

Course name- Analog VLSI Design (108104193)
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Week- 7
Lecture- 20, Module-1

Welcome back, this is lecture 20. So, in the previous lecture we saw that when we bias our transistor with a constant current source when the source when the current source was connected to the source of the transistor, we could use this configuration to our advantage in order to realize multiple topologies. One topology was obviously the common source amplifier and the other topology was to design a common drain amplifier where the resistance with the load resistance R_L is connected to the source and the drain is incrementally grounded, right. Okay, so this is R_S , this is V_i , C_1 , this is C_2 . Then we also saw what are the constraint on C_2 , we also saw what are the constraint on C_2 in order to ensure that this behaves like a common drain amplifier. We also discovered one interesting property of the MOSFET and the interesting property that we discovered was the fact that the resistance looking into the source of the MOSFET is equal to $1/g_m$, right.

So, in case of an ideal MOSFET biased in saturation, right, so biased in saturation, right. So, it means gate is with some voltage, drain is with some voltage, source is with some voltage, the incremental resistance looking in, right, the R_{TH} looking in is equal to $1/g_m$ of the transistor and we also knew that the R_{TH} looking into the drain of the transistor, right, when the transistor is biased in saturation R_{TH} is equal to infinity because its Y_{22} was 0 or its g_{ds} was 0. So, hence we were able to use common source amplifier to get out, to drive all the current that was coming from the MOSFET into the load resistance and we got some voltage gain, in the process we also got some power gain. However, in this architecture we saw that since the resistance looking in into the source of a transistor, the incremental resistance looking into the source of the transistor is not infinity, in fact it is $1/g_m$ which can be small with respect to the resistance R_L , right.

So, in that case it seems like we have a contraption whose output resistance is lower than the load that it is driving. If the output resistance is lower, so if R_{TH} or R_{out} is much much smaller than R_L , then we could, we can use the block as a voltage source, right, as a voltage source. And using this property we try to see if we connect R_L to the source of M_1 , obviously through AC coupling capacitor C_2 , then whether this can behave like a voltage source and we saw that, right. We saw that as long as g_m of M_1 times R_L is much much greater than unity, which is another way of saying $1/g_m$ is much much smaller than R_L , then we saw that R_L was not loading, R_L was not loading the transistor, which essentially means that the transistor is now acting as a voltage source while driving R_L , right. So, this is where we stopped.

So, we would like to explore this architecture a bit more and we would also like to see what are the swing limits of this architecture. If you recall we saw the swing limits of a common source amplifier, few lectures back. We would like, we would also like to see what is the maximum swing that you can apply, maximum input voltage that you can apply to this transistor in order to establish its limits, right. So, if we, obviously I cannot apply infinite amplitude sine wave at the input. So, what we would like to see what are the, what are the swing limits for this architecture, right.

So, let us go about analyzing the same. Ok, so what should we do to establish swing limits? What is the first thing that we need to do? We need to first find out the quiescent drain and quiescent voltages and currents. So, what is the quiescent voltage? So, we establish that I have some current I_{DQ} and we have some voltage V_{GQ} , right. So, let me use a different color. So, we have a voltage V_{GQ} , this current is I_{DQ} , ok.

So, what is this voltage at the source? This voltage at the source is V_{GQ} minus V_{GS} for a current of I_{DQ} . So, let me call this V_{SQ} , ok. What is the, what is the current, quiescent current through R_L which is clearly 0, right. What is the quiescent current through the source, right, through the input V_i which is also clearly 0. Let us assume V_i is equal to $V_p \sin \omega t$, ok.

Further we can assume that in this particular instance we can, let us assume that C_1 and C_2 are large enough to be treated as short circuits, ok, at frequency of ω . And now if that is the case, so what is the next step to do? Since the quiescent point is established we would also like to see what is the incremental currents and voltages at the different terminals of the transistor and also through the transistor, the currents through the transistor. So, the next step will be to figure out what will be the incremental equivalent of this. So, let us do that what is the incremental equivalent of this? So, V_i R_s capacitor shorts, right, because we assume they are to be large enough for to be treated as a infinite valued. This is R_1 parallel R_2 , ok.

So, this is the gate voltage V_g , this is our transistor, right. This is the source, this is V_s , ok. What is connected at the source? At the source clearly I have a infinite value capacitance C_2 and then a resistance R_L to ground and what happens to the current source I_{DQ} that opens up, right. So, this I have R_L to ground. So, just to ensure that I am drawing symmetrically I will just connect all the grounds together.

That is always a good practice to avoid mistakes, right. Now, this value is g_m times V_g , the drain is clearly a short circuit. This goes here and this is my incremental V_{naught} , ok. So, what is V_g ? V_g is equal to V_i times R_1 parallel R_2 by R_s plus R_1 parallel R_2 , right. So, which is approximately equal to V_i if we size R_1 parallel R_2 to be much much greater than R_s and since R_1 parallel R_2 is in our control we can decide to do that.

So, let us assume that we have done that, ok. So, V_g is established. What about V_s ? What

about V_s ? So, in order to figure out V_s what we need to do? We need to solve the KCL at the node V_s , right. KCL at V_s will lead to g_m times V_{gs} will be V_s times g_l , but g_l is of course 1 over R_L , ok, which is g_m times V_g minus V_s which is V_i minus V_s which is equal to V_s times g_l . Again, even though it is obvious it is instructive to note that this is g_m times V_{gs} , this is not g_m times V_g because your source is not grounded, ok.

$$\begin{aligned}
 \text{KCL @ } v_s &\Rightarrow g_m v_{gs} = v_s G_L \\
 &\Rightarrow g_m (v_i - v_s) = v_s G_L \\
 &\Rightarrow g_m v_i = v_s (g_m + G_L) \\
 &\Rightarrow v_s = v_i \left(\frac{g_m}{g_m + G_L} \right) \\
 &\Rightarrow v_s = v_i \frac{g_m R_L}{1 + g_m R_L}
 \end{aligned}$$

So, now if I rearrange the terms what do we get? We get g_m times V_i is equal to V_s g_m plus g_l and hence V_i V_s over V_i or simply V_s is equal to V_i g_m by g_m plus g_l and if you are comfortable now writing in terms of R_L we can write it as V_i times $g_m R_L$ by 1 plus $g_m R_L$. So, this is V_s and this is exactly what we had got in the previous lecture, ok. So, we got the gate voltage, we got the source voltage, what is the other thing that we need? We need the incremental current, right. So, I_{ds} incremental is g_m times V_{gs} which is g_m times V_g minus V_s which is V_i minus V_i times $g_m R_L$ by 1 plus $g_m R_L$ which means I_{ds} is equal to g_m times V_i by 1 plus $g_m R_L$, ok. So, before we proceed let me draw your attention to one more level of detail.

So, in the previous lecture we saw that if $g_m R_L$ is much much greater than 1 , right then V_s is independent of R_L and hence this was a voltage source, right or in other words in this case and V_s in this case was is equal to V_i . So, this was acting as a voltage buffer, right. So, if we use the same constraint, if we use the same constraint here what we get? If $g_m R_L$ is much much greater than 1 , what we get I_{ds} becomes $g_m V_i$ by $g_m R_L$ which is V_i over R_L , right. So, this seems to be a very simplistic result, right. It seems like I mean after doing all these things and the current the incremental current through the transistor is essentially the input voltage divided by R_L , right.

$g_m R_L \gg 1$
 V_s is independent of R_L
 $V_s =$

$$\Rightarrow V_s = V_i \frac{g_m R_L}{1 + g_m R_L}$$

$$i_{ds} = g_m V_{gs} = g_m \left(V_i - \frac{V_i g_m R_L}{1 + g_m R_L} \right)$$

$$\Rightarrow i_{ds} = \frac{g_m V_i}{1 + g_m R_L}$$

So, is this only mathematics or is there a reason behind it? I would like to draw your attention to the fact that there is in fact a reason behind it and the reason is as follows. So, if we honor the condition of $g_m R_L$ to be much much greater than 1, what do you think the value of V_s or V_O would have been? V_s or V_O would have been equal to V_i , right. So, V_O , this voltage would have been equal to V_i if $g_m R_L$ was much much greater than 1, right. If this voltage, if the incremental voltage at the output V_O would have been equal to V_i , what do you think the incremental current through R_L would have been? So, in this case, I_{R_L} would have been V_i over R_L and since this would have been V_i over R_L that current has nowhere to flow but through the transistor. Hence, the incremental current through the transistor would also have been V_i over R_L .

Very often you will see that in case of a voltage buffer we often make the approximation that the incremental current through the transistor is V_i over R_L without getting into all the details of whether $g_m R_L$, whether the details of output is g_m times V_i by 1 plus $g_m R_L$, you will get some amount of details through that but very quickly in order to develop intuition you can assume that under the condition that $g_m R_L$ is much greater than 1, the incremental current through this source follower is V_i over R_L and this is the genesis of that argument. Okay, so let us not get sidetracked, let us come back to the requirement of swing limits, right. So, let us sketch the circuit once more. Okay, so what is the total current or rather let us do the easy one first, let us do the voltage, right. Let us do the condition for linearity of M_1 , right.

So, for M_1 to go, M_1 to be away from linear region. What is the condition? The condition is that total drain voltage has to be greater than equal to total gate voltage minus threshold voltage, correct. What is the total drain voltage? Total drain voltage is V_{DD} because nothing is connected, I mean the drain is already connected to V_{DD} . So, condition is V_{DD} should be greater than equal to, what is the total gate voltage? Total gate voltage is a quiescent voltage,

right, quiescent voltage V_{gQ} plus the incremental voltage at V_g , correct, minus the threshold voltage. So, in other words, V_{DD} should be greater than V_{gQ} .

What is the incremental voltage at V_g ? We saw that the incremental voltage at V_g under the condition that R_1 parallel R_2 is much greater than R_s is equal to V_i minus threshold voltage which essentially means that the condition that we end up with is V_i should be less than equal to V_{DD} minus V_{gq} plus threshold voltage or since V_i is a sinusoid, we end up with V_p max should be less than V_{DD} minus V_{gQ} plus threshold voltage, ok. So, this is as far as the linearity condition, this is as far as keeping the transistor in saturation condition is concerned. What about the cut-off condition? Again, I would like to remind you the fact that we are not talking about absolute cut-off here, we are talking about the case in which the quiescent current plus the small signal incremental current goes to 0 and we saw that that is a far more prudent condition to use. You would anyway not want to drive your transistor to 0 current because your signals will be far more distorted and also we saw that the condition for small signal approximations to hold is that your the incremental V_{gs} has to be much lesser than the overdrive of the transistor, right. So, since that has to hold we cannot anyway shut the overdrive to 0, right.

So, we will use the, throughout the course we will use the condition of quiescent plus incremental, right. So, what is what was the quiescent current? So, what is the condition? The condition is i_D , as it has to remain greater than 0. So, what is the quiescent current? Quiescent current through the transistor in this case i_{DQ} , right. Quiescent current through the transistor was I_0 . What was the incremental current? Incremental current is i_{DQ} , right.

$$\Rightarrow v_i \leq V_{DD} - V_{GS} + V_{TH}$$

$$\Rightarrow \boxed{V_{pmax} \leq V_{DD} - V_{GS} + V_{TH}}$$

Cut-off condition

$$i_{DS} \geq 0$$

$$\Rightarrow I_0 + i_{DS} \geq 0$$

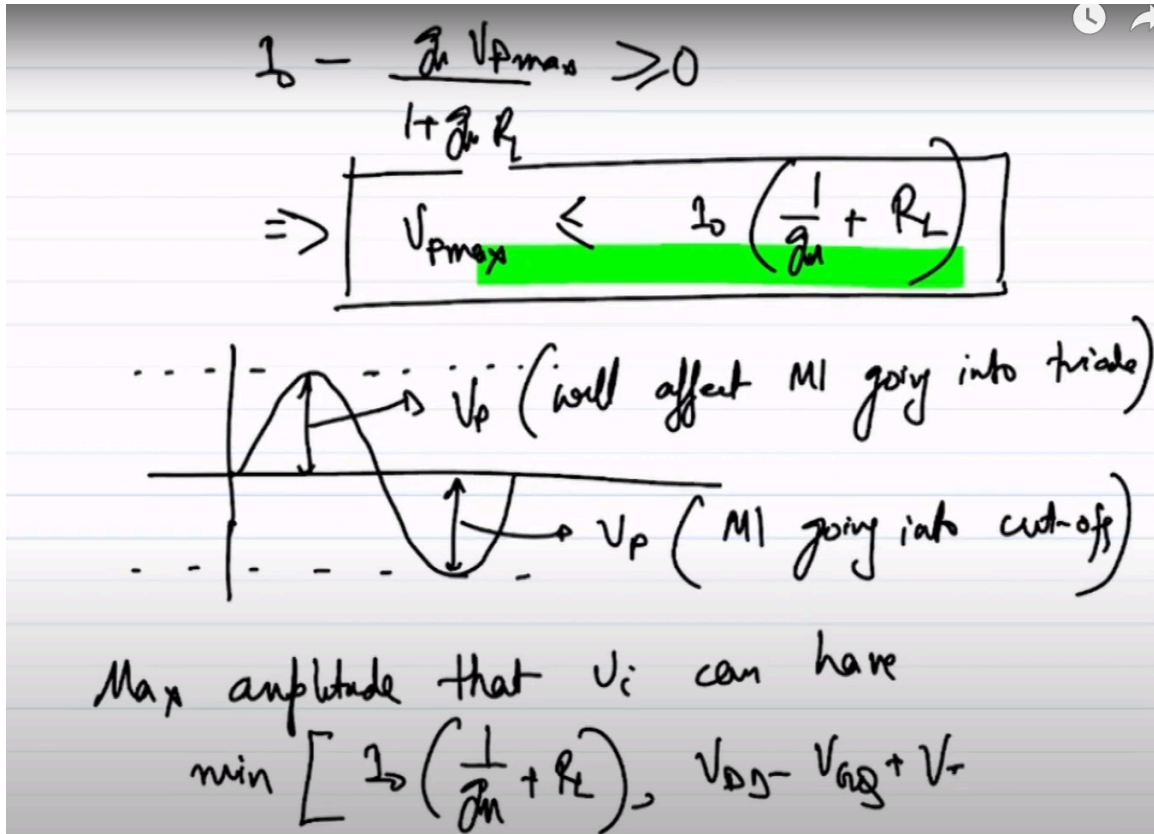
$$\Rightarrow I_0 + \frac{g_m v_i}{1 + g_m R_L} \geq 0$$

$$\Rightarrow v_i \frac{g_m}{1 + g_m R_L} \leq -I_0$$

i_{DS} has to be greater than equal to 0. What was i_{DS} ? i_{DS} was $g_m v_i$ by $1 + g_m R_L$ this has to be greater than 0. In other words, in this case $v_i g_m$ by $1 + g_m R_L$ has to be less than equal to minus I_0 . If we directly replace v_i is equal to minus V_p . So, this condition is likely to occur when v_i is equal to minus V_p , right.

Goes at the lower end of the cycle. So, this becomes I_0 minus g_m times V_p by $1 + g_m R_L$ or V_{pmax} in this case, right. $g_m R_L$ should be less than equal to 0, sorry, greater than equal to 0. In other words, V_{pmax} has to be less than equal to $I_0 / (1 + g_m R_L)$, ok.

Ok. So, this is essentially the constraint that you will have to honor while trying to figure out what is the max, what is the V_{pmax} when the sinusoid goes in the negative direction, right. So, essentially what we are saying is if I have, if this is, if this is my sinusoid, right. This is V_p , this will be V_p , this will also be V_p , but this V_p will affect M1 going into triode. The lower V_p will affect M1 going into cut-off. Now, if you want to ensure that it neither goes into triode nor into cut-off, then you will have to choose your V_p in such a way that it honors both the condition of this and this.



So, whichever is the smaller value, right, whichever is the smaller value will ultimately end up being the, will end up being the maximum amplitude of the sinusoid that you can apply. So, in other words, so V_p max amplitude that v_i can have will be minimum of this condition that is $I_0 \frac{1}{g_m} + R_L$ and this condition, right, $V_{DD} - V_{GS} + V_T$, $V_{DD} - V_{GS} + V_T$, ok. Thank you.