


Course name- Analog VLSI Design (108104193)
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Institute – Indian Institute of Technology Kanpur
Week- 3
Lecture- 8, module-01

Now, welcome back this is lecture 8. So, in the previous lecture we saw we saw the use of incremental Y parameters, incremental short circuit admittance parameters or the Y parameters which are supposed to help us in analyzing two port networks, supposed to help us in analyzing non-linear two port networks right. And what did we do? We simply said that if I have a non-linear two port network and these were Biostat voltages $V1q$, $V2q$ and if these currents are $I1q$ and $I2q$ and if we perturb the port voltages right, we perturb these port voltages by small $v1$ and small $v2$ then these currents would change, these currents would change and the new incremental currents and voltages that is if we represent them as $I1$ and $I2$ right. So, we could represent $I1$ as $\frac{\partial f_1}{\partial V_1}$, this one $F1$, $V1$, $V2$ simply let me simply write these non-linearities as $F1$ and $F2$ with understanding in that both $F1$ and $F2$ are functions of $V1$ and $V2$ right. So, this becomes $\frac{\partial f_1}{\partial V_1}$ around operating point $V1q$, $V2q$ times $V1$ plus $\frac{\partial f_1}{\partial V_2}$ around operating points $V1q$, $V2q$ times $V2$ right. Similarly, $I2$ we could express as $\frac{\partial f_2}{\partial V_1}$ again around operating point $V1q$, $V2q$ plus $\frac{\partial f_2}{\partial V_2}$ around operating point $V1q$, $V2q$.

Lecture 8



$$i_1 = \left. \frac{\partial f_1}{\partial V_1} \right|_{V_{1q}, V_{2q}} v_1 + \left. \frac{\partial f_1}{\partial V_2} \right|_{V_{1q}, V_{2q}} v_2$$

$$i_2 = \left. \frac{\partial f_2}{\partial V_1} \right|_{V_{1q}, V_{2q}} v_1 + \left. \frac{\partial f_2}{\partial V_2} \right|_{V_{1q}, V_{2q}} v_2$$

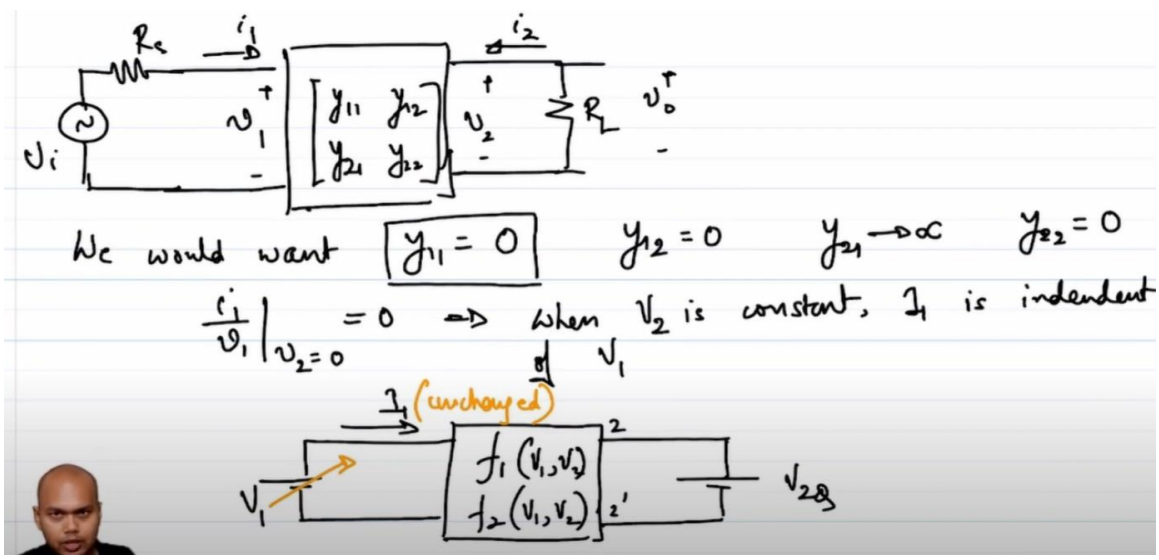
$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2$$

And then we further say that instead of writing these differentials around the operating point repeatedly there is a more compact way of writing these expressions or these currents and voltage relationships and that we could write in terms of short circuit incremental y parameters right. And simply we rewrote these as $I1$ equal to $y11 V1$ plus $y12 V2$. Similarly, $I2$ plus $y21 V1$ plus $y22 V2$. And why are these called short circuit admittance parameters? It is called short circuit admittance parameters because in order to find out one of the parameters let us say I want to find out $y11$ right.

y_{11} is a relationship between the current at the input port and the voltage at the input port under the condition that V_2 is 0 that is incrementally O_2 is short circuited. Similarly, if I have to find out y_{12} I will have to short circuit the incrementally short circuit the input port and find out the relation between I_1 and V_2 that is I have to excite O_2 right, I have to excite O_2 and figure out what will be the incremental change in the current in I_1 under the condition that the voltage at I_1 is held constant or incrementally shorted ok fine. Then we also said that now we if we now go completely into the incremental domain and apply an incremental current voltage to these y-parameter, to this y parameter network which is incrementally expressed right. We noticed few things we found out we find we found out the relationship between V_0 and V_i and we noticed few things and we derived few conclusions and we notice that in order to get an ideal amplifier right in order to if we were to get an wish list of an ideal amplifier what what eventually realized what eventually was an ideal wish list for our amplifier as far as the incremental parameters were concerned the wish list was we would want y_{11} would be equal to 0. y_{12} to be equal to 0, y_{21} should be as high as possible and also y_{22} should be 0 ok.

So, now time has come to now time has come to dig a bit further into what these conditions actually mean right ok. So, let us start off with the first one the easier one y_{11} equal to 0. So, what does y_{11} equal to 0 imply? What does y_{11} equal to 0 imply? It is it implies that I_1 over V_1 when V_2 is equal to 0 is 0 right implies that what does it imply? It implies that in plain English when the voltage across port 2 is held constant right when V_2 is constant right I_1 is independent of V_1 right that is what it means right. So, effectively what this means is if I have if I take this network hold the output port that is the port 2 with a constant voltage right and I apply certain voltage V_1 at the input port and I observe this current I_1 even if I change this voltage V_1 I_1 remains unchanged. What that this is the total voltage and the current that we are referring to right.



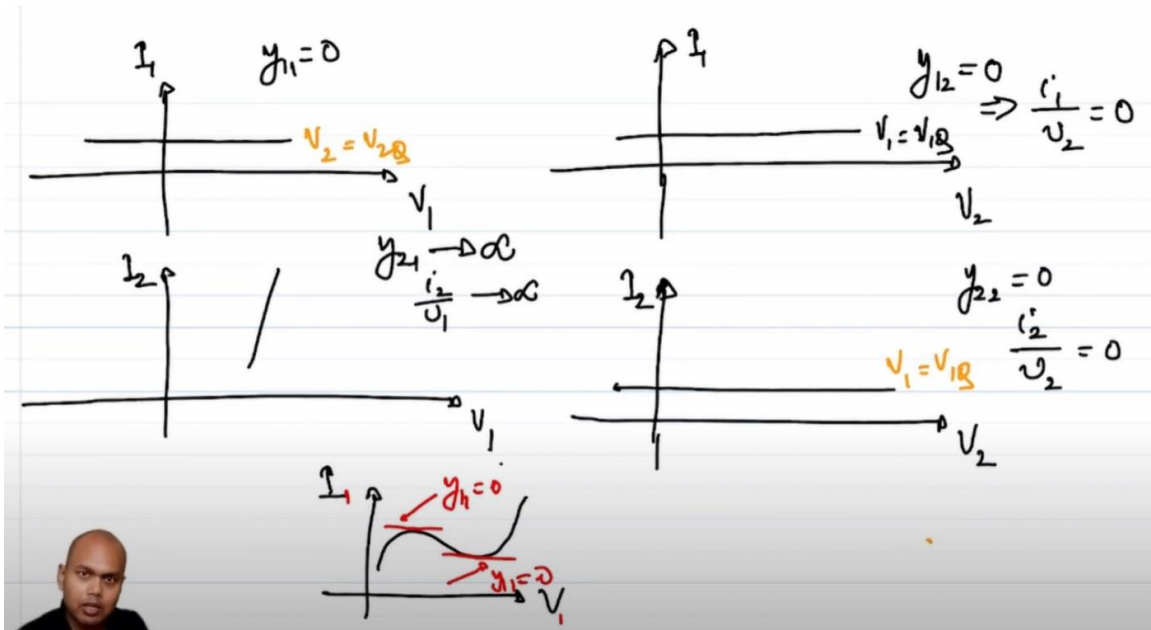
This is what this is what the actual meaning of $y_{11} = 0$ means. So, what will be what will be the current voltage characteristics of I_1 and with respect to V_1 ? So, if I sketch so this for $y_1 = 0$ what does the current voltage characteristics of I_1 and V_1 look like? $y_{11} = 0$ means the derivative of I_1 and V with respect to V_1 is constant right. So, all it means is that the derivative of I_1 which means that I_1 is constant with respect to V_1 and this is probably for $V_2 = \text{certain } V_2$. That is all this is saying nothing else ok. Similarly, if we turn our attention to we turn our attention to y_{12} what is our y_{12} ? y_{12} also should be 0 and what does y_{12} signify? y_{12} signifies the relationship between the incremental I_1 and the incremental V_2 or in other words this signifies $\frac{\Delta I_1}{\Delta V_2}$ is equal to 0 right.

So, again similar to similar to the curve of $y_{11} = 0$ this will also be a straight line right that is all that is all we can conclude and this is for some particular value of V_1 which is equal to let us say $V_1 = \text{ok}$. Similarly, if we turn our attention to $y_{22} = 0$ what does it imply? It implies $\frac{\Delta I_2}{\Delta V_2} = 0$. So, from which plot would I from which IV characteristics plot should I get y_{22} ? So, I should be plotting I_2 versus V_2 and again what should it signify? Same old same old like the like the other two it should also be constant right and this is for certain $V_1 = \text{correct ok}$. And similarly if now if we turn our attention to the last parameter right so last parameter is y_{21} and I want y_{21} to be as high as possible right y_{21} should be as high as possible which means that which means that $\frac{\Delta I_2}{\Delta V_1}$ should be as high as possible correct. So, what does it signify I mean what should I be plotting? I should be plotting I_2 with respect to V_1 and what does it mean when I say $\frac{\Delta I_2}{\Delta V_1}$ should be infinity? All it means is that the slope should be the slope of I_2 over V_1 should be as high as possible right the slope of I_2 over V_1 should be as high as possible right.

Now note that all I have done is extrapolated the total IV characteristics from the slopes right I have not done anything else when I said when I assume the statement that I made here right the statement that I made here that when $I_1 = 0$ means that when V_2 is constant I_1 is independent of V_1 right and we essentially do a straight line parallel to the x axis or parallel to V_1 to signify that the slope is 0. Now note that it did not necessarily be that the slope has to be 0 everywhere right what does slope 0 mean? You can have let us say I_1 you can let us say have an IV characteristics a generalized IV characteristics which has a characteristics like this quite possible. So, note that there are two places in this IV characteristics where the slope is 0 right had this been I_1 and had this been V_1 then I could have as well said that at these two points $y_{11} = 0$ right. This could as well have been another $I_1 - V_1$ characteristics right. The note that the four characteristics that I have drawn did not necessarily be the only possible solution right.

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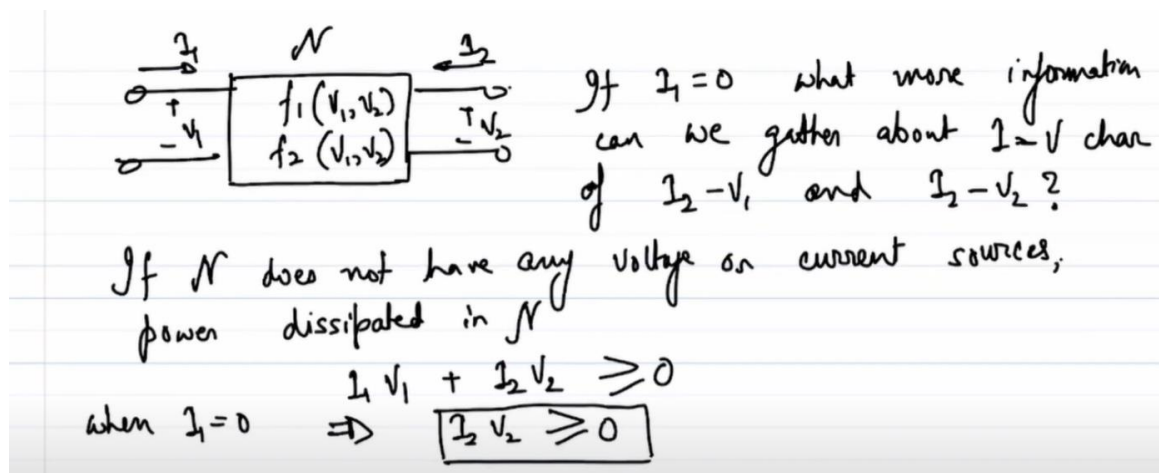
there can be many other solutions also why because we have simply extrapolated what we could have gotten from the slope and did not it need not necessarily be that the slope needs to be constant right for all values of I_1 and V_1 . So, that is something we should keep in mind right. Now with that disclaimer with that disclaimer let me now take some specific cases right. Now as it turns out right as it turns out this is just by the statement that I am going to make right now is based on the devices that we already have at our disposal right. So, as it turns out as it turns out in there are many devices in which in which I_1 is close to 0 ok.



So, this is some devices have this for all V_1 right. This is again the fact of I mean this is again the characteristics of certain device ok. So, now if I get I_1 equal to 0 for any V_1 equal to V_2 I need not bother about the top two plots right. If I do not have to bother about the top two plots then I can as well say that forget about them let us concentrate on the other two plots ok. So, what are the other two plots? So, again so if I sketch our if I sketch the two port network in a generalized form and let me mark these as V_1 , V_2 as we have been doing throughout.

So, as it turns out I_1 is equal to 0 right. So, let us say if I_1 is equal to 0 what more information can we gather about port, IV of I_2 versus V_1 and I_2 versus V_2 . So, this is the question that we would like to delve into next right. So, now going from this information to the next information right what we are saying what we are essentially saying is that let us assume the stuff inside this box right the stuff inside this box does not have any extra power sources right. So, let us assume if N does not have any voltage or current sources right.

If N does not have any voltage or current sources then you might say if I mean I am probably violating one of the original assumptions that we made I am not violating because I am not saying There can not be any other voltage sources right. So, the voltage sources the extra DC voltage sources can be connected to can be connected to port 1 or port 2 right we are taking a special case right. If N does not have any voltage or current sources then what do you think will be what do you think will be the total power dissipated in the network can we can we make a comment on the total power that is dissipated inside the network right. So, the power sources and the power dissipated in N can be represented as I_1 times V_1 plus I_2 times V_2 right and if since N does not have any power sources which means it is a passive network right it cannot generate a power on its own which means the power dissipated will always be positive right. So, this has to be greater than or equal to 0.



Now given that I_1 is equal to 0 that is the assumption that we took or that is a that is something that I just that I just took out of the hat right then under the condition that when I_1 equal to 0 right then your $I_2 V_2$ should always be greater than 0 right. So, let us let us take a note of it because this will give us some important insight into the $I-V$ characteristics from which we are getting Y_{22} and what are those insights let us see. Now what when we when we said that my Y_{22} is equal to 0 what did we say again what is Y_{22} ? Y_{22} is the slope of the plot slope of the characteristics between I_2 and V_2 and when we said Y_{22} equal to 0 all we are saying is that right there is some region there is some region in this $I_2 V_2$ characteristics where I_2 is constant with respect to V_2 right that is all we are saying this is for this is for certain value of V_1 let us say V_{1Q} right. So, this is for V_1 equal to V_{1Q} ok So, now now what do you think will happen right what do you think will happen if I extend this plot to the right and to the left right. So, let us I mean can you comment on the generic trend of this plot if I extend it to the left right.

So, essentially what I am asking is that can you comment whether this plot in principle can go to the second quadrant. I would like to draw your attention to the fact that we have already we have already come up with a constraint that I_2 times V_2 has to be greater than

equal to 0 right. So, we have already come with a constraint that I_2 times V_2 has to be greater than equal to 0. What does it mean? It essentially means that the product of I_2 and V_2 is always positive which means that I cannot get into the second or the fourth quadrant right. So, I_2 V_2 characteristics always have to stay between the sec first and the third quadrant right.

So, all this is saying is all this is saying is you might have a flat region of I_2 and V_2 somewhere in between some range of I_2 and V_2 , but ultimately this guy has to draw right. Ultimately this guy has to draw. Note that it does not say anything about how it is supposed to drop right. It can drop like this, it can drop like this, it can do this, it can be anything. It does not really comment on how it is going to drop, but all it says is that this current has to change from whatever value it had to 0 and maybe it can go down into the third quadrant also if necessary right.

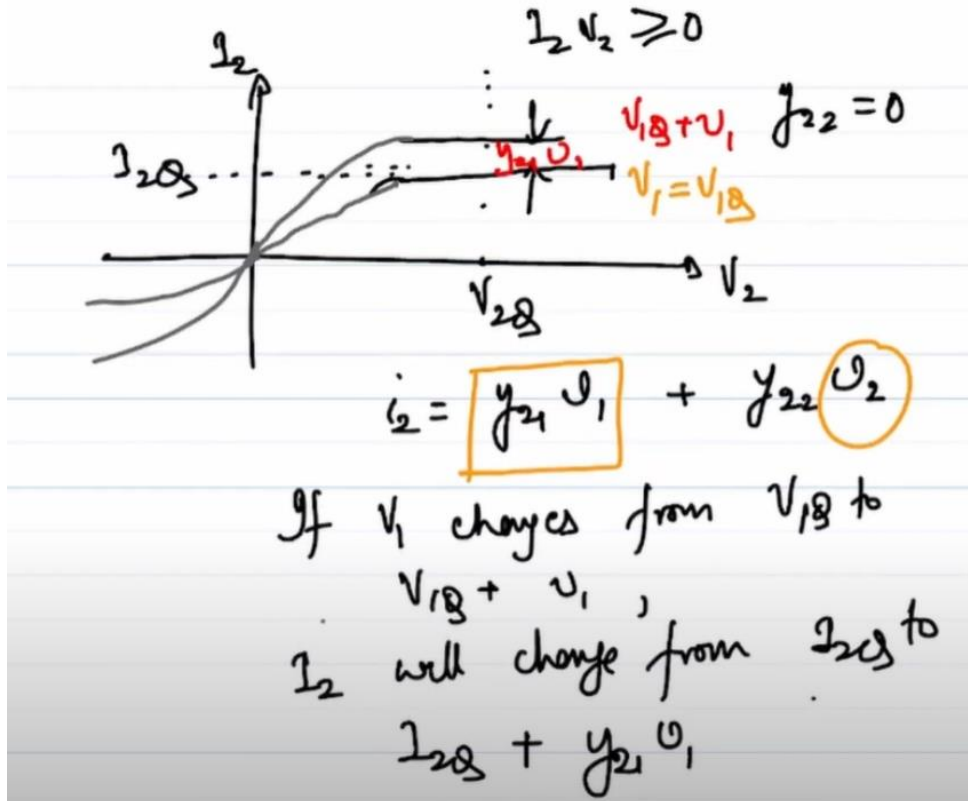
That is all that this is telling us ok. So, now let us say that I have changed this voltage V_1 from V_1 cube to V_1 cube plus delta V_1 right. So, if I have changed this voltage let us say from V_1 to V_1 cube plus small v_1 what comment can you make about current I_2 . Let us say this current I_2 here is I_2 cube right. So, what comment So, what comment can you make on the current I_2 , I_2 cube, I_2 will it be the same I_2 cube, will it be a different I_2 cube, what do you think? In order to understand this we will have to go back to see what happens if what happens to I_2 if V_1 changes right.

That is what we have done here, that is what we are asking correct. So, incrementally what is the relationship between I_2 and the port voltages I_2 is incrementally I_2 is $Y_{21} V_1$ plus $Y_{22} V_2$. Now, if we are concentrating on a particular value of V_2 let us say V_2 cube right. If we are concentrating on a particular value of V_2 that is V_2 cube then what is small v_2 ? We are not changing V_2 cube, if you are not changing V_2 cube and we are trying to figure out what will happen to the current V_1 changes what is small v_2 ? Small v_2 is essentially 0 right. So, delta V_2 is 0 which means small v_2 is 0.

So, which means that this current I_2 is going to change by this amount. So, this will be the delta I_2 right. So, if V_1 changes from V_1 cube to V_1 cube plus V_1 I_2 will change from I_2 cube to I_2 cube plus this change which is Y_{21} times V_1 . And what you think Y_{21} is or the desired Y_{21} , is the desired Y_{21} is a large value right. So, for small changes in V_1 we would expect we would expect this curve to shift upwards assuming Y_{21} is greater than 0 right.

So, which means that this curve will shift upwards and this change will be Y_{21} , this change will be Y_{21} times V_1 . And similarly this curve will also have to somehow nose dive into 0 and maybe go into the third quadrant. Now note that again how it will go to the third

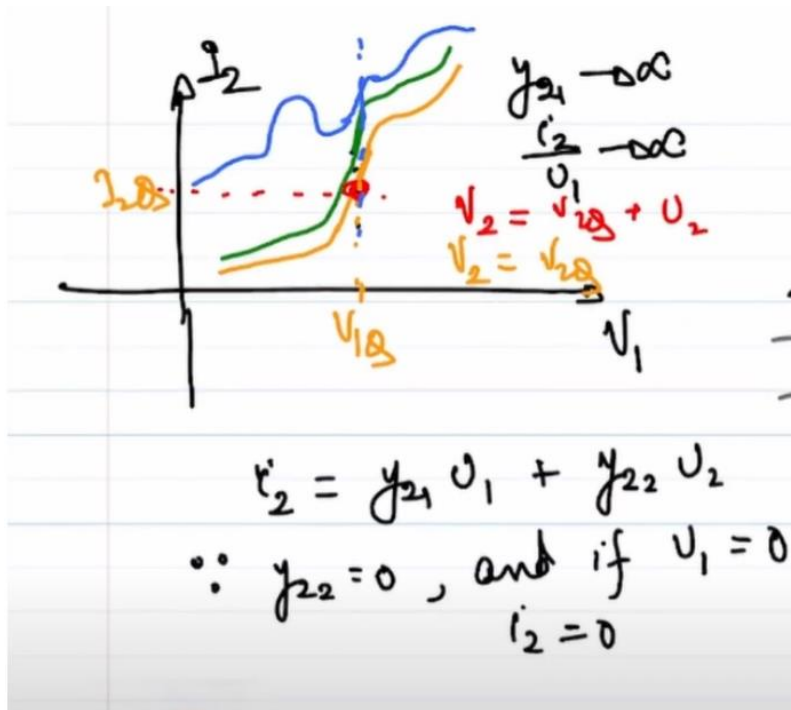
quadrant we do not know, but all we know is that it will go ok. Similarly, if I now further if I decrease if I decrease V_1 from V_1 cube to V_1 cube minus some small delta this entire curve will shift to the shift downwards and we will have the we have the same mechanism ok. So, now let us shift our focus to the other curve the last remaining curve.



So, this is $i_2 v_1$ and we are saying that y_{21} tends to infinity. Again y_{21} means small i_2 over small v_1 tends to infinity which means the slope of i_2 slope of the curve defining the relationship between i_2 and v_1 has to be very high right infinity means vertical line ok. So, which essentially means that all we are saying is around for let us say for some fixed v_2 like v_2 equal to v_2 cube and let us say around some operating point v_1 cube the curve needs to be very steep that is all we are saying right. Does it say anything what will happen if I deviate from v_1 cube? It does not it does not say anything if I the wish list does not say anything if I deviate from v_1 cube the wish list only tells us that if we all we know is that all we need is a is a is a is a is a place in the IV characteristics of i_2 versus v_1 where the slope is very high. It does not say what the slope needs to be if I deviate from that particular operating point right.

So, which means that it may it may as well be right. So, it may it may as well be that this goes like this right it may as well be that this goes like this right. So, can you now can you comment on what will happen like in case of in case of the plot on the right if I change v_2 from v_2 plus v_2 v_2 from v_2 cube to let us say v_2 cube plus I go from this. So, let me

mark this. So, let us say this curve is like this and we we change V2 from V2 V2 cube plus an incremental V2.



So, what is going to happen? What is I2 incrementally? My I2 is Y21 V1 plus Y2 to V2 right. If we are not changing V1 Q that is we are sitting on V1Q and we have changed V2 and I want to see I want to see what is going to be the what is going to be the change in I2 if I change V2 right. What is what is our Y22 right. My Y22 is 0 right. So, since Y22 is 0 and if V1 equal to 0 that is we are talking at we are sitting at a fixed V1Q then I2 will change in I2 will be 0 right.

So, your I2Q will still remain as I2Q right. So, this was I2Q. So, even with change in output voltage V2 this current will not will not change and it will remain exactly where it is ok. But note that note that this slope this curve will have the exactly same Y21 as another curve let us say which is shifted upwards, but having the same slope as V1Q as the as the as the orange line or maybe another curve will just come something completely crazy, but as it appears around V1Q has the same slope and goes something like this. All three of them will have the same Y21 because the Y21 has been evaluated by the slope at V1Q.

Now, if V1Q changes right if you bias your if you bias your network with a voltage which is different from V1Q is quite possible not even possible with the sketches that I have made you can see that you will have a completely a different value of Y21 ok. Thank you.