

**Course name- Analog VLSI Design (108104193)**  
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**Lecture- 36, Module-1**

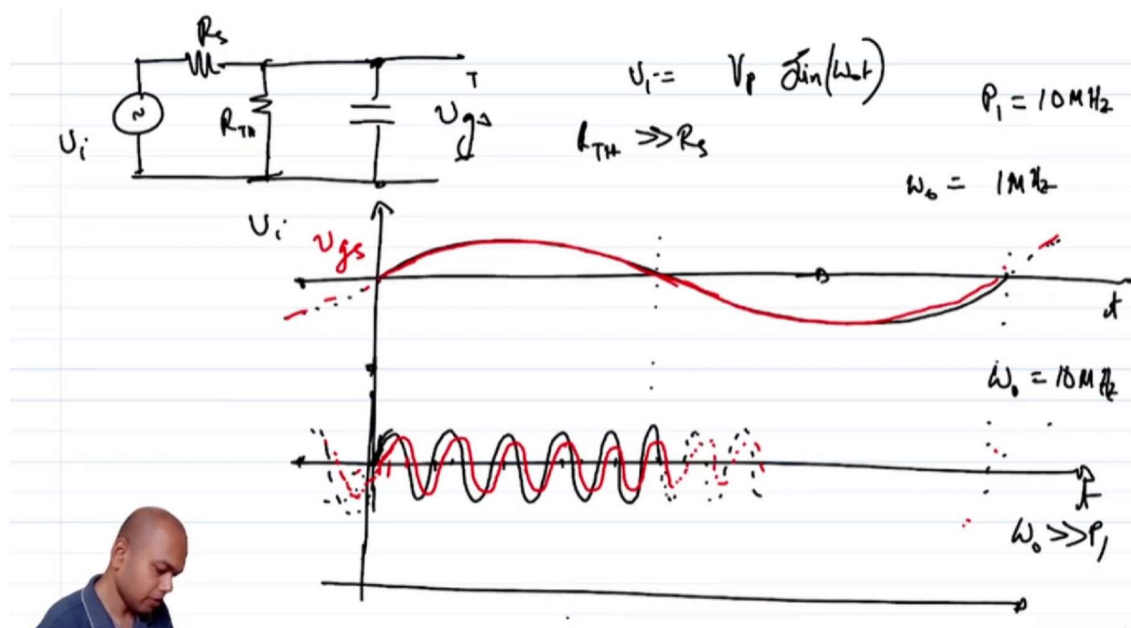
Welcome back, this is lecture 36. So, in the previous lecture we started seeing the frequency response of our common source amplifier and genesis of the frequency dependent characteristics of the common source amplifier was due to the inherent capacitance associated with the MOSFET. So, before yesterday's lecture or before the last lecture we kind of ignored the capacitance of the MOSFET, but in a design we cannot really ignore them right. So, since we cannot ignore them we need to understand the contours till which the common source amplifier will work with regards to the frequency with respect to if the input is frequency varying ok. So, then we saw that there are capacitance associated with the gate to source, capacitance is associated with gate to drain, drain to body and so on right. So, and then we sketched our small signal equivalent of the common source amplifier right and this is what we saw and we ignored the channel length modulation I mean you can as well put that back just that what will happen is the load resistance  $R_L$  will get slightly modified is  $R_L$  parallel and the  $R_{out}$  ok and we also have  $C_{gd}$  and then we said that I mean we will treat the let us figure out what is going to happen due to the capacitance  $C_{gs}$  and  $C_{db}$  and we will talk about  $C_{gd}$  in one of the later lectures right.

So, let us continue doing that and then we spend some time understanding the frequency response of  $V_{gs}$  with respect to  $V_i$  right. So, and we saw that  $V_{gs}$  in the Laplace domain can be represented as the DC gain or the gain at the or the transfer function at DC which was  $R_{th}$  by  $R_{th}$  plus  $R_s$  times  $1$  by  $1$  plus  $s$  by  $p$ ,  $p$   $1$  where  $p$   $1$  was  $1$  over the time constant associated with the capacitance  $C$   $1$  right. So, this is  $p$   $1$  tau  $1$  which is  $1$  over  $C_{gs}$  times  $R_s$  parallel  $h$  right. Then we saw the Bode approximation of the same.

So, essentially let me write the thing down once again  $V_{gs}$  over  $V_i$  was and the Bode approximation of this was if this is  $\omega$  right if this is  $\omega$  and y axis is our is in dB right this is  $20 \log$  of  $h$  e for  $V_i$  and this is in dB right. So, this is  $20 \log$  ok. So, at DC we will have this will be  $20 \log$  of  $R_{th}$  by  $R_s$  plus  $R_{th}$  and this is less than less than  $0$  dB right. I mean if this is  $0$  dB this would be this is since this is less than  $1$ . So, in dB scale this will be less this will be negative and this will be flat till some frequency  $p$   $1$  and then it will go down at a slope of minus  $20$  dB per decade right and this will be the magnitude plot and what will be the phase plot? The phase plot will also be something similar that

is it will start from it will start from 0 right.

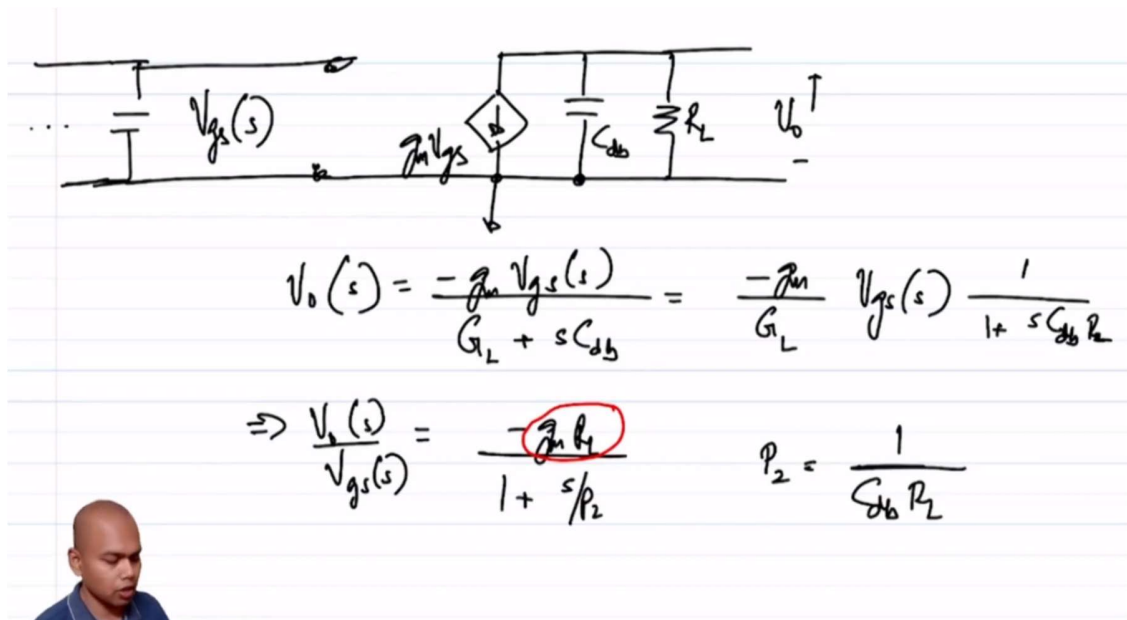
So, it will start from 0 at p 1 at pole p 1 it will get minus pi by 4 and at infinite frequency this will saturate at minus pi by 2 right ok. So, and the exact expression for the frequency characteristics is the following ok. So, if we spend a couple more minutes on this topic. So, if I tell you if I tell you that what will be what will be  $V_{gs}$  what will be  $V_{gs}$  if  $V_i$  let us say  $V_i$  is equal to  $V_p \sin(\omega t)$  right and I want to plot  $V_{gs}$  in time domain with respect to time right. So, let us say this is  $V$  this is the input.



So, let us say this is so let me draw one cycle right and this is going on forever right. So, it is a steady state response. So, we do not assume that the input has just started input has started some long time back ok. So, let us say this is  $V_i$  this is  $V_i$  and let us say  $\omega$  in this case  $\omega$  is equal to let us make some assumption let us say  $p_1$  is equal to 10 megahertz right and let us assume in this case  $\omega$  is 1 megahertz ok fine. So, what do you think will  $V_{gs}$  look like right.

So, I want to plot  $V_{gs}$ . So, clearly we are in the bode plot where are we we are somewhere here right we are somewhere this side we are low below  $p_1$  right we are 10 times below  $p_1$  which means if we have to do bode approximation what are we doing what are we essentially saying we are essentially saying that the output is the transfer function is equal to the DC characteristics right the  $V$  whatever  $V_{gs}$  will observe when we are at much lower frequency than the first pole is equivalent to the case where the pole did not even exist which means the capacitor did not exist. So, the capacitor did not exist

what would have been the amplitude of the output it will be the input amplitude times  $R_{th}$  by  $R_{th}$  plus  $R_s$  and let us let like further simpler and we say that  $R_{th}$  is much much greater than  $R_s$  which means in this case the input will be amplitude of the input will be exactly equal to the amplitude of that of the output all good but what about the phase what about the phase where are we in terms of phase what is the phase phase is angle is minus  $\tan^{-1} \omega C_{db} R_L$ . So, if you are much much at a lower frequency than  $p_1$  then we can always say that the phase is also minus  $\tan^{-1}$  a very small number which is much less than 1 which means you can approximate it as 0 right. So, which essentially means that the output will will essentially track the input all the way all the way right ok.



So, so this is for  $p_1$  equal to 10 megahertz sorry this is an omega naught equal to 1 megahertz and  $p_1$  equal to 10 megahertz right ok. So, let us do another one let us say omega naught is equal to 10 megahertz that is omega naught is exactly at the pole frequency or rather exactly yeah exactly at the pole frequency right. So, so let us sketch the same thing again. Let us re sketch right ok. So, the first let us draw the input first ok.

So, if omega naught is a 10 is 10 megahertz if I have to draw the same thing in same scale what will my input look like. So, thus this case on the top is omega naught equal to 1 megahertz the case that I am trying to draw is omega naught equal to 10 megahertz. So, in the same scale I will have 10 cycles of of the input right. So, if so essentially or in other words I should have 5 cycles within the one half cycle of the of the top plot right. So, so let me break up 1 2 3 4 5 ok.

So, let us see let us say which essentially this will mean that the amplitude will be the same 1 2 3 4 5 I guess right. So, approximately we have 5 cycles and this is sorry I think I made a mistake and drawing is pretty bad here. So, you have to break this guy up into 5 cycles 5 parts. So, 1 part this is 2 part 3 part 4 part that is 5 part. So, 1 2 3 4 5 right still does not look good, but you get the idea ok.

So, if this is the input what will the output what will be the output amplitude if  $\omega = \omega_c$  if I use Bode approximation then at  $\omega_c$  also it seems like it seems like just below  $\omega_c$  I have the gain is that of DC gain, but just above  $\omega_c$  the gain is kind of dropping, but we know we know better we know that at the frequency of  $\omega_c$  I will have a 3 dB drop right. So, the amplitude will be  $1/\sqrt{2}$  times that of input right, but more critical thing is happening in terms of phase. What is happening to the phase? The phase at  $\omega_c$  is  $-\pi/4$  which means that our output will be delayed output will be delayed by 45 degree right. So, in other words we will have something like this right. So, this is delayed by 180 degree this is delayed by 90 degree somewhere I have delayed by 45 degree.

So, we will have something like this. And so on right ok fine and the amplitude will be  $1/\sqrt{2}$  times that of input ok. And what is going to happen let us say at very high frequency right at  $\omega \gg \omega_c$  or much much greater than  $\omega_c$  or much much greater than 10 megahertz. What is going to happen? In that case obviously I mean I am physically not equipped enough to sketch the sketch the input with such such high number of oscillations within a same period right. So, let us just assume that is that assume that this is it.

So, this is the input right and so on. So, what will the output look like? Firstly where am I talking about now? I am talking about the scenario where I am far away from  $\omega_c$  right. Let us say if I say that this is like  $10 \times \omega_c$  right  $\omega = 10 \times \omega_c$  let us say  $\omega = 100$  megahertz let us say. So, then for all practical purposes I can say that the input has almost died down there will be a very small very small amplitude right. However, there will be a delay there will be a phase lag and how much will the phase lag be? The phase lag will be that of 90 degree right.

So, the phase lag cannot be more than 90 degree because this is the first order system that is all it is saying right. So, essentially we will have something like this and so on ok. So, if I have to blow this up if I have to blow this up this is what is going to happen let us say we are still skating  $\omega = 100$  megahertz case. So, let us say your input is one cycle is something like this your output will be 90 degree delayed, this quarter cycle delay right will be of very small amplitude.

Ok. So, you get the idea right. So, this is 90 degree delayed and reduced in amplitude right. So, this phenomena of different delay and different amplitude will be of lot of interest as we move forward when we talk about a stability of feedback systems. So, this I wanted to give you a intuitive feel of what we mean when we say that there is a frequency dependent component and we have certain tails and we have certain amount of certain magnitude response right. This is obvious and we have all done phasor systems phasor analysis in the past, but this is this is a quick refresher ok fine.

So, this is as far as this is as far as the in as far as the dependency of the input is concerned. So, now, let us concentrate on the output side right. So, let us say this is  $V_{gs}$  and we know what is the  $V_{gs}$ . So, let us say this is  $V_{gs}$  of  $s$  right. So, this  $V_{gs}$  of  $s$  got generated because we have some stuffs connected to it right.

Now, we want to find out now we want to find out what is the final output because that is what the goal is ok. This is  $C D D$  ok. So, if we use the same framework as earlier right what should we see what we should see we should see  $V$  we should basically say that  $V$  naught  $s$  will be  $g_m V_{gs}$  of  $s$  this is the current minus of that divided by where the conductance at the output and the conductance is  $G_L$  plus  $S C$  right ok. We can also express this in terms of in terms of DC gain DC transfer characteristics and the associated poles and if you have to do that what we need to do we basically then say that this is equal to minus  $g_m$  by  $G_L$  times  $V_{gs}$  of  $s$  times  $1$  by  $1 + S C R_L$  right. So, in other words this becomes or in other words  $V$  naught over  $s$   $V$  naught  $s$  over  $V_{gs}$   $s$  becomes minus  $g_m$  over  $G_L$  or minus  $g_m R_L$  by  $1 + s$  by  $P_2$  where  $P_2$  is equal to  $1$  by  $\tau$  by capacity.

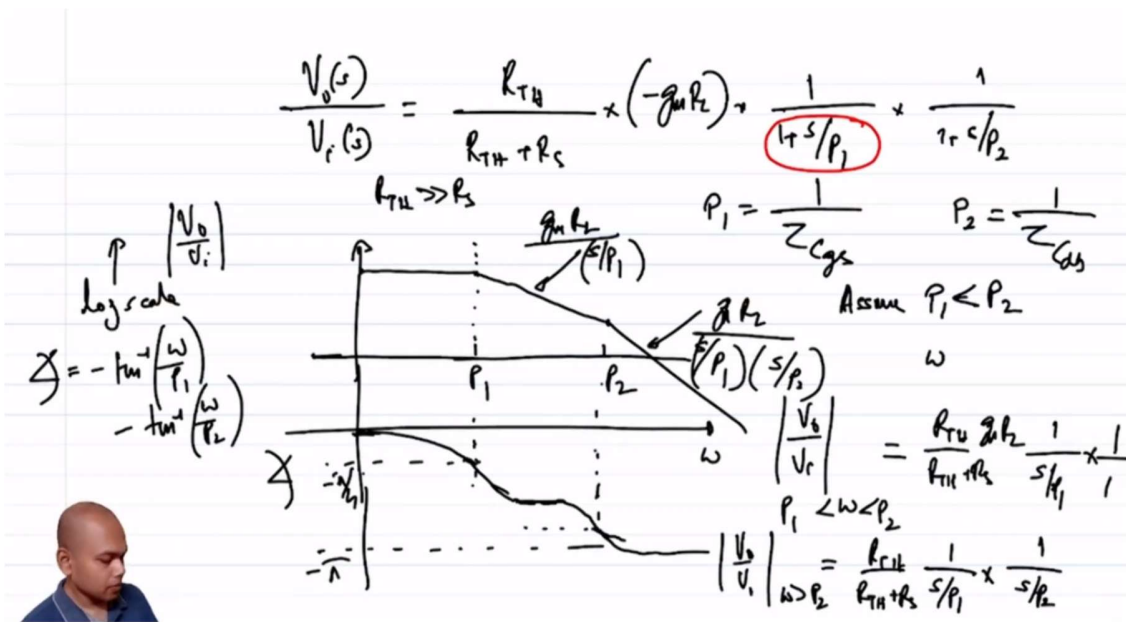
Or  $1$  by the time constant associated with the with that node times  $1$  over the time constant associated with the node the time constant is essentially  $C D B$  times  $R_L$  right ok fine. So, what is this again so can could we have written this without writing out the entire transfer function could we have written this in terms of the framework that we are developed. Yes we could have because what is this is the gain this is the gain at very low frequency this is the is the transfer function at DC times some frequency dependent part and the frequency dependent part can have a  $0$  and can have a pole right or can have multiple zeros and multiple poles. We haven't yet dealt into we haven't yet talked how to identify if there is a pole  $0$  or not so but we since we know that this guy doesn't have a  $0$  so let's take keep the discussion on on the  $0$  aside for the time being. But do we know how many poles we have yes we know because we have this is a one capacitor circuit one capacitor and bunch of other non frequency dependent elements which means we have one pole what is the frequency of the pole the frequency of the pole is that of  $1$  over the time constant we know how to find out time constants and time constant in this particular case is  $C dv$  times  $R_L$  which means we know the transfer function correct.

So, what at the end of the day what becomes  $V_0$  over  $V_i$  what is  $V_0$  over  $V_i$  this becomes  $R_{th}$  by  $R_{th}$  plus  $R_s$  times minus  $g_m R_L$  times  $1$  by  $1$  plus  $s$  by  $P_1$  times  $1$  by  $1$  plus  $s$  by  $P_2$  right where  $P_1$  is equal to  $1$  by excuse me  $1$  by time constant associated with  $C_{gs}$  right so this is  $\tau_{C_{gs}}$  and  $V_2$  is a time constant associated with  $C_{dv}$  this is  $\tau_{C_{dv}}$  which I know  $\tau_{C_{gs}}$  is equal to  $C_{gs}$  times some equivalent  $R$  some Thevenin equivalent resistance and  $C_{dv}$  times  $C_{dv}$   $\tau_{C_{dv}}$  is  $C_{dv}$  times  $R_L$  ok. So, so now can we sketch can we sketch the bode plot. So, if I have to sketch the bode plot what should it look like this is let us say I sketch mod of  $V_0$  over  $V_i$  in log scale that is  $20 \log$  or I will drop writing log scale every time we will assume that this is log scale right. So, we will assume log scale right. So, this side is log scale if that is the case then what is the what is the gain at at very low frequency and let us further assume that  $P_1$  is less than  $P_2$  right let us assume.

So, there is no no guarantee that  $P_1$  has to be less than  $P_2$ . So, let us assume  $P_1$  is less than  $P_2$  this is assume. So, it helps us in in in marking of the points. So, let us say this is  $P_1$  and let us say this is  $P_2$  right. So, if  $g_m R_L$  times  $R_{th}$  by  $R_{th}$  plus  $R_s$  is I mean is much greater than  $1$  right or at least greater than  $1$  and let us further assume  $R_{th}$  is greater than  $R_s$ .

So, this does not load. So, that the gain and DC is  $g_m$  times  $R_L$ . So, that is what we get here  $g_m$  times  $R_L$  till the frequency  $P_1$  then between  $P_1$  and before  $P_2$  what is the new transfer function. So, using bode approximation. So, what is  $V_0$  over  $V_i$  is  $\omega$  is greater than  $P_1$  and less than  $P_2$  what will that be.

So, that will be the D I mean I. So, what we are essentially saying is that in this case I am  $\omega$  is greater than  $P_1$  which means that  $s$  plus  $P_1$  term  $s$  by  $P_1$  term will dominate using bode approximation right. So, this becomes  $R_{th}$  by  $R_{th}$  plus  $R_s$  and if I am doing mod then this big times  $g_m$  times  $R_L$  this is let us say mod right then it becomes  $1$  by  $s$  by  $P_1$  times  $1$  by what  $1$  plus  $s$  by  $P_2$ , but in this case  $\omega$  is less than  $P_2$ . So, using bode approximation I am neglecting the  $s$  by  $P_2$  terms this becomes this right. So, essentially it still is a  $20$  dB per decade roll off right similar to what we had earlier. So, this become right then what is going to happen then when we are at  $P_1$  over  $V_i$   $\omega$  is greater than  $P_2$  then what will be the transfer function the transfer function will be  $R_{th}$  by  $R_{th}$  plus  $R_s$  times  $1$  by  $s$  by  $P_1$  times  $1$  by  $s$  by  $P_2$  which means I have a  $s$  square term right and so that the roll off will be at  $40$  dB per decade.



So, this will be something like this ok. So, here the transfer function essentially becomes  $g_m$  times RL by  $s$  by  $P_1$  and here this becomes  $g_m$  times RL by  $s$  by  $P_1$  times  $s$  by  $P_2$  right. Now what about the phase what about the phase what about the phase the phase will start from 0 as usual at  $P_1$  if  $P_1$  and  $P_2$  are separate far away I mean the phase at  $P_1$  let us say is not getting affected by the phase of  $P_2$  right. So, we will get so let me also write out the phase first phase expression will be minus tan inverse omega by  $P_1$  minus tan inverse omega by  $P_2$  and if we assume that  $P_1$  and  $P_2$  are separated and they are not interacting with each other which means that when I am at omega equal to  $P_1$  I am far away from omega equal to  $P_2$  which means the phase will be 45 minus 45 degree. So, this is minus pi by 4 and then it will reach minus pi by 2 then again it will go by additional minus pi by 4 and settle at minus pi right ok.

So, that is what the phase will do and what impact will that have on our transient response just that you will see that. Now, the maximum phase delay that we could have had at very high frequencies could have been only 90 degree, but now we can have 180 degree phase maximum phase delay which means there can be a half cycle delay of the of the input at max at very high frequencies what is very high frequency in this case it is that frequencies much higher than  $P_2$  right ok. .