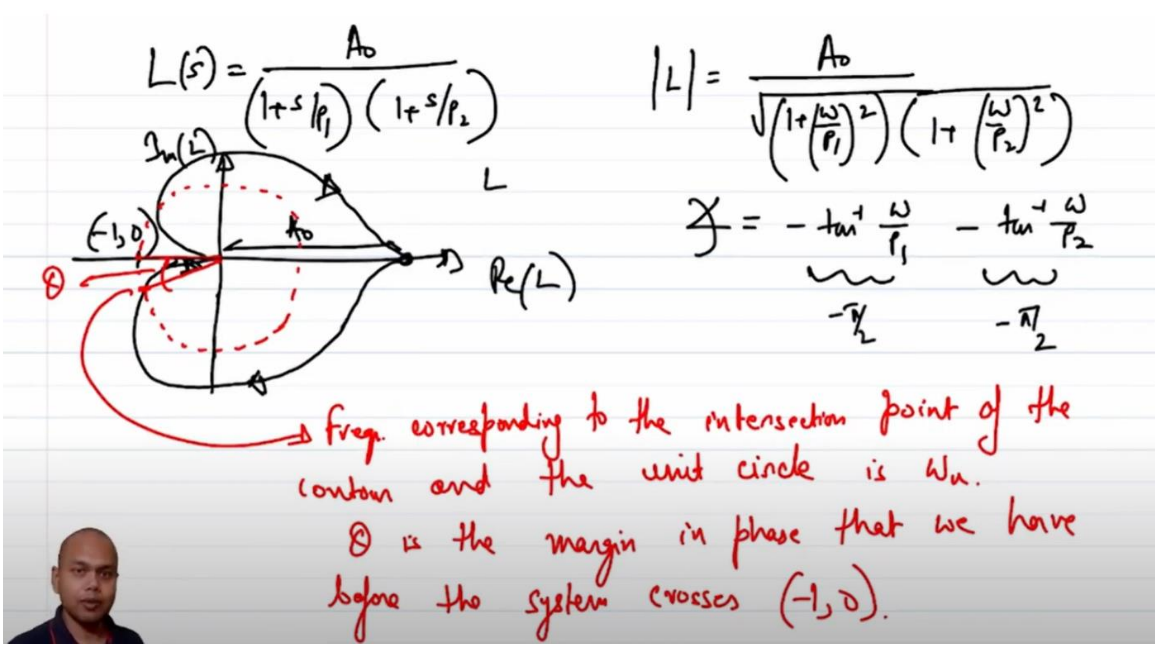
So, we are saying what about a second order system? Let us say L of S is a0 by 1 plus b 1 plus 1 plus b 2 1 plus S by b 2 right. So, let us plot the Nyquist plot once again. So, what is mod of L? Mod of L a0 by under root 1 plus omega pi P1 square times 1 plus omega pi P2 square right. And what is angle? Angle is minus tan inverse omega by P1 minus tan inverse omega pi ok.

So, again at omega equal to 0. So, this is L plane right this is L plane mod of this is real of L this is imaginary L. So, at omega equal to 0 where are we? We are far out on the on the real axis that is a0 right. So, this is a0 then as you increase frequency right I am coming closer to the origin, but from which angle I am again going down right. But now at very high frequency right at omega equal to infinity I know that we will go to 0, but at what? Through what angle I will go to 0? I will reach the origin at omega equal to infinity you get minus pi by 2 from here and you get another minus pi by 2 from here. Essentially you have a total phase lack of minus pi right.

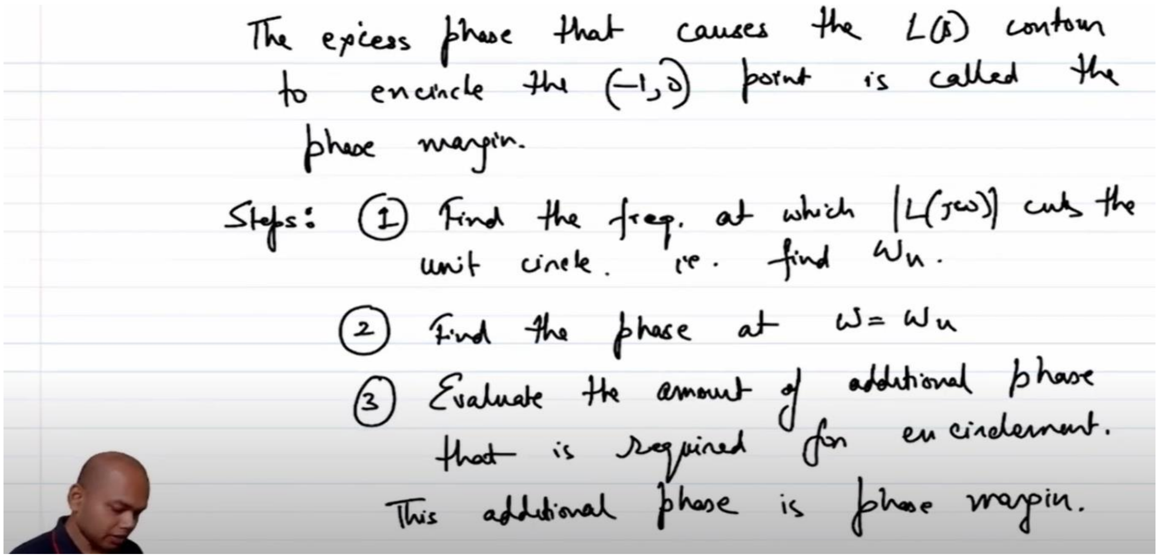
Essentially we will reach 0 from the third quadrant right. So, essentially this will do something like this. And if we have (-1,0) point here you can see that we are kind of getting closer to to trouble ok. So, we are still not encircling we are still not encircling, but we are getting closer and closer to trouble ok. So, now when we are getting closer to trouble we need to ensure that how far are we from the trouble ok.

So, how do we ensure how far are we from the trouble? So, as it turns out people have figured out a way of quantifying that exact thing and the quantification is as follows. So, ultimately we have to ensure that wherever we are away from this (-1,0) point and whenever we are cutting this (-1,0) point right this frequency the frequency corresponding to this (-1,0) point is omega. So, the frequency to the frequency corresponding to the intersection point of the contour and the unit circle right. So, when I say unit circle what do I mean? What I essentially did in this case is I drew a circle of magnitude of 1 centered at the origin right. So, whenever you are cutting this point what is the magnitude of this radius? This radius is 1.

So, wherever you cutting this point I know the loop gain is 1 right. So, frequency corresponding to the intersection point of the contour and the unit circle is omega u right that we can also get from Bode plot and the angle right and the angle that we are away from the (-1,0) point. So, this let us say this angle is theta. So, theta is the angle or theta is the margin in phase that we have before the system crosses (-1,0) right. In other words if we take this system and give it an additional phase lag of theta I will end up insert I end up cutting the (-1,0) point and if we give slightly more phase lag than theta I will end up encircling the (-1,0) point right.



So, then our system will become unstable right? So, this margin is often called is more popularly called phase margin right? So, this margin in phase or let me write it in a more formal way. The excess phase that causes the a log s contour to encircle the (-1,0) point is called the phase margin ok. So, ok so what is the what are the steps to find out the phase margin-right. So, the steps to find out phase margin is the steps to find out phase margin is number 1 find the frequency at which L of s or L of mod of L of j omega cuts the unit circle right ok.



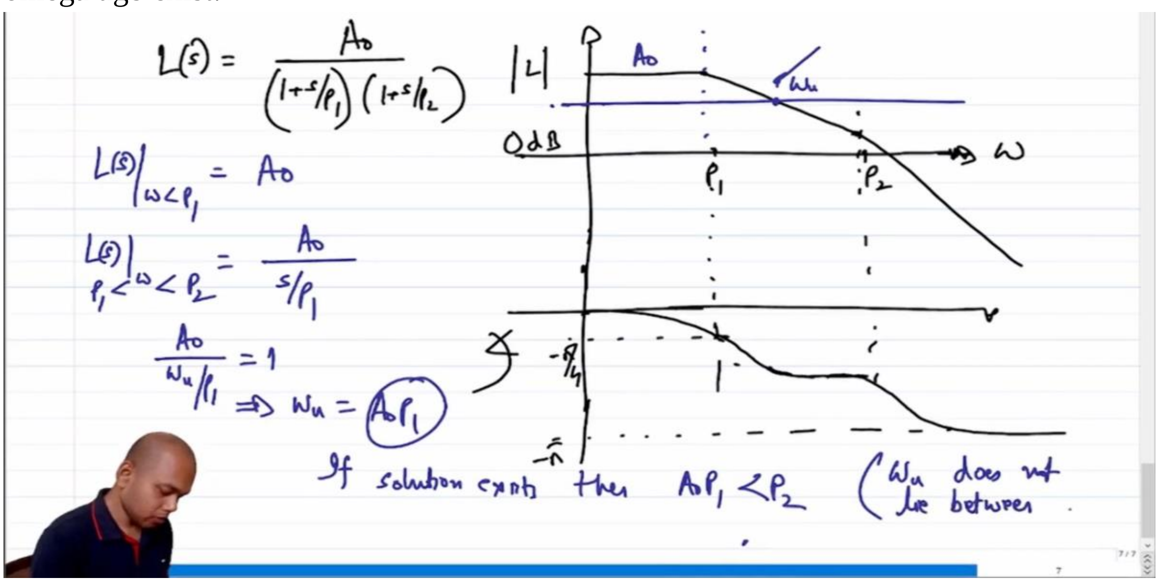
So, in other words I find omega u ok. SteP2 is find the phase at omega equal to omega u. Step 3 is evaluate the amount of excess phase of X or amount of additional phase that is required for encircle. This additional phase is phase margin ok. So, again I mean it is better shown with an example than writing it in words. So, let us do that and that will help us in our understanding right.

So, let us do the let us do the same second order transfer function in terms of that the bode plot in terms of bode plot. Let us say L of s is N odd by 1 plus s by P1 times 1 plus s by P2 ok. So, if I do if I sketch a bode plot so, what should I see mod of L I should see somewhere I should see I would know the locations of the poles in advance. So, let us say this is P1 let us say this is P2. So, I will sketch the transfer function right I will sketch the transfer function something.

So, this will be something like this ok and the phase of this will also be also to start from 0 will go like this to minus pi by 4. Then 4 log 2 minus pi by 2 then and this will go this will go like this ok. So, what is the steP1? The steP1 is to find out omega u right. So, in other words the steP1 is to find out where this L of s plot cuts the 0 dB line right. So, how should I find that out? So, this seems to be a piecewise linear plot.

So, how should I find that out? In order to find in any piecewise linear plot we have to take a piecewise we have to find out solutions in each of the each of the line segments you know in each of the possible possible segments and see go back and see whether the solution makes sense or not right. So, what is the equation what is the equation of L of s when when omega is less than P1 right. So, L of s for omega less than P1 is equal to a0 right because this is what this is a0. So, if a0 is greater than 1 which obviously it is because we are making an amplifier. So, clearly your omega u or the unity gain frequency does not reside below P1 right.

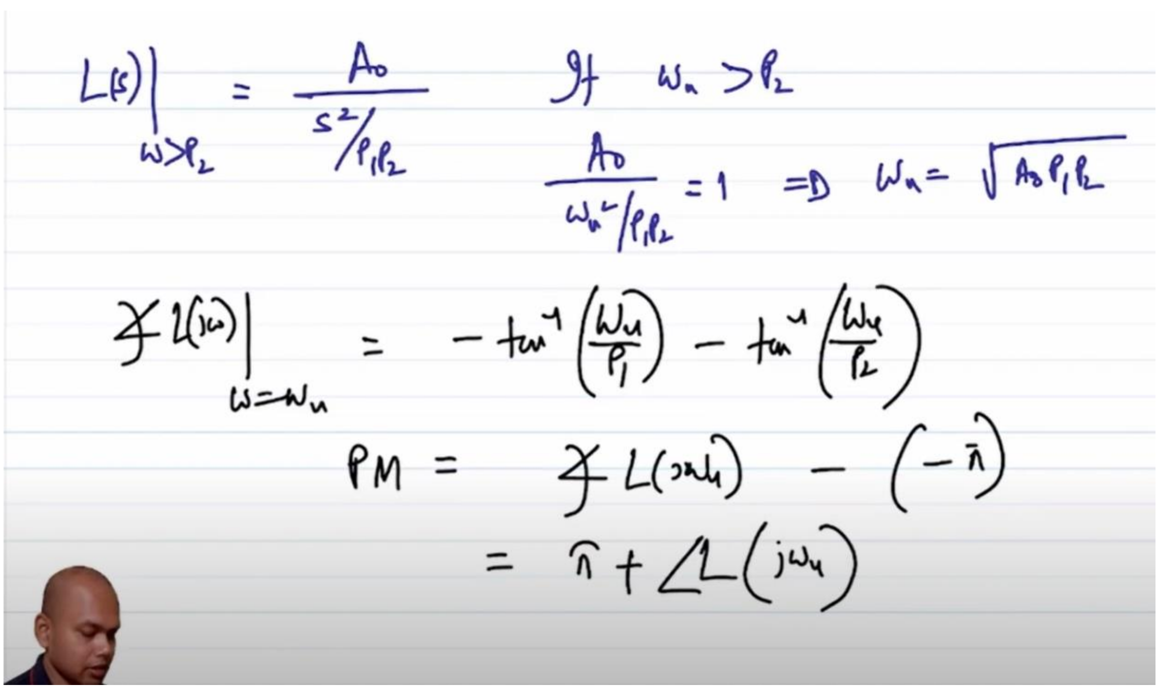
So, that is good. So, what is the what is the equation of L of s when omega is greater than P1, but less than P2. This will be a0 by s by P1. So, if L of if the unity gain frequency lies in that somewhere here right if the unity gain frequency lies here let us say if this is omega u g b if this is omega u this point right then what should I see I should be able to equate a0 by omega u by P1 to 1 and I should get a solution of this omega u which is equal to a0 P1 and this a0 P1 has to be. So, if solution exist then a0 P1 will end up becoming less than P2 right. If the solution does not exist then a0 P1 will end up more than P2 which means my initial assumption will not be correct right and if this is not correct then obviously the omega ugb exist.



So, this means that omega u does not lie between P1 and P2 right and if this is not true then obviously you have to evaluate L of s in the third part third segment. So, L of s for omega greater than P2 is a0 by s square by P1 P2 and if omega ugb lies here if omega u is greater than P2 then the solution will be a0 by omega u square by P1 P2 to P1 which means omega u will be under root of under root of a0 P1 P2 right. So, this way we can find out omega u right. So, let us say we found out omega u and let us say the omega u we found out is somewhere here ok. So, what should we see then we should see the phase we should see the phase associated with omega u right phase associated with the transfer function at omega u.

So, step is to find phase. So, angle of L of s at s equal to or at angle of L of j omega at omega equal to omega u which is minus tan inverse of omega u by P1 minus tan inverse omega u by P2 correct ok. So, maybe that is somewhere here. So, we get this phase this is this is this angle omega u use a different color. So, this is the angle at omega u. So, how much margin do I have? I have this much of margin before which right if I push this if I keep that if I keep the magnitude plot identical and I push the phase plot down by this much amount theta right.

So, then at omega u I will be getting a I will getting a phase of 180 degree also that is I will be cutting that (-1,0) point and that is unacceptable right. So, then how much margin do I have? So, the margin is the phase margin that I have the phase margin in this case is how much this theta the theta is this minus let me write it in a more generic term this will be angle at omega u minus of minus pi right because this is minus pi right and this is the angle. So, the angle minus of minus pi is your is your margin right. So, essentially the margin that we have is pi plus the angle at pi plus the angle of the loop end at the unity gain frequency right right. So, if we can ensure this then we are good else I mean if we can ensure that this phase margin is positive right if phase margin is positive system is stable right.



So, if phase margin is negative system is unstable right. So, note that again let me put in a caveat the analysis that we have done in this case is for this is for all pole systems right. So, strictly speaking this analysis is true for all pole systems. However, we will also be using this similar analysis even if we have even if we have 0s and we will see that we will see that going forward right. So, to summarize to summarize whatever what did we do till now we saw that a negative feedback system is susceptible to instability right. We saw that using intuition and the intuition was if whatever in the open loop case if the return if the return voltage is exactly or is exactly equal in magnitude to the input voltage, but 180 degree out of phase then our system can become unstable right .

(For all pole systems) If PM is +ve system is stable.
" " -ve " " unstable.

Then we related this condition using a Nyquist stability criteria and we said that we need to avoid encircling the (-1,0) point in the in the phasor plot of L of s or L of j omega and then we move to Bode plot right because Bode plot is much easier to plot using pen and paper than a Nyquist plot right. So, we move to Bode plot and we said that we will use a 3 step process to figure out if the system is stable or not and if it is stable how much margin do I have for stability and what is the criteria that we use we essentially said that we will find out we will find out the we find out the frequency at which we are getting unity n right right. So, this is like unity gain frequency you have to find out at what frequency we are getting unity gain unity gain right then we find out the phase of the transfer function at that unity gate frequency and then we figure out how far we are from minus 180 degree point or minus 180 degree phase at the unity gain frequency that difference is the margin and that margin is phase margin ok. So, it will become clearer as we do more examples and that is what we are going to do ok. Thank you.