Course name- Analog VLSI Design (108104193)
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Lecture- 35, Module-2

Ok, so let us proceed with the analysis of this network. Before we proceed with the analysis of this network, let us do one, simplify the circuit a bit. I mean just like if you recall when we did our y parameter analysis long time back, we said that we did some analysis and we found that y12 had some issues, we assumed y12 to be 0 and all right. Because there is what was the reason, I mean we wanted y12 to be 0, we wanted y12 to be 0 because we did not wanted to avoid instability right. So, but in this case as you can see the y12 will not be 0 because of this capacitance Cgd right. What is y12? y12 is a short circuit current at the input port if you excite the output port with some voltage right.

So, clearly in this case in the presence of Cgd y12 is not 0. So, Cgd is going to have an impact right. So, we will deal with the impact of Cgd later on. For the time being, for the time being we will assume that Cgd does not exist or Cgd is small enough to be neglected and we will proceed.

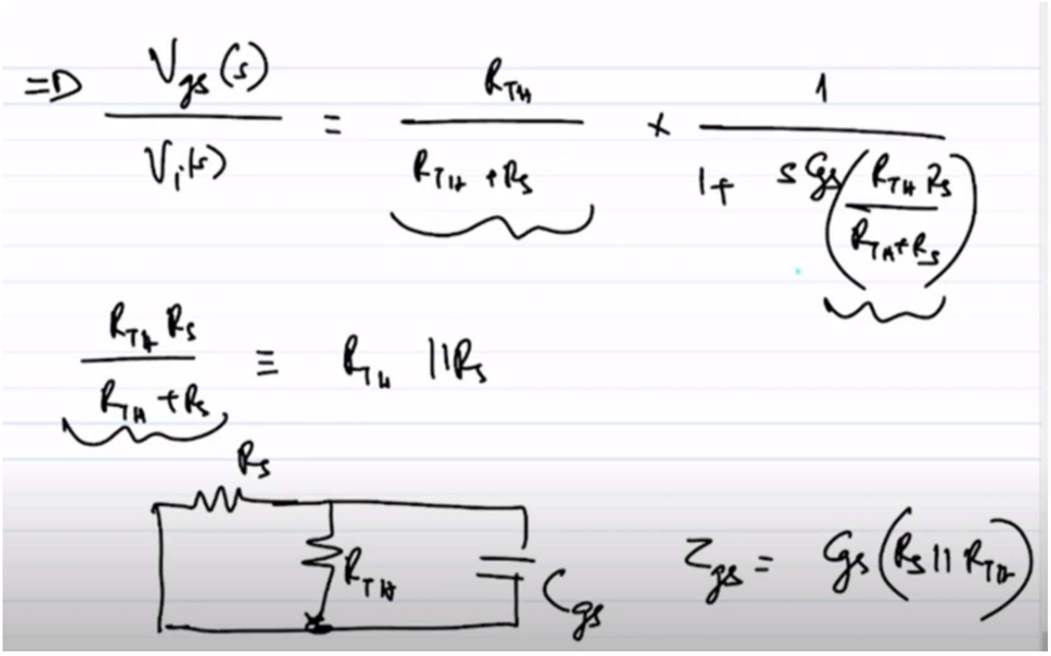
And in the next lecture hopefully right. So, we will go ahead and see the impact of Cgd right. For now let us assume the Cgd does not exist. So, if we assume that what does our small signal model of the common source amplifier reduce to, so this is Rs, this is right let me call this Rth Rth in which is R1 parallel R2. So, we have Cgs, then we have Cdb, we have RL, this is Vgs, this is gm times.

$$V_{gs}(s) = \frac{\dfrac{R_{TH}}{1+sR_{TH}C_{gs}}}{\dfrac{R_{TH}}{1+sR_{TH}C_{gs}} + R_s} V_i(s)$$

$$= \frac{R_{TH}}{R_{TH} + R_s + s\,C_{gs}R_{TH}R_s} V_i(s)$$

So, this is Vgs, this is V0 right ok. So, now as usual what should we do, we should now find out what is Vgs because now Vgs is not a not only a register divider between Rth and Rs right. There is a frequency dependent element of Cgs ok. So, what will be Vgs? So, Vgs in Laplace domain right if we evaluate this in Laplace domain what will we see, we will see it is a voltage divider between the parallel combination of Rth and Cgs and Rs right. So, what will that reduce to? So, this will become Rth by 1 plus sigma gm.

So, this is the parallel combination of Rth and Cgs divided by the same thing gm by 1 plus s times Rth and Cgs plus Rs ok. So, this means what? This is equal to t h by 1 plus Rs plus s times Cgs times Rth times Rs right. So, let me replay let me express this in another format sorry I missed vi right. So, this is vi of this. Also please mind the notation now we switch back to the we switch back to capital V here we are switch switching back to capital V and small subscript because we are in the phasor domain now right.



We are in the transform domain these are Laplace domain or Fourier domain we will soon go to Fourier domain. So, this which means that in Vgs s by vi s becomes Rth by Rth plus Rs times 1 by 1 plus s Cgs times Rth Rs right Rth plus Rs right. So, why did I choose to write in this form? I chose to write in this form because if Cgs were not there right if Cgs would have been 0 what would Vgs have been? Vgs would have been Rth divided by Rth plus Rs right. So, if Cgs would have been 0 our expression should reflect that right. So, in this expression if you plug in Cgs equal to 0 that then you see that the stuff that is remaining is a scenario where we do not did not have a we did not have a capacitance in place.

So, this kind of this kind of checks out right, but what is this thing what do you think is s Cgs times that what is that doing? So, this seems like a parallel combination of Rth Rs by Rth and Rs right. So, that is what this is this Rth Rs by Rth plus Rs what is this? This is equivalent to parallel combination of Rth and Rs. So, where do you see a parallel combination of Rth and Rs and how does it how does it come about here? So, if you stare at this loop if you stare at this at this input loop right this input side what is the time constant associated with the capacitance Cgs? The time constant associated how will you find out the time constant? The time constant associated with this with the capacitance is you short the input. So, Vi goes to 0 right. So, we had Rs, we had Rth, we had Cgs and we had Vi.

So, short Vi what is the time constant associated with Cgs? So, tau gs is Cgs times Rs parallel Rth right. So, that is what is that is what is appearing here that is what is appearing here right. So, note that we can express this term Vgs over Vi also in this format right. We can also express in this format that is Rth by Rth plus Rs is 1 by 1 plus s by p is by p 1 right. What does this mean? What is p 1? So, here p 1 or rather let me express in the more formal way s minus p 1.

So, what is this 1 minus s by p 1 telling you? This means that there is a pole. So, this implies there is a pole at frequency p 1. What is a pole? Mathematically what is a pole? Pole is the root of the denominator of the transfer function right. So, what is a pole? Pole is the root of the denominator of a transfer function right. So, in a first order so, what is the order of the system? This is clearly the first order system I have one capacitor and one equivalent resistance is the first order system right.

So, in a so, what is the location of the pole? Now for in our case p 1 is equal to minus 1 by C gs times Rth parallel Rs right. So, that is what we see right. So, 1 plus s times C gs times Rth parallel Rs is equal to 1 minus s by p 1. So, basically we replace p 1 with whatever we got and this is what we this is what we get right. So, what does this mean? What are the observations? What are the key observations here? So, firstly so, p 1 is a left hand side pole right.

It is a left hand plane pole. So, what is meant by left half plane pole? So, let me quickly refresh your basic control theory your basic control theory concepts. So, when we are doing Laplace transform right when you are putting when you are evaluating anything in s domain when you are doing Laplace transform what are we essentially saying? We are essentially saying that in the s plane right. So, in the s plane in this is let us say s plane where s is equal to sigma plus j omega this is sigma this is j omega for if we evaluate our transfer function right if we evaluate b 0 s over vi s right for all values of s ok. So, if we evaluate b 0 v 0 s over vi s for all values of s right.

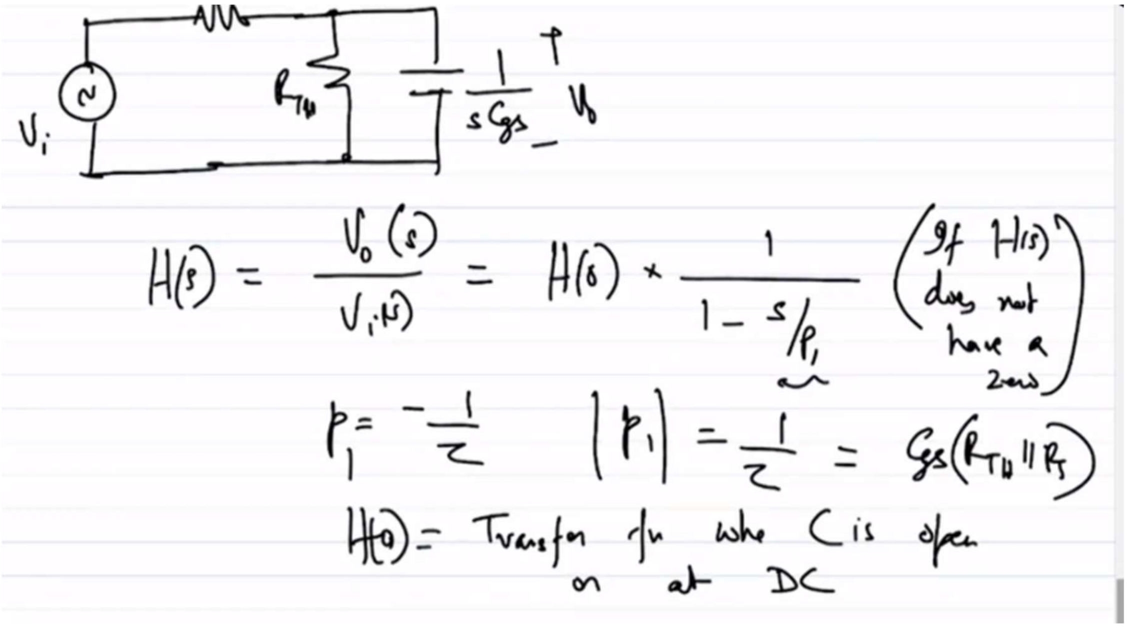
So, which means essentially that we are evaluating our transfer function for this entire s plane somewhere in this somewhere in this s plane the transfer function will go to infinity right. That is all it is saying right. So, somewhere in the somewhere in this in this s plane right the transfer function will go to infinity and let us say it goes to infinity here right. So, if it goes to infinity there we call this guy a pole let us we call this b 1 right. So, pole is basically that that location in the in the s plane where the transfer function goes to infinity.

Now, what is this what is this telling us where is the location of P 1? The location of P 1 is firstly P 1 is real P 1 does not have in the first order system that we have the P 1 is real it does not have any imaginary part there is no j omega which means that P 1 lies on the x axis and P 1 is negative right. So, we have P 1 here ok and also what does a negative P 1 mean? For a stable system for a stable system you poles have to be poles must be in the left half plane right. So, pole has to be here. So, this is the mark of a stable system right because if you give a if you give an impulse input to a stable system its output should not blow up right. So, its bounded input bounded output stability criteria from there we we know that the pole should be on the left half plane right.

So, that is fine. So, in this case the poles are on the left half plane that is good our common source amplifier is stable that is number 1. What is the second observation? We see that the pole is so also the second observation is P is equal to minus 1 over the time constant associated with that right. So, your P 1 was tau of tau ds right. So, what essentially means that which essentially means that for a first order system pole is minus 1 over time constant associated with the data system right. So, this this is a very critical piece of information that we should keep at the back of our mind because we will be coming back to this multiple times.

You remember at the beginning of the course we spent a lot of time finding out transfer time constants of different different types of circuits right different types of first order circuits. So, we leverage those learnings here why because if we know the time constant we automatically know the poles right ok that is good right. So, that is good. So, let us go back to our circuit once again let us see if we are given a circuit like this right. So, sorry if I giving a circuit like this how should we go about finding out the transfer function without without writing the KVL, KCL and everything right ok.

So, so in this case if I have to write the transfer function what should I do. So, what is V 0 s by V i s. So, we are we say that this transfer function will have two parts one is one is the part where V 0 where will be will have. So, let us say I call this H of s right let us call this H of s. So, this transfer function will have two parts one where which will be equivalent to the case where capacitance is 0, but also note that that is equivalent to the case when s equal to 0 because what is where is the s count is coming from s terms is coming from because we have a capacitance whose whose impedance is 1 over s C gs right.



If s were not there if s equal to 0 is equivalent to the case of capacitance equal to 0 right. So, essentially we will have we can express this V 0 over V i term in this form we will have some H of 0 right times 1 by 1 minus s by V 1. Why why can we express this in this form because this is a first order system. A first order system will have one pole right what is the I mean by definition what is the order of the system is the number of poles is the order of the system say first order system will have one pole. In this case we have we have we have assumed that there is no 0, but that is not obvious we will come back to 0 later on right.

So, if a first order system right does not have a 0 right this is an under the case of your if H of s does not have a 0 right. So, then we can express it in this form else we will have to go back and see if these have a 0 or not right. So, we will consider the 0 later on let us assume that we know that this circuit does not have a 0. So, let us go ahead ok. So, if this is the case what is the pole associated with the first order system.

So, so P will be P will be minus 1 over time constant right. So, which means that P 1 will be or mod of P 1 that is the exact location of the pole will be 1 over time constant. We know the time constant time constant is C g s times R th parallel R s. We know the pole what is H of 0? H of 0 is that is that is a transfer function when the capacity when s equal to 0 which means is a transfer function when the capacitors are open circuited right. So, H of 0 is transfer function when C is open or at DC right.

So, we know how to figure out the DC transfer function this is simply a voltage divider. So, we know what is H of 0. So, we can simply write of H of 0 as R th by R th plus R s right and we know what is the pole we can find out the pole by inspection. So, if we know the DC transfer function and we can find out the pole by inspection and we know that let us assume that we already know that there is no 0 then we will be able to write out the transfer function by inspection only ok fine great. So, now if that is the case so we know what is the transfer function.

So, now we know that H of s is some H of 0 1 by 1 plus s by P 1 or rather 1 by s 1 plus s by mod of P 1 right because P 1 is negative. So, what is why are you going into the Laplace domain because ultimately the goal is to find out find out because capacitor is a frequency dependent element we need to find out what is the what is the effect of high frequencies right what is the effect of the capacitor on non-zero frequencies right. So, in other words we are trying to find out what is H of j omega right. So, what is H what does H of j omega mean H of j omega means V 0 j omega by V or rather Vgs j omega in this case Vgs j omega by Vi j omega right ok. So, what does this mean in English this essentially means this implies.

So, what is or let me just say also H of j omega is Vi of this Vi of j omega. So, what is Vgs of j omega Vgs of j omega in plain English means this implies this is equal to the response or the phasor response at node there is a response across Vgs when a sinusoid of frequency omega or let us say omega is applied at Ei ok. So, clearly so what is in our case what is H of s H of j omega rather in our case H of j omega is Rth by Rth plus Rs times 1 by 1 plus j omega by P 1 omega by mod of P 1 right. So, this is a complex term. So, it will have a magnitude and a phase part right.

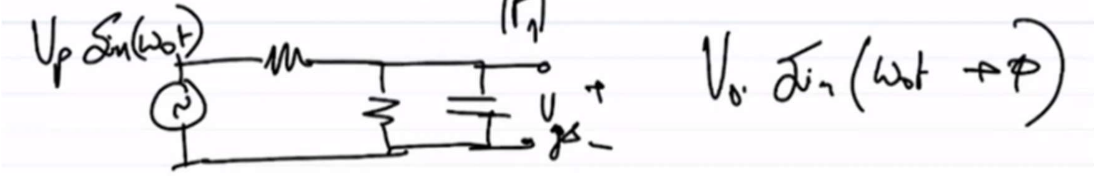
So, this can be expressed as Rth by Rth plus Rs. So, or let me simply say that what is the mod of H of j omega mod of H of j omega is Rth by Rth plus Rs mod of the second term which is 1 by under root 1 plus omega squared by mod of P 1 squared in this case P 1 is I mean real. So, it does not really matter you made mod or not and what is the angle, angle is the phase associated with it the phase associated with it is minus tan inverse omega by mod of P 1 right. So, what does this essentially imply? This implies that if I when I have the input here let us say I have an input and I apply an input of V P sin omega naught t and this is where I am observing the output Vgs. So, what will be Vgs? If input is V P sin omega naught t and I know the transfer function is this.

So, what will be the what is the amplitude of the sinusoid that I am going to see at the output? So, at the output I am going to see the sum V o sin omega naught t plus phi right. It is a linear system. So, what if I apply a frequency of omega naught I am going to see a frequency of omega naught right, but we but what will change what might change the amplitude of the sinusoid might change and the phase of the sinusoid might change right these are the two possibilities. So, given that I know what is the transfer function right given that we know what is the transfer function what will be V naught and what will be phi. So, clearly by definition that h of mod of h of omega gives us gives us what gives us the mod of output by input which means I can simply scale the amplitude of the output by that mod of h of j omega.

$$H(j\omega) = \frac{R_{TH}}{R_{TH} + R_S} \quad \frac{1}{1 + \frac{j\omega}{|P_1|}}$$

$$|H(j\omega)| = \frac{R_{TH}}{R_{TH} + R_S} \quad \frac{1}{\sqrt{1 + \omega^2/|P_1|^2}}$$

$$\angle = -\tan^{-1} \frac{\omega}{|P_1|}$$



So, V naught will be in our case V naught will be V P, V P is the amplitude of the input times mod of h of j omega naught which is R th V P times R th by R s plus R th times 1 by under root 1 plus omega naught square by mod of P 1 square right and what will be phi? Phi will be equivalent to the phase the angle of the of the transfer function. So, phi will be equivalent to minus tan inverse omega naught by mod of P 1 ok fine. So, why is this critical? This is critical because this helps us in understanding the sinusoidal steady state output response by the way these are all sinusoidal steady state output response right this is assuming that this is assuming that the input has been is a sinusoid forever we are not assuming that you have turned on the signal generator now and you are trying to figure out what is going to happen right from then. So, this is all steady state waveforms these are all steady state waveforms for a when you observe the sinusoid for infinite amount of time what is going to be the response ok. So, now then so that is why that is why the phase and the angle of mod of H is important right.

So, what is going if I plot mod of H with respect to omega what am I going to see? So, I am going to I am essentially saying that I am planning to plot this guy right I am planning to plot this guy with respect to omega right. So, what is going to happen at very low frequency? Let us say omega is close to 0 what am I going to see? I am going to see H of 0 right. So, I am going to see Rth by Rs plus Rth right at very high frequency what am I going to see right at omega tends to infinity what should I see? I should see 0 right because denominator of the transfer function dominates. So, at omega tends to infinity it should go to 0. So, somewhere between this dc value and 0 it will roll off right.

So, it will do something like this ok that is all good, but if I locate this value this pole location P1 on this frequency axis by the way note that this is frequency in radian per second right this is not H is radian per second. So, if we denote a frequency of P1 here right or mod P1 here right. So, what happens when omega naught equal to P1? As you can clearly see when omega naught is equal to P1 the amplitude of or rather transfer function magnitude goes to 1 over root 2 of that of the dc value right. So, essentially if I mark off right this is 1 by root 2 of H naught.

So, this is H naught right. So, wherever it hits 1 over H naught will be the location of the pole right fine. So, similarly what is going to happen to the angle? So, what is going to happen to the angle of H? What is the angle of H? Angle of H is minus tan inverse omega naught over P1 mod of P1 ok. So, which means it starts off from omega equal to 0 it starts off from 0 and then as omega increases it goes negative right. So, it goes negative what is the value of phi at omega naught equal to at P1? What is the value of phi at P1? This tan inverse minus tan inverse of 1 that is minus pi by 4. So, this is minus pi by 4 and what is the value of phi at omega equal to infinity minus tan inverse infinity that is minus pi over 2.

So, this will go to and saturate to minus pi over 2 ok. So, you see that these type of plots have lot of significance because once we have these plots we do not have to go back and write out the transfer function again. Once we have these plots we do not have to struggle to find out what will be the amplitude and the phase of a sinusoid at any frequency. If I tell you what if I ask you what so tell me what is the sign what is the amplitude of the output if my input is at a certain frequency 10 times P1 right. So, let us say I if omega if the input frequency it is at 10 times P1 all you to have to do is to read off this plot and find out this value whatever that value is that will be the amplitude of the sinusoid at the output right.

So, that is a power of this of this phasor plot of this magnitude and the phase plots right ok. So, as it turns out we further simplify this right we further simplify this and we use another approximation which we call a Bode approximation. So, what is a Bode approximation? So, we use a Bode approximation. What Bode approximation essentially tells us is if a transfer function h of s is of the form some a naught by 1 plus s by P1 right. I mean let us assume P1 is positive now because we know that it is a stable system.

So, there is no point in writing mod. So, we will just drop that negative sign and we will always assume that poles are in the left half plane right. So, we will make our notation simpler right. So, we will so from now on we will use P1 instead of mod P1 right. Simply because we know that it is a I think this will be I mean the first order system is cannot have a pole on the left half plane because I mean system is stable right by default ok. So, what is Bode approximation for a first order system? All this says that all this is saying is that if right if omega which is I mean s equal to g omega we know right s equal to j omega right.

I mean for sinusoidal steady state we know that s equal to j omega this is for sinusoidal steady state. So, if omega is much much less than P1 right if omega is much much less than P1 what is H of s? H of s is approximately equal to you know right. If omega is much much greater than P1 what is approximately H of s equal to H of s is approximately equal to a naught by s by P1 which means a naught P1. So, what Bode came along and said that I mean it will make our life really simpler if we drop this much much less than much much greater constraint and we say that as long as omega is less than P1 H of s is a naught and as long as omega is much greater than P1 H of s is a naught by s plus by P1. Why? Because then you will see that we can express these smooth curves with a straight line segment right.

So, essentially what we as what Bode came along and said that I mean we are good in finding out solutions of straight line segments. So, instead of instead of dealing with smooth curves well let us say this is P1. So, what we will say is that as long as P as long as we are operating with omega which is which is much less than P1 or which is less than P1 we will consider this to be a straight line of value H of 0 and beyond P1 will again assume it to be a straight line but with some slope right. The slope will be right a0 P1 by s right.

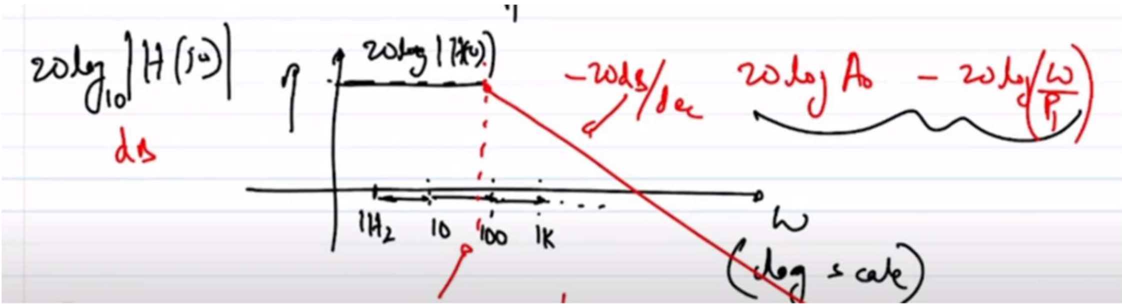
So, essentially this is a hyperbola. So, this will become something like something like this right. So, this is some this will be something like this ok. Then people I mean then people figured that there is no point writing I mean when things become really part apart right when you have to deal with very numbers which are orders of magnitude apart right because I mean H0 can I mean at some at very high frequency of omega right at very high frequency this magnitude of H can go to really small values and if you want to then plot the same small values in the same scale with respect to what is happening at omega equal to 0 then you will not be able to show them. But there is a solution but solution is if we express this in logarithmic scale right if we express in logarithmic scale then this is a linear scale that I have drawn if we express it in logarithmic scale then there is a hope right. So, if we express this in logarithmic scale what will happen? So, if we express as log of 20 log of mod of H of j omega right.

So, this is mod of yeah. So, log 10 of this. So, what will happen at very low frequency this will be 20 log of H of 0 and what will be the other guy? So, this equation is what this equation is a naught t1 over s. So, log of this is what? So, log of this is 20 log a naught t1 minus 20 log omega right 20 log I mean if I am if I am concerned with mod then basically this is mod right. So, it will become omega right fine. And if we also express this axis in log scale by log scale what do I mean? What I essentially mean is that if let us say this is 1 Hertz and I go some increment and I say this is 10 Hertz right then if I go equal increment again to the right this becomes 100 Hertz if I go another same equal increment to the right it becomes 1 kilo Hertz and so on right. So, equal increments on the x-axis multiplies the frequency that is what log scale essentially implies.

If I if x-axis is log scale which means x-axis is also increasing exponentially right and y-axis is also in log scale right this is also in log scale then this becomes a log-log plot right. So, for every 10x increase in omega how much does this guy reduce? For every 10x increase in omega this guy reduces by 20 dB right. So, by factor of 20 which is often referred to as dB right. So, there is another popular way of referring this instead of doing this we keep the P1 and S together.

So, we call this this. So, essentially then this becomes this right. So, at omega tends to infinity where will this red line go to? The red line will go to minus infinity dB right. So, this is now in dB the units are in dB ok. So, in the top plot if omega tends to infinity mod of H will go to 0, but because we are doing a log-log plot at omega tends to infinity this will go to minus infinity right. So, essentially what is the what is the moral of the story? The moral of the story is that if we do a bode approximation and we further use that bode approximation to use to plot everything in a log scale right.

So, we can replace these smooth curves with straight line segments. So, moment we can we can we can replace them with straight line segments things become easy right. So, because this break point here what is this break point? This break point will become the pole and what is a pole for a first order system? The pole for a first order system is 1 over tau right. So, if you see that I mean you everything you now we are is clicking right. So, at the beginning of the course we found out what is how to figure out a pole at the beginning of the course and throughout the course we have been dealing with DC transfer function because we have been neglecting capacitor right. So, now if you have to put two and two together if I have a frequency dependent component at least a first order frequency dependent component in a first order low pass frequency dependent component first order low pass frequency dependent response then I should be able to look at the circuit and predict its output right.



What will be the output? The output will be the gain at DC will be the bode approximation is telling us the bode plot will be flat till we hit the first pole what will be the flatness what will be the value of the flat portion the value of the flat portion will be the DC gain then you hit the first pole why is the location of the first pole the location of the first pole is at 1 over time constant and what is happening after the location of the first pole it rolls off at at minus 20 dB per decade right this is minus 20 dB per decade right. And another reason bode approximation is immensely useful is because now we don't have to deal with the entire transfer function we can deal with piecewise line segments of the transfer function okay since we can deal with the piecewise line segments of transfer function we will be able to make our life a lot easier we don't have to use higher order equations right. So, I mean the whole part I mean the whole thing that we are doing in circuit design is to make the analysis simpler so that we can make quick and easy decisions and we can make quick and easy decisions if the analysis is simpler right and we know how to deal with first order circuit we are trying to break everything down into system of first order circuits that is all we are doing right. So, let us stop here in the next lecture we will see I mean when we put the entire common source amplifier together what will be the eventual frequency response okay. Thank you.