Course name- Analog VLSI Design (108104193)
Professor – Dr. Imon Mondal
Department – Electrical Engineering
Institute – Indian Institute of Technology Kanpur
Week- 2
Lecture- 10

Now, welcome back this is lecture 10. So, in the previous class we took a dive into the structure of the MOSFET device which we will be using to achieve our goal of amplification right. And we also we also spend some time in getting ourselves familiarized with the current voltage equations of a MOSFET. So, quickly a recap. So, if this is our n MOSFET and these terminals are marked as gate, drain and source and the current between the drain and source if we mark it as Ids right. What do we get? We get the fact that Ids is equal to there are three regions of operation.

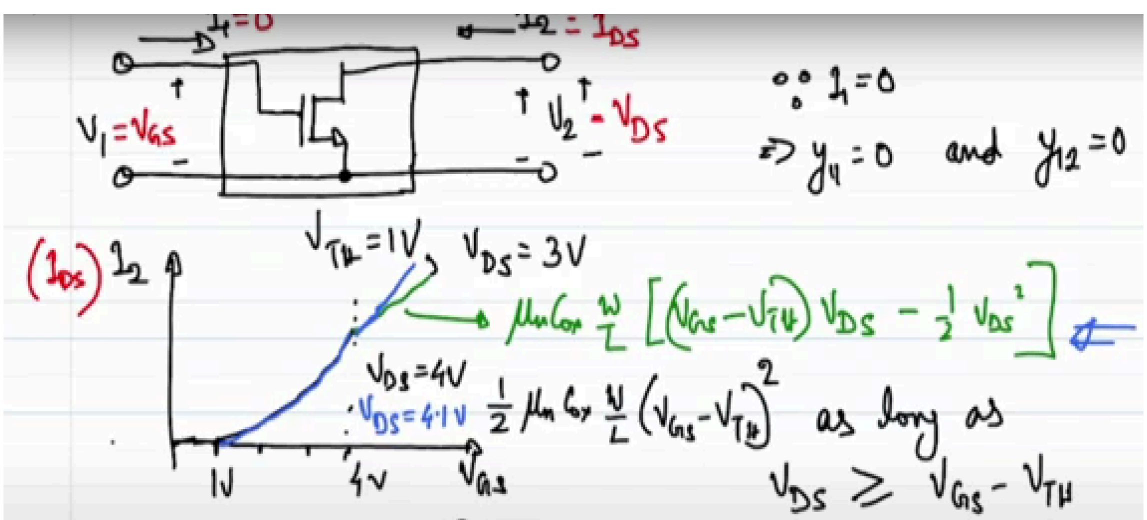
$$I_{DS} = \begin{cases} 0 & \text{when } V_{GS} < V_{TH} \\ \mu_n C_{ox} \frac{W}{L}\left[(V_{GS}-V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2\right] & V_{GS} > V_{TH} \text{ and } V_{DS} < V_{GS} - V_{TH} \\ \frac{1}{2}\mu_n C_{ox}(V_{GS}-V_{TH})^2 & V_{GS} > V_{TH} \text{ and } V_{DS} \geq V_{GS} - V_{TH} \end{cases}$$

$$I_G = 0 \quad \text{always @ DC.}$$

So, Ids is equal to 0 when Vgs is less than the threshold voltage right and Ids is equal mu n Cox W/L[(Vgs-Vth)*Vds- Vds/2 is squared] . This is when of course, Vgs has to be greater than threshold voltage, but an additional condition is to be satisfied and what is that? That Vds should be less than Vds minus the threshold voltage and we have one more region of operation which is where the current is half mu n Cox (Vgs - Vth)^2 under the condition that of course, again Vgs has to be greater than Vth, threshold voltage and Vds has to be greater than or equal to Vgs minus the threshold voltage right. The other thing that we did not touch upon was what about this current, what about the current that is going into the gate, what about Ig? Now note that if we go back to the previous lecture, if we go back to lecture 9 what we saw is that the MOSFET structure right by the way the MOSFET is made, we have a oxide or insulator between the gate and the channel right. So, this these parallel lines in a MOSFET symbol essentially symbolizes the fact that there is a capacitance between the gate between the wherever you have the gate terminal and wherever you have the channel that is the other side of the of the capacitance.

So, naturally the DC current DC Ig is always equal to 0 regardless of any other condition right. So, then I g is equal to 0 always at DC ok. So, now if these are the complete current voltage characteristics or complete current voltage DC characteristics of a MOSFET how does this help us, how does this help us in achieving amplification that is the root that is the root question right that is what we are after. So, if before stepping into answering that what we will do is we will plot this current voltage characteristics just like we plotted the current voltage characteristics of a of a two port network from where we hypothesized that the two port network needs to have certain type of characteristics in order to, in order to give us amplification of power right. So, we need to know so, what we will do next is we will sketch the I V characteristics of this MOSFET because this MOSFET in itself can behave like a two-port network and we will see whether this satisfies our requirement right.

So, what we will do? So, let us mark the terminals properly first. So, let us say so, this is the box right this is the box this is one port this is one terminal that comes out one terminal is common this is this terminal the first port is this the second port is this right and we call this V2 and we call this V1 this current that is going into port 1 we call this I1 and this current that is going in port 2 we call this I2. Now, note that now that we know that this is a MOSFET inside what do we know this I1 is always equal to 0 and I2 is essentially Ids right and V2 is Vds and what is V1? V 1 is Vgs ok. So, now that we know many things about MOSFET let us quickly sketch out the current voltage characteristics. So, what will be, I1 versus V1? I1 versus anything is essentially 0 right.



So, I1 is always 0. So, I do not have to, I do not have to plot the first two characteristics right I do not have to plot I1 versus V1 and I1 versus I2 which naturally means that Y11 is equal to 0 and Y12 is equal to 0. So, since I1 equal to 0 these are the natural consequences right. So, we are off to a good start because what we expected from an ideal from an ideal 2 port is the fact that I should have Y11 equal to 0 and Y12 should also be equal to 0. So, what are the other two things that we need to bother about Y21 and Y22 right.

$$\Rightarrow V_{GS} \leq V_{DS} + V_{TH} = 4V$$

$$y_{21} = \frac{\partial I_2}{\partial V_1} = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \quad (V_{DS} \geq V_{GS} - V_{TH})$$

So, why do I how do I get Y21 from which plot should I get Y21? Y 2 1 in order to get Y 2 1 I need to find out the current voltage relationship between I 2 and V 1 right. I need to find out the relationship between I 2 and V 1. Now, what is I 2 for a MOSFET by the way we have sketched I 2 is nothing, but I d s and what is V 1? V 1 is nothing, but Vgs right. So, what is the current voltage equation that is what is what is the relationship between Ids and Vgs? Now, you might turn around and say that a I mean there are two relations two equations governing the I two equations governing my relationship between Vgs and Ids which one should I plot right. So, what are these equations and under what conditions are they valid? Firstly, if we start from Vgs to be equal to 0 let us assume the threshold voltage is all positive let us assume threshold voltage is positive.

So, assume Vth is greater than 0 right. So, let us say I mark off the point Vth on the x axis. So, what can you comment what will be the current when V1 is less than Vth the current is absolutely 0. So, that is an easy part right. So, we will have 0 current no problem what happens after I increase Vgs beyond the threshold voltage if I increase Vgs just beyond threshold voltage you might turn around and ask me that is not a sufficient information you need to tell me what is Vds? What is the voltage difference between the drain and the source? So, that I can understand which equation I should use right ok.

Let us take some numbers. So, let us assume Vth is equal to 1volt. So, this is 1volt and let us further assume some value of the drain voltage right. Let us assume Vds is equal to 3 volt ok. So, now, what can you comment on, what can you comment on when my Vgs is just above 1 volt when Vgs is just above 1volt then what is Vgs minus Vth? The Vgs minus Vth is close to 0.

So, Vds is greater than Vgs minus Vth right. So, I will be in saturation region of operation right I will be in saturation region of operation. So, this equation will hold correct if that equation holds what should my curve look like? It should look like a parabola right. So, this should look like a parabola. So, what is this equation? This equation is of mu n Cox W/ L Vgs minus threshold voltage whole squared right.

So, till when, till what value of Vgs I should be able to or rather the question is till what value of Vgs will this will this parabola persist? If I keep on increasing Vgs till infinity will this keep on going? Obviously, no because you must have already figured out that when I am increasing Vgs what am I doing? I am going closer and closer to the condition of Vds is equal to Vgs- Vth right In other words saturation region is true right. So, this equation is valid what is the validity of this equation? This equation is valid as long as as long as Vds is greater than equal to Vgs minus threshold voltage. What is Vds in our case or in our case Vds is 3 volt which means this will be true as long as Vgs is less than equal to Vds plus threshold voltage that is that is 4 volt right. So, till if I mark off like if this is 1, this is 2, this is 3, this is 4 right if I mark it mark of 4 volt here so this parabola let me, let me remark them otherwise I will run out of space. Let us say this is 1 volt, this is 2 volt, this is 3 volt, this is 4 volt right.

So, it was 0 then it became a parabola and it went like this. This is 4 volt, this is 1 volt Vgs then what will happen? Which equation will take over? Which equation will take over clearly? The linear region equation will take over the equation 1 will take over and in equation 1 what can you comment? What is the relationship? What is the type of relationship between the current and Vgs? It is a linear relationship right. So, from here on from here on the current will increase linearly right. So, this will be mu n C ox W over L Vgs minus Vth times Vds of Vds square right. So, that will be the linear region condition.

Note that this curve is for this curve is valid as long as Vds is held constant you have a battery between the drain and the source constant at 4 volt right. So, this is for Vds is equal to 4 volt. Now, let us say I increase Vds from 4 volt to 4.1 volt. What do you think is going to happen? So, let us say I increase Vds from 4 volt to 4.1 volt.

What do you think is going to happen to the curve? What do you think will happen to the saturation region curve? You must have already you must have already figured it out that the saturation region current equation is independent of Vds which means there will be absolutely no change as far as the saturation region condition is concerned. However, what is going to happen? What is going to happen to the linear region condition? In the linear region condition the current is going to change it will increase or decrease. It will it will increase simply because I mean initially it will start increasing because when you, when you I mean when you increase your Vds right if you stare at this equation you will see that, you will see that beyond when you are in linear region right when Vds is less than Vds minus Vth when you increase Vds your current increases so which means you have slightly it will still be linear the slope will still be linear I mean the curve will still be linear with respect to Vds, but it will be a steepest curve fine. So, now what is this is as far as, this is as far as II2 versus V1 or Ids versus Vds curve is concerned. This is the large signal plot of I2 versus V1 or I2 versus Vgs, but we are interested in incremental Y parameter right we are interested in Y 2 1.

$$y_{21} = \frac{\partial I_2}{\partial V_1} \qquad \Rightarrow \boxed{V_{GS} \leq V_{DS} + V_{TH}} = 4V$$

$$= \frac{\partial I_{DS}}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \quad (V_{DS} \geq V_{GS} - V_{TH})$$

$$y_{21} = \left.\frac{\partial I_{DS}}{\partial V_{GS}}\right|_{linear} = \mu_n C_{ox} \frac{W}{L} V_{DS} \quad (V_{DS} \leq V_{GS} - V_{TH})$$

So, what is Y 2 1? Y 2 1 is nothing, but del I 2/ del V 1 obviously under the condition that V 2 is constant or Vds is constant in this case. So, this is nothing, but del Ids /del Vgs which is mu n C ox W/L Vgs minus threshold voltage right this is under under saturation condition right this is under saturation this is when for as long as Vds is greater than equal to V g s minus threshold voltage. What about the case what about the case when I have crossed I have crossed the cross reached the limit I have keep I have kept on ok sorry I made a mistake here I mean you some of you might have already figured it out let me let me correct the mistake before I go ahead. So, what, what mistake did I do when I went from 4 volt to 4.1 volt right, I like to draw your attention to this curve that we drew when you went from 4 volt to 4.1 volt right what is, the what is the cutoff frequency for so for threshold what is a new cutoff frequency for threshold it is not sorry so this was for V d s is equal to 3 volt and I should have said I should have said what is going to happen for V d s equal to 3.1 volt beg your pardon apologies for the mistake. So, now the question that I am asking is that what how should that curve that I drew the the the blue curve look like when I change from V d s equal to 3 volt to V d s equal to 3.1 volt clearly the saturation region current would not have would not have changed right, but what about the what about the limit of saturation region should it change? Obviously, it should change because not that the limit of saturation region is this if the V d s has changed then the limit of saturation region should also have changed which means the new limit should have been 4.1 volt right the new limit should have been 4.1 volt which means this current should have continued increasing till 4.1 volt and then should have started to linearize right ok.

So, given that we corrected that in time let me ask you this what is going to be my Y2 1 when I am in that linear region of operation, and it is straight forward. So, we can simply do Y2 1 is equal to del I d s del V d s in linear region which will be which will be nothing, but mu n C ox W over L times V d s ok. Now, can you comment can you comment on which which y 2 1 is higher the y 2 1 in in in saturation region or y 2 1 in linear region.

Now, note that this also has a limit the limit is V d s is less than V d s minus the threshold voltage right. So, clearly the y 2 1 in saturation region is higher right because even though this equation is telling you that it might look like if I keep on increasing V d s y 2 1 will increase right. I mean when you are in linear region you can if you keep on increasing V d s you will hit saturation region and you will switch over from this equation to this equation right. You will switch over from the bottom equation to the top equation and naturally which means that you are switching over from one region of operation to the other region of operation. So, clearly you always get at least from the equation that we the governing equation that we are using you will always get higher y 2 1 you will get higher y 2 1 as long as you are in saturation region.

Now, if I take you back few lectures if we take you take you few lectures back what is the expected y 2 1 what is our wish list what what is what is our wish list for a good amplifier y 2 1 should be as high as possible right y 2 1 should be as high as possible which means that I would not only we need to be in saturation region right we need to be at we need to have high W by L we should need to have high V d s minus V t h and so on and so forth right. So, that is that is as far as that is as far as maximizing y 2 1 is concerned right ok. So, so to maximize so in a nutshell to maximize y 2 1 we need large W over L and large or I should not say maximize I should say to get large y 2 1 we need to have large W over L and large V d s minus threshold voltage while keeping MOSFET in saturation. So, this is crucial this is crucial because because so what will happen let us say if I keep on increase looking at this curve what will happen if I keep on increasing V d s minus V t h. So, this this portion of the curve here so this portion of the curve is denotes saturation region and beyond this this denotes linear region.

So, if you keep on increasing V g s minus V t h right what what are you what is the trouble that you are inviting the trouble that you are inviting is you are going closer and closer to the to the edge of saturation region right. So, you can only do so much. So, they are in a there in essence is a limit of how much you can increase V g s minus V t h before you run out of any voltage room right ok. So, you will see that we will see that these constraints will come up quite frequently as we move ahead with the course ok. So, this is as far as the Y21 for our MOSFET is concerned we also need to figure out what is Y22 right.

So, in order to figure out Y22 what should we what should we sketch we are right we should sketch I2 versus V2 while keeping V1 to be constant what is I2 again I2 is nothing, but Ids what is V2, V2 is nothing, but Vds we need to, we need to sketch the Id V characteristics or Ids versus Vds characteristics while keeping V1 constant or while keeping Vgs constant right V1 is nothing, but Vgs. So, let us say we sketch it for Vgs is equal to let us say we sketch it for Vgs equal to 2volt. So, if Vgs is equal to 2 volt right what is the, what is that boundary of Vds or what is the boundary of V2 when we hit where one side of the boundary you will be having linear region and other side of the boundary will have saturation region. So, clearly we need to figure that out. So, the boundary is Vds should be greater than the boundary for saturation or the condition for saturation region is Vds should be greater than Vgs- Vth again threshold voltage is 1 volt.

So, this is this becomes 1volt. So, if I mark off let us say 1 volt here if I mark off 1 volt here this side, this side of the plot right any side of the plot where Vds is greater than 1 volt will be saturation region and this side will be linear region for the condition of Vgs is equal to 2 volt ok. So, which plot is easy to draw? Clearly the saturation region plot is easy to draw because in saturation region my current does not depend on Vds right. So, since my current does not depend on Vds I can simply say that this is a essentially a straight line and let me call this Id sat or Vgs equal to 2 volt right. So, what is I d sat? Id sat is half mu n C ox Vgs is 2 volt minus threshold voltage is 1 volt whole square which is half mu n C ox times 1 right.

So, whatever this value might be. So, this is my Id sat or Vgs is equal to 1 volt ok fine. What will happen when I drop from 1 volt Vds to 0 right. Essentially what I am asking is that what is going to happen in linear region. So, linear region clearly my current equation becomes mu n C ox W over L Vgs minus threshold voltage times Vds minus half Vds square right. So, let us do let us see what will happen at very low very small values of Vds that is near the origin.

So, I clearly at very at near the origin. So, if Vgs or Vds is much much less than Vgs minus threshold voltage right. If this is the case right. So, I can essentially I can essentially neglect this term and my Ids becomes mu n C ox W over L Vgs minus threshold voltage times Vds right. So, which means it becomes a straight line.

So, near the near the origin this will become a straight line and minus half Vds square term will start picking up will start to dominate as we go away as you go away from origin towards higher value of Vds and clearl y this is also a parabola and this will, this will essentially go and merge with the Id sat right ok. So far so good. What is going to happen now if I say I change the value of Vgs let us say I go from Vgs equal to 2 volt to let us say Vgs equal to 3 volt what is going to happen to Id sat clearly I d sat will increase and it will increase by how much the new I d sat will be new I d sat will be half I d sat or Vgs is equal to 3 volt here the Vds is equal to 2 volt right. So, I made a mistake here is pardon me. So, this V for Vgs equal to 3 volt will be half mu n C ox 3 minus 1 whole square that is 4 times whatever we had earlier.

So, so this I d sat will increase to some value let me call this 4 times I d sat of Vgs equal to 2 volt. So, this will be equal to I d sat for Vgs equal to 3 volt. So, this again will be constant, but now till what voltage will this be constant, what voltage of Vds will this be constant in other words I am asking what is the limit for which beyond which you have the demarcation line of saturation region a linear region clearly if Vgs is 3 volt that line moves from 1 volt to 2 volt right. So, 3 minus 1 is 2.

So, that line goes through here. So, your saturation. So, it will be flat beyond Vgs equal to Vds equal to 2 volt and after that it will die it will nose die will become something like this ok. That is great similarly I mean I can do this exercise for I can do this exercise for all values of I mean continuous values of Vgs as Vgs is a parameter and if I will get similar plots I will get one plot for lower Vgs which will which should look like this. Similarly, I will get a higher plot. So, here lower the Vgs that the demarcation the demarcation voltage the demarcation voltage of Vds goes closer and closer to the origin right.

So, now, if I let us say I ask you what is going to happen if I join these demarcation voltage dots right. If I join these demarcation voltage dots I will get a curve what will this curve look like or rather what will this curve mean? I leave it up to you to figure it out right. I leave it up to you to figure it out and we can discuss this in the discussion session when we meet next ok. So, I am asking this find the equation of this curve ok great. So, now, I mean what was the motive of sketching all these plots? The motive was to find out y 2 2 right.

In saturation $$y_{22} = \frac{\partial I_{DS}}{\partial V_{DS}} = 0$$

In linear $$y_{22} = \frac{\partial}{\partial V_{DS}} \mu_n C_{ox} \frac{W}{L}\left[(V_{GS}-V_{TH})V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$= \mu_n C_{ox}\frac{W}{L}\left[V_{GS}-V_{TH} - V_{DS}\right]$$

So, what is y 2 2 now? So, y 2 2 so, let me so, y 2 2 is del I 2 del V 2 which is nothing, but del I d s del V d s right. For a particular value of V d s correct. So, in saturation y 2 2 is del I d s del V d s where I d s is independent of V d s right. So, I d s is flat in saturation your I d s is flat does not change with V d s which means y 2 2 is y 2 2 is 0 right. What about in linear region what is y 2 2? y 2 2 will be del del V d s of the linear region current equation what is that mu n C aux W over L V d s minus threshold voltage times V d s minus half V d s squared which is nothing, but mu n C aux W over L V d s minus threshold voltage minus V d s ok.

So, now, what is the wish list for our amplifier what should y 2 2 be ideally? Ideally y 2 2 should be 0 right to get a good amplifier characteristics ok. So, means that saturation region is good as far as the region of operation is concerned for our MOSFET right. So, this is so, just like what we concluded to get large y y 2 1 to get large y 2 1 we needed to get into saturation region with some condition. Similarly, to get to get small y 2 2 which is 0 in our case very good that is the best I can do we need to be we need to bias we need to operate the MOSFET in saturation region right. So, for y to get large y 2 1 we need to operate the MOSFET in in saturation region to get small y 2 2 also we need to operate the MOSFET in in saturation region right.

So, seems like saturation region is the good region of operation in order to get in order to get the revise operate in a manner in which the incremental parameters that we get are are in the lines of an ideal two porting right. So, so, so which means that to conclude saturation region is the region of choice of a MOSFET to get amplification of power right great. So, now, before we move on before we move on there is couple of a jargon that we need to be accustomed to. So, as it turns out in in a MOSFET we do not call y 2 1 y 2 1 we call y 2 1 the trans conductance or the g m right. So, y 2 1 we call it as g m and what is the equation of y 2 1 y 2 1 equation is mu n C ox W over L Vgs minus threshold voltage.

One more piece of jargon is Vgs minus threshold voltage is often called overdrive voltage or V o V right. So, the reason we call it the overdrive voltages because you need not only you need the Vgs to be at least equal to the threshold voltage you need Vgs to be higher than the threshold voltage in order to get the MOSFET to work. So, you need to have you need to have a drive on top of you need to drive on top of threshold voltage. You can assume that threshold voltage to be like a friction right you need to a static friction you need to at least give some force to get going, but in order to achieve certain velocity you can keep on you have to keep on applying more and more force. So, the minimum force that you require or minimum voltage that you require is threshold voltage and anything that you apply on top of threshold voltage is called overdrive right.

$$y_{21} = g_m = \boxed{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

Overdrive voltage (V_{ov})

$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \times \frac{2}{V_{GS} - V_{TH}}$$

$$= \boxed{\frac{2 I_D}{V_{GS} - V_{TH}}}$$

So, that is why we call it V o V or V overdrive. The curious among you would also notice that I mean this equation can be written in multiple forms. If we notice that I ds is equal to mu n C half of mu n C ox W over L Vgs minus V d whole square I can turn it around and say that this is equal to half mu n C ox W over L Vgs minus threshold voltage whole square times 2 over Vgs minus threshold voltage this becomes I d. So, this becomes 2 I d over Vgs minus threshold voltage correct. So, this becomes one expression for gm this becomes another expression for gm similarly if I eliminate Vgs minus Vth right if I eliminate Vgs minus Vth from this equation I can also write it in the form of under root 2 mu n C ox W by L times I d right.

This is also an expression for gm right. So, you might you might wonder right what to do which expression to use under what condition or even more poignantly are all of them giving us the right information right. For example, you might look at the first equation and say that in order to increase gm I need to increase the overdrive I need to increase Vgs minus Vth, but then if we look at the second equation it looks like in order to increase gm I have to reduce Vgs minus Vth right and it looks like in a third equation whatever I do to Vgs minus Vth gm does not change right. It seems like all three equations are giving me three different intuitions right, but here you have to be careful and you have to understand that not only that all three equations are giving you the same information because they have come from the same mother equation right. They have come from the same mother equation of current in saturation region and then the question arises that why it seems like they are different for these three different. Why does it seem like the first equation in case of first equation when I increase the overdrive gm increases in the second equation, I increase overdrive the gm decreases.

All you have to understand is the fact that in the first equation or rather in the second equation when you are when you are saying when you are implying that I am decreasing the threshold voltage, what are you doing to Id? Will Id remain constant? No right because Id is a function of the overdrive right or I should have since I have used Id s everywhere I should have used Id s here just to ensure that I am not have using any notations right. So, Id s is a function of overdrive right so if you have set up your MOSFET in the way we have set it up right. You have applied voltage at the between the gate and the source right and you have generated some current Id s and you now say that I am changing Vgs, but I will use the second equation and I will keep Id s same whatever it was earlier then obviously you are doing something wrong because you have to understand that moment you change Vgs minus Vth right your Id s will also also change right. Similarly if you say that a looking at the third equation it seems like if I change the Vgs minus Vth gm is not changing at all again that is interpretation will not be correct because we know that if you change Vgs minus Vth right if you change the overdrive Id s will change. So, all the informations are hidden inside the variables or they are embedded inside the variables you need to know what will change what might change and what is constant right.

You might have a circuit you might have a circuit in which the overdrive is constant overdrive will not change you might have a circuit in which Id s is constant or Id s will not change. So, you have to choose you will have to choose your equations your equations appropriately ok great. So, you will see the you will see this more and more as the course progresses as to based on different configurations of circuit one of these equations might be more useful than the other ok. So, the final thing final jargon for today s lecture is Y22 we call Y22 as Gds right and Y Gds is equal to 0 in saturation which is great for us right ok. We stop here we will start from the we will continue in the same way from the next class. .