

Course name- Analog VLSI Design (108104193)
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Week- 10
Lecture- 28

Now, welcome back, this is lecture 28. So up until now, we have been discussing various flavors of amplifiers, right. We also we discussed various flavors of control sources. And in the last couple of lectures, we also saw that the same amplifiers that I mean the same set of circuits that we were designing with an NMOS transistor can also be designed with a PMOS transistor, right. And I am sure you took the opportunity and went through the, went through this week's assignments, right, where we saw the same configurations of amplifiers using NMOS's which were earlier done with NMOS's, right. Okay, so far so good.

So in this lecture, I would like to draw an attention to one, an important non-ideality of all the amplifiers that we have been dealing with till now, okay. So this has kind of crept in, because we did not address it right up front. The moment I tell you what the non-ideality is, it will immediately hit you as to why this is a problem. So what is in our common source amplifier, what is the gain? So A_v in a CS is minus g_m times R_L , right.

If it is loaded with an active resistor, right, then it is minus g_m times r_{ds} , okay. Now the issue is g_m is dependent on mobility variation, threshold voltage variation and one might say that if we bias a transistor using constant current source, its g_m is not dependent on threshold voltage variation granted, but still is dependent on mobility variation, right. Similarly, r_{ds} is also dependent on λ , which is again a function of the device which might change based on the structure of the device, right, based on ambient conditions also. And λ is also proportional to i_{ds} . Again if it is biased with constant i_{ds} , λ is less likely to change, but the mobility of the transistor will change, right, mobility of the transistors will change if we change the temperature.

So what is the issue then? The issue is this amplification factor, right, is not constant, right. And what is the implication? The implication is, a very practical implication is you can consider your music system, right, when you slide the volume bar of your music system to some, maybe to some level, you expect the sound, the volume of the sound that is coming out to be of certain value, certain level, right. You do not expect that volume to go up and down based on the temperature outside, right, which essentially is another way of saying is that you want a constant gain regardless of the ambient conditions around the amplifier. So even though your, even though our transistor here

has a gain, a common source amplifier can give us a gain, right, but this gain is not very constant, right. So this is, this gain is dependent on ambient variations, right, okay.

This is often called, this is often referred to as process voltage and temperature variations, right. So this is called process voltage and or PVT variations. So the gain of our amplifier is dependent on PVT variations. Why P? P is process, by process I mean, so you make your transistor in batches, right, you make your transistor in batches of wafers, so it is quite possible that one batch of wafer has a different mobility than the next batch, right. So if your design, if your amplification factor is dependent on mobility, then obviously your amplifier is susceptible to process variation.

What is temperature? Temperature is quite evident, right. Mobility again depends on temperature. So the gain is dependent on mobility. What about voltage? This is not very obvious right away. This essentially implies that if the quiescent voltage, if the voltage, if the supply voltage changes, right, or even if the quiescent voltage changes, is your amplifier gain likely to change, right.

As we saw in our case, it is likely to change, right. If for example, the I_{DS} changes, right, instead of voltage you can think of current, right. If the current changes, g_m will change, if g_m changes, then you will have variations of gain, right. So that is a problem. What is the other problem? The other problem is a common source amplifier, this is problem number 1, right, this is problem number 1.

What is the other problem? Common source amplifier is dependent on, the gain of the common source amplifier, so this is ideally R_{DS} , let me write it in this way, mod of this is $g_m R_{DS}$ parallel R_L , right. So the gain of the common source amplifier is dependent on R_L , right, gain dependent on R_L . So let me put a cross points before this because these are all negative attributes, ok. So if gain depends on R_L , which means that if I change R_L , the gain changes and we have talked about this. Why does gain depend on R_L ? Because a common source amplifier that is very cold is a voltage control current source, ok.

But what is the good thing about a common source amplifier? The good thing is it can give high gain or even decent gain, it can get decent gain, right, ok. What about a common drain amplifier? What about a common drain amplifier? The common drain amplifier that we saw could have, was giving us a gain of approximately 1 under the condition of $g_m R$ to be much greater than 1, right. So under the, if $g_m R_L$ is much, much greater than 1, right. So what are the positive attributes? The positive attribute is since as long as we can ensure $g_m R_L$ is much greater than 1, so this is PVT invariant, right. What is the other positive attribute? The gain is independent of, gain is

independent of R_L , right, as long as you satisfy the inequality.

Now what is the negative attribute of this? Obviously the big negative attribute is you do not get gain, right, gain is less than equal to 1, right. In fact gain is always less than 1, you cannot get more gain than 1, right. So this becomes a problem, ok. So what we would like to do next is to build, is to try and build an amplifier, right. So what we would like to do next is goal, we modify our goal and say that we would like to build an amplifier that is PVT invariant can drive or let me say R_L invariant and what is the last thing, gain greater than 1, right.

So this is our new goal using the structures that we have already seen, ok. So let us target the first guy, let us target this. So we would like to build an amplifier whose gain is PVT invariant. So forget about amplifier for the time being, can you think of something in an IC on an integrated circuit that is that does not change with temperature variations, right. So what a resistance, value of a resistance does it change with temperature? Yes it does.

If we think of trans conductance does it change with temperature? Yes it does. So it looks like apparently everything in an integrated circuit changes with temperature, right. However, if I take a ratio of two components, right, let us say I take the ratio of two resistors, right. So let us take the ratio of two resistors, let us say R_1 , let us say I take a voltage divider, ok. Let us say this voltage is V and this is R , this is R .

What is this value? This is always V by 2, correct. So let us say now temperature changes, what is going to happen? This R is going to change and go to R plus ΔR , but the bottom R is also going to change because in an integrated circuits all the R , all the resistances are placed in their neighborhood, right. So if the wafer hits up, everything hits up. So this guard is also going to change by the same amount because they are made out of the same material and they are sitting next to each other, which means the ratio is invariant of PVT, right. So what is the first observation? Ratios of identical structures or ratios of identical electrical elements are PVT invariant, ok.

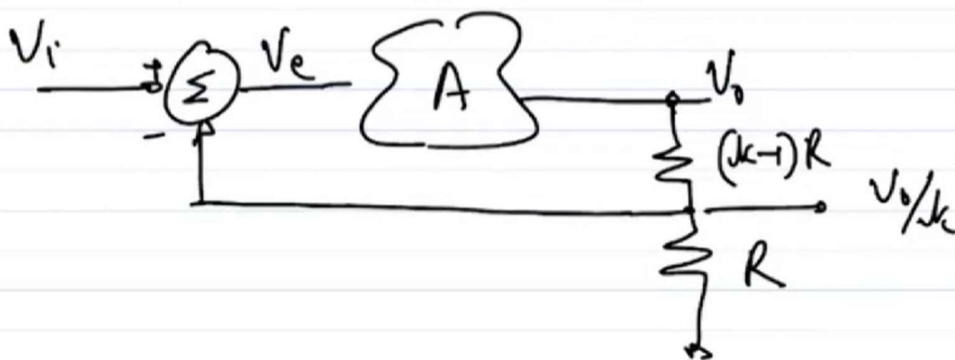
However, when you take a ratio, you will always get something which is less than 1, right. If you take a resistor divider, you cannot get a voltage of greater than V , you will always get a voltage less than V , right. So this is a positive attribute, this guy is a positive attribute and the negative attribute is ratio is always less than 1, right. However, we saw a different problem with the common source amplifier. In a common source amplifier, what did we see? We saw that the gain is more than 1, but the gain is PVT dependent, right.

So if we stop and think that looks like I have two different, I have two structures which have different problems. One structure is a common source amplifier where we can get high gain, but the gain is PVT in PVT variant, right. In other case, I have a ratio of resistances where we get low gain, right, gain less than 1, but the ratio is PVT invariant. So the question is can I marry these two together? Can I marry these two properties and get a gain of more than 1, right, while making the gain PVT invariant, right. So that is what we will explore next, ok.

Build a ckt that drives $V_e \rightarrow 0$

$$V_e = V_o - kV_i \rightarrow 0 \quad (\text{goal})$$

Define $V_e = \frac{V_o}{k} - V_i \rightarrow 0$



So in order to explore the same thing, we will resort to negative feedback and I will take you back to few lectures back where we introduced negative feedback while trying to design a current controlled voltage source, right. So a quick refresher, what is negative feedback? In negative feedback, what is the principle? The principle is observe the output, compare with expected output, right, drive the output with the knowledge of the difference between observed output or actual output and expected output, right. So this is step 1, this is step 1, this is step 2, this is step 3, ok. So in order to build an amplifier, what is step 1? We have to first observe the output and compare with the expected output, right, expected output, ok. So what is the expected output in an amplifier? What is the expected output? So we need to build an amplifier of V_o equal to let us say K times V_i where K is greater than 1, correct.

So what is the expected output? Is equal to K times V_i , ok. What is the actual output or

observed output? It is V_0 , right, so which means that I have some output V_0 and I have some expected output K times V_0 , K times V_i , right. I have some expected output K times V_i . So everything is in some voltage node because we are dealing with voltage quantities. So what is the difference between the expected output and the actual output? The difference is I call it V_e , right, let us say we call it error voltage.

The V_e is V_0 minus K times V_i , right. So based on this difference, we have to take some action. What action should we take? Based on this difference, we should take an action such that the error voltage goes to 0, right. So what action should we take? We should take an action such that build a circuit that drives the error voltage V_e to 0, right. So essentially what we need to do? We need to compare V_0 with KV_i , right.

We need to generate V_e , compare by generating V_e means I am comparing V_0 with KV_i , then I am generating the difference and I am doing something to drive the difference to 0, right. So now do you see a problem in this formulation? What I am saying is to get V_e , I need V_0 which obviously I have because V_0 is some output node but we also need K times V_i . So if we already had K times V_i , then why would we bother doing all these things, right? The whole purpose is to generate K times V_i . Now if I require K times V_i to find out V_e , then looks like there is no point in pursuing this line, this approach, right? You are right but note that what is the goal? The goal is to drive this to 0. This is the goal, right? This is the goal.

If we can drive this to 0, then V_0 will be actually be equal to KV_i but we do not have to do that. But can we just modify this equation and say that we will define V_e as V_0 over K minus V_i and then drive this to 0 where K is more than 1. Is it possible? Yes, it is possible. Why? Because K is more than 1 and I can generate V_0 over K by using a simple resistive divider, right? Okay, right? Great.

So let us do that. So block diagrammatically, what is going to happen? So now what is the, we have an output V_0 and we have to now compare it with, compare the V_0 over K with V_i , right? So how do I get V_0 over K from V_i ? I put a resistor divider, right? Okay. So value of let us say K minus 1 R and R , right? So the voltage here will be V_0 over K , right? So this voltage here is V_0 over K and I have an input V_i . I need to compare V_i with V_0 over K , right? Block diagrammatically, what is comparison? It is nothing but taking a difference. So let us say I take a difference. I take a difference between V_0 and between V_i and V_0 over K , then this becomes V_e , correct? So we have to take some action on V_e and derive it and put some block so that some, once we take some action on V_e and drive V_0 , it will drive V_0 towards our desired output, right? So that is the whole feedback system, right? This is the whole feedback system that we need to build, okay? Okay, fine.

So that is as far as the block diagrammatic representation is concerned, but we have left out some critical pieces here and the critical piece that we have left out is we do not know what is the transfer function of this block, right? So in a feedback system, we often call this G , right? So let me call this A for the time being, right? A beta, right? So let me call this A for the time being. We don't know what A is, right? Okay. But before we proceed, let's see whether this loop actually makes sense or not, right? What do I mean by that? So what I am essentially trying to get to here is how do I know that this loop is indeed a negative feedback? So what is the property of negative feedback? One of the properties of negative feedback is the output, whatever happens, the output will be stable, right? So output V_0 will be K times V_i , correct? Other way of stating the same thing is if there is some excitation in this loop anywhere, the output will still remain K times V_i , right? Okay. So if the output is supposed to be steady in the presence of any excitation, then is there a way to figure out by looking at the loop that whether the sense of the loop is in the right direction or not? There indeed is because we can inject some hypothetical injection anywhere in the loop and see whether the action of the loop is to suppress the injection or increase the injection, right? So let me illustrate that with an example. So let's say you have set this loop in the way I have shown here and let's say we inject, somehow we inject an input.

We yank this voltage up. What happens if I yank this voltage up? What happens to V_e ? Note that it goes through a negative sign. So this voltage goes down, right? If A is positive, what is happening to V_0 ? V_0 goes down. So what is happening to V_0 over K ? V_0 over K also goes down, right? Which means that the loop is trying to correct any excitation that I am trying to inject anywhere. You can do that anywhere in the loop, right? So let's say I inject, I yank this voltage up.

This voltage is going to go up. This voltage is going to go down. Since sorry, this voltage is also going to go up. Since we have a negative sign now, so this voltage is going to go down and the loop is trying to suppress the initial excitation that we give. Now whether the loop will be able to sufficiently suppress the excitation or not is a matter of detail that we will have to look into, but at least the sense of the loop is in the direction to correct itself. So what happens, let's say for example, I have a gain of minus A , if the gain is negative.

So let's play this game again. So let's inject an excitation here, right? If I inject an excitation there, this voltage goes down. What happens to V_0 ? This voltage goes up because I have a negative gain, which means this voltage goes up, right? See this is a positive feedback. I have injected an excitation and when you come around the loop, it reinforces the, it has a tendency to reinforce that excitation, right? Which essentially

means that this is a positive feedback. This is not a negative feedback loop, ok? How should I change the sense of the loop? There are two ways to change it. One of course is to change the sign of A and the other is to say, I will change the sign of the summer, right? If I change the sign of the summer, then what happens? Let's say I inject, I inject, I make an hypothetical injection.

This goes up, this goes down, this goes down, right? So sense of the loop seems to be, seems to be fine now, ok? So let's, so let's go back to our original loop. So now if we are satisfied that this loop is in, this loop indeed is in negative feedback, the next thing is to find out what is the expected value of A which will give us the negative feedback, right? I mean which will give us what we want. It's a negative feedback with the expected value of A for which V_0 will be equal to K times V_i , right? Other way of saying the same thing is that what is the expected value of A for which V_E will tend to 0 because ultimately we would want the V_E to tend to 0, right? So ultimately we would want this V_E to tend to 0. If the loop is operating properly, if we have designed it properly, then V_E should tend to 0 and that is what we will see next, ok? So let's see. So it's a simple loop and let's find out what is V_0 ? V_0 is equal to A times V_E .

What is V_E ? V_E is V_i minus V_0 over K , correct? So V_0 $1 + A$ over K is equal to A times V_i which means V_0 is A times V_i by $1 + A$ over K and if I bring K, if I multiply everything with A over A for example, so this becomes V_0 is K times V_i by $1 + K$, correct? So can you, now what does this tell you? This is essentially telling us that if we set A to be infinity, right? If we have an infinite gain amplifier, if V_0 limiting value out of V_0 when A tends to infinity is K times V_i , right? So this is what this expression is telling us and if this is achieved, what will be V_E ? The limiting value of V_E when A tends to infinity is 0, correct? So essentially, if we tend, if we are able to tend A to infinity, right? V_E will tend to 0, right? So V_E will tend to 0 and if V_E tends to 0, what happens to this node voltage? This node voltage tends to V_i , if this node voltage tends to V_i , V_0 over K tends to V_i , then V_0 tends to K times V_i , right? So this is a typical way of analyzing a negative feedback loop. We never analyze a negative feedback loop through the forward path. We always analyze a negative feedback loop through the reverse path and what is the starting place? The starting place is the fact that if the loop works properly, which means that we have a very high gain amplifier, in this case, A tends to infinity, if it's a negative feedback and we have a very high gain amplifier, then the job of the loop will be to suppress the input of the amplifier and it will go close to 0, right? So negative feedback with high gain amp pushes the input of the amp to 0, right? So since this pushes the input of the amp to 0, which essentially means that these two voltages, V_i and this voltage, right? V_0 over K, the input voltages of the summer, they track each other. In other words, they are virtually shorted, right? These two nodes become virtually shorted, right? So I am sure you have seen this concept in different

domains and V_0 over K , this is V_i . So if these two nodes are virtually shorted, what happens to V_e ? So V_e goes to ground, goes to 0.

So this is not an actual ground, but this is a virtual ground, right? So V_e becomes a virtual short if A tends to infinity. Okay? Okay. So now our job will be to replace this summer and this A with real transistors, right? Before we move on, can you tell me what happens if A changes? Let us say I start from, let us say A is finite. Assume if you cannot get infinite gain, you have to settle for something, right? So if A is finite, what is going to happen? What is going to happen to V_0 ? So V_0 we saw was A times V_i by 1 plus A by A . So this is what is the expected value? What is ideally what is the expected value? So V_0 ideal is A times V_i .

Now we have an error, right? What is the error? By error, I do not mean V_e , I mean the error between the expected and the actual output, right? So error in V_0 is A times V_i minus A times V_i by 1 plus k over A , which is equal to A times V_i , 1 plus k over A minus 1 by 1 plus k over A , right? So what is error percentage? You divide by the expected, you divide the error divide with the expected value that is k times V_i . So the error percentage becomes A over A by 1 plus k over A , right? So if A is much greater than unity, then or A is much greater than k , this becomes approximately equal to k by A , if A over k is much greater than 1, right? So let us say you shoot for A to be equal to 1000, right? And k to be equal to let us say 2, you want a gain of 2, you build an amplifier of again 1000 or let us say 100, you build an amplifier of gain 100. How much will be your error? It will be 2%, right? And let us say because of ambient variation, A changes from 100 to let us say 200. How much will be your error? Error will be 1%, right? So still, even if you have gain is varying, even if the gain of your amplifier is varying by 2x, right? Even if the gain of the amplifier is varying by 2x, your output is accurate to 98%, right? Your error is so essentially what is the implication? The implication is the error, steady state error or the error percentage is times 100, right? I forgot about the times 100. The error is inversely proportional to the gain, okay? Since the error, note that the output is not proportional to the gain, correct? In a common source amplifier, right? In an open loop common source amplifier, the output is proportional to the gain, correct? So in this particular case, error is inversely proportional to gain, not the output, right? So output is not proportional to gain, correct? In open loop common source amp, output is proportional to gain.

Output is proportional to gain. So if gain changes by certain factor, the output also changes by certain factor. In a negative feedback loop, if the gain changes by the certain factor, the error percentage changes by certain factor, not the output, right? Output is almost steady. So for example, in this case, in a common source amplifier, let us say if the gain goes from 100 to 200 or if gain drops from 200 to 100, your output can or if the

gain drops from let us say 6 to 3, your output can change by a factor of 100%, right? Or like 50%. But in case of a feedback amplifier, if the gain changes by let us say from 200 to 100, your output only changes by 1%, right? So that is the critical difference, right? So that essentially ensures that a negative feedback way of building an amplifier, right? A negative feedback way of building an amplifier gives you a PVT invariant way of designing an amplifier, ok? Ok, so what about if I put a load resistance R_L , what is going to happen? So let me sketch the circuit somewhere again and then we can talk about it. So, what if I put a resistance R_L , is the gain dependent on R_L ? I mean everything is dependent but is the gain proportional to R_L , does it change drastically if I change R_L ? So let us see.

So essentially what we are trying to figure out is that is this a voltage control voltage source or a voltage control current source? A common source amplifier was a voltage control current source. So is this guy a voltage control current source or a voltage source? So what is the test? The test is to check output impedance, right? Let us check the output impedance. To check output impedance what should we do? We should desensitize the input. So I rounded the input, ok? I do not need this R_L , all I need is a test voltage, right? And I need to figure out what is the I test, ok? Ok, so let us see. So if I put a test voltage here, what is this voltage? This voltage is V_{test} over K .

What is this voltage? The other terminal is grounded, so this is minus V_{test} over K , right? So now we have a problem because the way we have sketched this amplifier is it is an ideal amplifier, it is an ideal A . So what we put a block diagram of ideal A which means it is a voltage control voltage source. So which means this can be any voltage regardless of what am I driving, what I am driving with. But note that in our case we have, we are driving the output with a voltage source which means there is a conflict, right? In this block diagram itself there is a conflict. But we need not worry too much because we know that this A , this A is some sort of a common source amplifier.

So that is what we have, that is the only high gain stage that we have, right? All the other stages that we have seen have low gain. So we know that this is the common source amplifier which means that it is a voltage control current source, right? So let us assume this is a voltage control current source. So let me, so let us say this is V_E , so I have a voltage source. Because the way I have sketched, let me draw this then we will talk about the because part.

Okay, so this is g_m times V_E . Note that I have put the arrow up in the direction opposite to that of a common source amplifier because in this case we assume the gain has to be positive. In a common source amplifier the gain is negative. You will see the implication of that shortly, right? Either in this lecture or in the lecture after this. But

because from the block diagrammatic perspective we know we want it to be, if V_E increases we want the output to increase which means I have to push current out, right? So that is all we are doing, okay? So if this is the case what is the current that I am drawing in? The current that I am drawing in is g_m times V_{test} over k , correct? Okay? Okay, so what is I_{test} ? I_{test} is g_m times V_{test} over k plus V_{test} by A_r , right? So what is V_{test} over I_{test} ? Is 1 over g_m by k plus 1 by A_r , right? So as long as we can set this to be much less than R_L , right? If we set, this is R_{out} essentially, right? So this is R_{out} . So R_{out} becomes 1 by, let me say we put the k on top instead of carrying it all the time.

Let me call this g , g_m plus g , right? So if this is much much greater than R_L , sorry much much lesser than R_L , if this is much much lesser than R_L then this is a voltage control voltage source. So in other words, all we have to ensure is that g_m times R_L plus g times R_L should be much much greater than, or much much greater than k or this should be much much greater than 1 . So is it in our control? Can we do this? Yes, we can do this because g_m is in our control, k is a kind of in our control because it's the gain that we are trying to set. Even if k is not in our control, g_m is in our control, g is in our control. So if we get certain values of R_L , we can make g_m times R_L by k plus R_L by k to be much greater than 1 .

$$\Rightarrow R_{out} = \frac{k}{g_m + G} \ll R_L \quad (V_C V_S)$$

$$\Rightarrow \frac{g_m R_L}{k} + \frac{G R_L}{k} \gg 1$$

If we do that, this becomes a voltage control voltage source, which means now we have a voltage control voltage source with a gain of more than unity, which can drive any load under certain condition and also whose amplification factor is independent of process voltage temperature variation. Right? Ok. Thank you.