

**Course name- Analog VLSI Design (108104193)**  
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So, up till now we have been dealing with whatever we have been dealing with in terms of differential amplifier, we can summarize as follows. So, number step 1 is we can essentially say that any arbitrary input  $V_1$  and  $V_2$  applied to the gates of  $M_1$  and  $M_2$  can be broken down into a common mode input and a differential input and we can analyze the circuit independently of each other and add up the sum, right. So, if we do that this is what we saw, we will have to apply common mode input  $V_{ICM}$  and  $V_{ICM}$  on both nodes, both the inputs of  $M_1$  and  $M_2$  and a differential we have to apply  $V_{DIP}$  by 2 and minus  $V_{DIP}$  by 2 and then we can find out the short circuit currents, right. We can find out the short circuit currents  $I_{SC}$  for common mode and  $I_{SC}$  for differential mode. So, this is CM, this is gate, right and then we can do all sorts of things, but what is the, what were the key differences? The key differences were, so in case of common mode, right, in case of common mode when we were only applying signal at  $M_1$  and grounding  $M_2$ , what was  $V_X$ , what was this, what was this node voltage? If I take you back, this voltage was  $V_{ICM}$  over 2, right, you see that this voltage is  $V_{ICM}$  over 2, but if I again apply the input at the, instead of  $M_1$  if I apply it at  $M_2$ , what will be that voltage? That will again be  $V_{ICM}$  over 2, right and using superposition when we apply both the inputs together at the gates of  $M_1$  and  $M_2$ , what will that voltage be? It will be  $V_{ICM}$  over 2 plus  $V_{ICM}$  over 2 which means this will be  $V_{ICM}$ , right, under common mode condition. Under differential condition, right, in the circuit on the right, what will be that node voltage? This we saw, right and what did we see? This voltage would have been 0, correct? So, this voltage would have been 0, ok.

So, what is so novel about that? This essentially, the novelty here comes from the fact that, so this is by the way, this is approximately, right, not exactly, right because if you, the tail transistor, if it is, if the tail transistor, if the tail node that is a  $V_X$  in the case of a common mode is actually  $V_{ICM}$ , then there is no incremental current flowing through  $M_1$  and  $M_2$ , but we saw that is an approximation, right because ultimately there will be an incremental current flowing, but that is not captured from this picture, ok, fine. So, in case of a differential mode, in case of a differential mode,  $V_X$  is actually 0. So, if  $V_X$  is actually 0 in the incremental picture, then it becomes easy to find out what is the incremental current because then the incremental current becomes  $V_{diff}$  over 2 times  $g_m$ , correct. If I switch my focus to the, if I switch my focus to the, so the common mode scenario once again, what is the incremental current? The incremental current through

the resistor  $R_0$  is  $V_{ICM}$  over  $2R_0$ .

What is the incremental current through the resistor  $R_0$  in the differential mode? This incremental current is 0 because  $V_X$  is not getting excited, right. Again in case of a common mode, this current  $V_{ICM}$  over  $R_0$ , where will it flow? This current will flow equally between two branches because again, symmetry, right, so it will flow equally over two branches, this would be  $V_{ICM}$  over  $2R_0$  and the other will also be  $V_{ICM}$  over  $2R_0$ . Now, this  $V_{ICM}$  over  $2R_0$  makes way into the short circuit and creates the short circuit current. In case of a differential picture, in case of a differential mode,  $V_{diff}$  over  $2$  times  $g_m$  makes way into the short circuit and creates a short circuit current which is  $V_{diff} g_m$  over  $g_m$  over  $2$ , ok. So, what is the common mode rejection ratio? So, what is CMRR in this case? This is, I mean since, I mean as well I can say this is a short circuit, I can evaluate this ratio of the short circuit currents between the differential and the common mode because the gain, while you are trying to find out the voltage, you have to multiply with the output resistance and output resistance is common in both cases, so they essentially cancel off.

$$CMRR = \frac{V_{diff}/2 \cdot g_m}{\frac{V_{icm}}{2R_0}} = \frac{V_{diff}}{V_{icm}} \cdot g_m R_0$$

$V_1 = V_i$   
 $V_2 = V_i \Rightarrow V_{icm} = V_i$

$V_1 = V_i/2$   
 $V_2 = -V_i/2$

So, we can deal with only the short circuit current and that becomes what? So, this becomes  $V_{diff}$  over  $2$  times  $g_m$  by  $V_{ICM}$  over  $2R_0$ , correct. So, essentially becomes  $V_{diff}$  by  $V_{ICM}$  times  $g_m$  times  $R_0$  and if  $V_1$  is equal to  $V_2$ , right, if  $V_1$  is equal to  $V_2$ , then for differential mode and  $V_1$  is equal to minus, so let us say in case of, if we say that  $V_1$  is equal to  $V_i$ ,  $V_2$  is equal to  $V_i$ , right. So then,  $V_{ICM}$  is  $V_i$ , but if  $V_1$  is equal to  $V_i$  by  $2$  and  $V_2$  equal to minus  $V_i$  by  $2$ , then what is  $V_{diff}$ ? Then  $V_{diff}$  is  $V_1$  minus  $V_2$  that is  $V_i$ , correct. So essentially, under this condition, your CMRR essentially becomes  $g_m$  times  $R_0$ , by the way this is  $R_0$  of the tail transistor, right, tail transistor. Okay, and since the tail transistor has an effectively infinite impedance, if we replace the tail transistor with a constant current source, ideal current source, then we get infinite, infinite gain, right, infinite CMRR rather, not gain, okay.

Okay fine, so what is, so what is it that we have learnt till now? What we have learnt till

now is that this is a differential amplifier, right, this is a differential amplifier and we have learnt the analysis of the differential amplifier with regards to having common mode and differential inputs and why is that relevant? That is relevant because moment we have common mode and differential inputs, we can split any input into common and common mode and differential modes and do our analysis. However, note that we are still not at the finish line because of what we have used RL, right. Note that in case of a common source amplifier also we saw that the maximum gain that we could have gotten would have been limited if we had to use a passive resistance RL. So how did we get rid of that problem? We got rid of the problem by replacing the RL with an active load or rather a current mirror load, right. So let we can do that also.

So how do we go about improving our design? So let us say we have, let us only concentrate on differential modes for the time being. Let us say we say this is  $V_i$  by 2 minus  $V_i$  by 2 and we are trying to find out what is the gain at this output node and we see that and for this particular purpose I will put back this  $I_0$  because having the  $I_0$  or having a resistance R is irrelevant as far as a differential signal, a differential operation is concerned because we saw that that node  $V_x$  will be approximately equal to 0, right or rather will be equal to 0 under differential operation, right. So we can as well make our life simpler, analyze it simpler by putting a constant current source, okay, okay, fine. So we would like to replace this RL with a, so I mean a quick recap. The problem that we are trying to address is this was a common source amplifier that we had seen earlier, right.

So  $V_b$  plus  $V_i$  and the problem with this structure was we could get limited gain with a finite  $V_{DD}$  because the transistor would have gotten into linear region. So how did we solve the problem? We solved the problem by putting an active load, a PMOS transistor, right. So this was  $V_b$  plus  $V_i$  anyways and the PMOS transistor needed to be biased in saturation so we used this structure, right. So you remember when we use this structure to get higher gain, right, okay. So why do not we do the same thing in case of our differential amplifier? So we had this RL, we will replace this RL with a PMOS transistor.

So let us do that. So we can replace this RL with a PMOS transistor, okay. How should I bias? Use some space here. We can use the same old bias that we have been using all along. What is the current through this PMOS transistor? Let me name them.

Let me name this as M4 and let me name this M4M which is a mirror transistor or M4 prime, okay. Okay, so what is the current through M4? Ideally what is the current that we need through M4, right? So the expected current through M4 is  $I_{naught}$  over 2 because the expected current through M2 is  $I_{naught}$  over 2. So if that is the scenario,

what is the expected current through M4 prime assuming  $W$  by  $L$  of M4 and M4 prime are same, the expected current through M4 prime is  $I_{naught}/2$  also, fine. Okay, great. So this kind of, this can give us a gain of more than, I mean this can give us gain higher than that we would have gotten through  $R_L$  because now what is the output impedance? What is the output impedance of this guy? The output impedance of this guy is whatever the impedance looking up in parallel to whatever the impedance looking down.

What is the impedance looking up? The impedance looking up is  $R_{DS4}$ . What is the impedance looking down? Impedance looking down is twice of  $R_{DS2}$ , right? So how does the Norton's equivalent get modified? Does anything change with regards to the short circuit current? Nothing changes with regards to short circuit current because the stuff on top, right, the  $R_L$  was not playing any part in short circuit, determining short circuit current. So in this case M4 will also not play any part in determining short circuit current. So what, so short circuit current does not change. So this remains  $g_m$  times  $V_i$ , right, or rather  $g_m$  times  $V_i$  by 2.

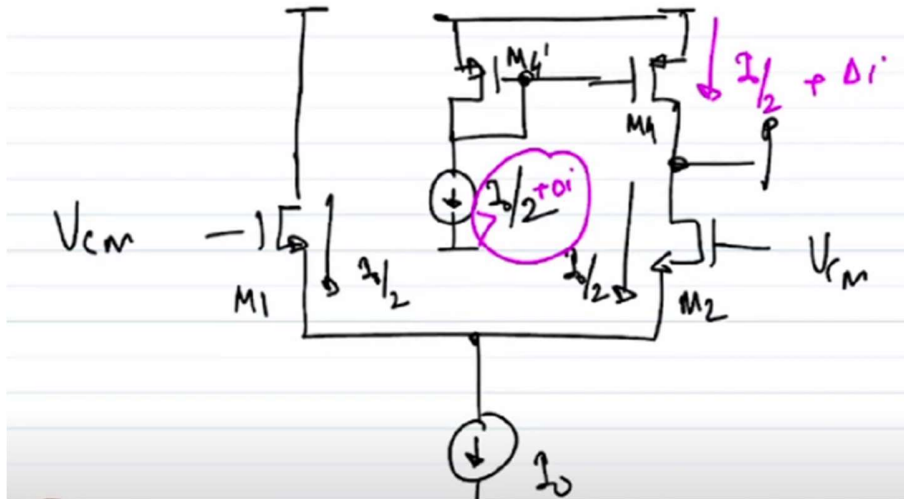
The only thing that changes is the output resistance instead of  $R_L$  parallel to  $R_{DS2}$ , now this becomes  $R_{DS4}$  parallel to  $R_{DS2}$ , right? So this becomes  $V_{naught}$ .  $V_{naught}$  is the drain, right? So this is  $V_{naught}$ . Okay? So this is as far as putting, replacing the, replacing our, our resistance with an active load is concerned. Nothing really changes, right? So but it is really instructive to look at it from a slightly different point of view. So let us do that also.

So let us say I have, so by the way, I should have been more careful here since we are dealing with, since we are dealing with biases, right? So we should, I should have probably put back the biases here, right? So this, let me call this  $V_{common\ mode}$ ,  $V_{common\ mode}$ , right? Okay. Okay. And on top of that, you will probably have  $V_i$  by 2 minus  $V_i$  by 2 plus  $V_i$  by 2.

Okay. Okay. So, so if we, if we do this here, so let me make more space so that we will have more vehicle room to draw. So let us say this is  $V_{cm}$ ,  $V_{cm}$ . So this under quiescent condition, what is this current? It is  $I_{naught}/2$ . What is this current?  $I_{naught}/2$ . What is this current? If we bias it properly and M4 and M4 prime are also having  $W$  by  $L$ , same  $W$  by  $L$ .

So this has to be  $I_{naught}/2$ , otherwise the current would not be biased properly. Okay. So now here is the interesting question. So let us say, if instead of  $I_{naught}/2$ , I have a current which is greater than  $I_{naught}/2$ . What do you think is going to happen? So if not that if this current is greater than  $I_{naught}/2$ , what is this current,

what will M4 try to drive? The M4 will try to drive  $I_{\text{naught over 2}} + \Delta I$ , right? So the M4 is trying to drive  $I_{\text{naught over 2}} + \Delta I$ , but M2 is only trying to sink  $I_{\text{naught over 2}}$ , right? So one current source is trying to drive higher current than the other current source is trying to pull.



So what is going to happen? So this cannot sustain. So what is going to happen to this voltage? You are trying to push in more current than you are trying to pull out. So what is going to happen to this voltage? This voltage will go up. And how long will it keep going up? It will keep going up till it crashes M4 and it moves into linear region, right? So M4 will stop being in saturation. So and it will keep on if the voltage will keep on going up till M4 goes out of saturation and into linear region till the current through M4 becomes  $I_{\text{naught over 2}}$ .

If the if M4 is in saturation, then the current cannot become  $I_{\text{naught over 2}}$ , right? Because the drain voltage does not determine the current of the transistor to the first order. If the drain voltage only has to affect the current, then the transistor has to move out of saturation, right? So that is what is going to happen. So we have to ensure that this current is  $I_{\text{naught over 2}}$ , okay? Okay, fine. So let us try to figure out what is going to happen to the short circuit current, right? Okay, so under Poisson condition, this guy is  $I_{\text{naught over 2}}$ , okay? So now let us say  $I$  increase, I am trying to figure out a short circuit current and let us say I have shorted this with the battery, right? Let us say this battery voltage is also VCM.

It can be anything. I kept it VCM just to ensure that M1, M2, M4 are all in saturation, right? And the incremental current will always flow into the battery, okay? And let us

say on top of this, I have applied some incremental voltage  $\Delta V$ , positive incremental voltage  $\Delta V$  to M1 and negative incremental voltage  $-\Delta V$  to M2, right? So what is going to happen now? Can you walk me through the steps of what is going to happen? So what is going to happen to the current through M1? Current through M1 has to increase and the current through M2 has to decrease, right? That is for sure. But how much will it increase or how much will it decrease by? That totally depends upon what is this voltage  $V_X$ . Now what is this voltage  $V_X$ ? This is the differential excitation. What is this voltage? That voltage will be 0, right? So this voltage will be 0. So how much will, so how much increase in current M1 will face? It will face an increase of  $g_{m1}$  times  $V_{gs}$ , incremental  $V_{gs}$ .

What is incremental  $V_{gs}$ ? Incremental  $V_{gs}$  is  $\Delta V$ . So this becomes plus  $g_{m1}$  times  $\Delta V$ , okay? How much current will reduce in M2? It will be reducing by the same amount. So the current that will be, that will eventually flow through M2 will be  $I_{O2}$  by 2 minus  $g_m$  times  $\Delta V$  in the direction shown, right? So in other words, the total current through M2, the total current through M2 will be  $I_{O2}$  by 2 minus  $g_m$  times  $\Delta V$ , right? Okay. What is the current that is flowing through M4? That is still  $I_{O2}$  by 2, right? So what is the current that is going into  $V_{cm}$ ? What is the incremental short circuit current that is going into  $V_{cm}$ ? This current has to be  $g_m$  times  $\Delta V$ . And this is the incremental short circuit current that we have been dealing with till now.

So this picture essentially combines what's actually happening in terms of the direction of the current flow with regards to the change in quiescent current, right? So you can see that we are able to combine the incremental picture on top of the quiescent picture to show you what, why actually you have a short circuit, okay? The short circuit current here is essentially the difference current between the current that is being pushed from the top and the current that is being sucked from the bottom, okay? So now the question is, can we do a bit better? Now some of you might have already noticed that instead of putting plus  $\Delta V$  by 2, I put  $\Delta V$ . I mean, we can as well go back and do  $\Delta V$  by 2. So everything goes by 2. So this will also be  $g_m$  times  $\Delta V$  by 2, right? So now the question is, can we do slightly better? Now so even before we ask the question, under quiescent condition, right? Under quiescent condition, what do you think is happening? Let us go back to quiescent condition. So what is the current to this branch? This is  $I_{O2}$  over 2.

What is the current to this branch? This is  $I_{O2}$  over 2, right? So the question that I am asking is, is it necessary for us to have a separate current mirror, right? Is it necessary for us to have a separate current mirror which sinks in current, which basically gives us this current  $I_{O2}$  over 2 when there is a adjacent branch to which  $I_{O2}$  over 2 current is already flowing, right? So the reason I am asking this question is, it seems like we are

unnecessarily using two current sources where we can get by using only one, right? Because the current through this guy is  $I_0$  over 2 and I essentially am using M4 prime to get a current of  $I_0$  over 2, right? So why do not I use the current in the left branch and mirror it? That I can always do, right? Okay, so if we do that, what is going to happen? So I get rid of this guy and say that since the current mirroring is here, current actually is flowing through this, so why cannot we simply connect it like this and let us call this M3? Okay, so what is the Poisson current through M3? What is the Poisson current through M3? Poisson current through M3 is exactly equal to the Poisson current through M1 which is  $I_0$  over 2 and this volt, because of the current mirroring effect, the  $V_{st}$  of M3 becomes exactly equal to the  $V_{st}$  of M4 which means M4 also gives us  $I_0$  over 2, so this gets biased perfectly. Correct? Okay, this makes sense, but now let us see something interesting that is going to happen. So this enabled us to not only reduce current, right, not only reduce the use of one less current source which is always a good thing, but it gives us another advantage and you will see that shortly, right? So let us go back and do that incremental analysis once more. So let us say I have applied  $\Delta V$  plus  $\Delta V$  by 2 here minus  $\Delta V$  by 2 here, right? Now again, what is  $V_x$ ?  $V_x$  is 0. So what is the Poisson current through M1? Now what is not the Poisson current? What is the total current through M1? Total current through M1 is  $I_0$  over 2 plus  $g_m$  times  $\Delta V$  by 2.

What is the total current through M2? What is this total current through M2? This is  $I_0$  by 2 minus  $g_m \Delta V$  by 2, right? So far so good. Everything is identical, but now tell me what is the total current through M3? What is the total current through M3? Total current through M3 is same as the total current through M1, correct? So total current through M3 becomes  $I_0$  over 2 plus  $g_m \Delta V$  by 2. Now note that M3 and M4 are current mirrors. So whatever flows through M3 has also, also needs to flow through M4, right? Which means what is the total current through M4 now? The total current through M4 is  $I_0$  by 2 plus  $g_m \Delta V$  by 2, right? So now if that is the case, what is the current, difference current that is flowing into this incremental short circuit or into  $V_{cm}$ ? Note that the current through the top is  $I_0$  by 2 plus  $g_m \Delta V$  by 2, current from the bottom is  $I_0$  by 2 minus  $g_m \Delta V$  by 2. So what is the short circuit current? The short circuit current is  $g_m$  times  $\Delta V$ , right? So note that we are getting a doubling of the short circuit current by simply using the current mirror method of biasing, right? So that is a good thing.

Why? Why is that a good thing? That is essentially, I mean without using any extra power, right? Without using any extra power source, we are able to increase the, increase the short circuit current by a factor of 2. So that is great, right? Okay. So in the next lecture, what we will do, we will analyze this particular structure in a bit more detail. So

we will see what is the CMRR of this, right? We will see what is the output resistance of this and we will take it from there, okay? Okay. Thank you.