**Course name- Analog VLSI Design (108104193)**
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**Week- 2**
**Lecture- 6**

Welcome back, this is lecture 6. So, up until now we have been discussing about a ways of making an amplifier. We have not yet reached that stage yet where we have been successful in making an amplifier. However, we could establish the fact that a linear network was useless to make a power amplifier, right. So, we saw that since we need to resort to a non-linear network, we needed to figure out a way of analyzing non-linear network. In other words, we needed a way of figuring out how to analyze non-linear equations.

And we know that from our experiences that there are no closed form solutions to a generic non-linear set of equations, which means that we needed to do some approximations. Another way of thinking about this process is that if you look back at your UG curriculum, undergraduate curriculum, so far you must have noticed that almost everything that you have solved or the curriculum that we are used to are geared towards solving set of linear equations. Even the circuit theory problems that you have solved till now are all linear circuit theory problems. Even the single sense systems courses that you might have taken which involves Laplace transforms, Fourier transforms and so on, they all work on the assumption that the system on which the transforms are getting operated on are also linear systems.

So, given that, we have given that we have so much exposure to linear systems set of linear equations, we would like to leverage that existing knowledge to understand how to deal with non-linear systems also instead of learning a whole bunch of new mathematics to deal with non-linear systems, right. So, in order to do that we understood that there are some tricks that we can play and what were the tricks? The trick was if we can use numerical methods to analyze the currents and the voltages or to figure out the currents and the voltages in a non-linear network once, then we could we could say that any further perturbation on of the network, if the perturbation is small enough then we could we could express the subsequent increments in currents and voltages in the non-linear network as a set of linear equations, right. And that essentially was the genesis of small signal approximations, but we needed to be careful that the small signal approximation is only valid when the signal is really small. Now, what is really small? The real what is really really small depends on what type of non-linearity that we are dealing with. For example, in the case of a diode let us say I take a forward bias diode, let us say the diode is connected to a broader network.

I do not know what the network is, but let us assume the network has some voltage sources, some registers and what not you can think of, right. And let us say this is Vdc and let us assume further that this Vdc voltage source has established a voltage of Vd across a diode and let us say that current is Id through the diode. And we have figured out this Vd and Id numerically graphically using some approximations, right. Then we say that what if I increase this increase this DC voltage, what if we increase this DC voltage by from Vdc to plus delta Vdc, then all we can say that all we can estimate is the fact that this diode voltage would have increased by some delta Vd and the diode current would have increased by some delta Id, right. Now, what is the relationship between Id and Vd? The relationship between Id and Vd was Id is equal to Is e to the power Vd over Vt, right.

$$I_D = I_s\, e^{V_D/V_T}$$

$$I_D + \Delta I_D = I_s\, e^{(V_D + \Delta V_D)/V_T}$$

So, this is the approximate forward bias diode equation, right. And given the knowledge of the voltage across the diode and the current through the diode, note that I have neglected the minus 1 term, I am which means that I am assuming that the diode is sufficiently forward biased. Now, what has happened after I have increased the the perturbation by delta Vdc? The voltage across the diode increased and the current through the diode also subsequently increased and we did this analysis in the previous previous lectures. But what I what I am trying to emphasize here is that is there a is there a maximum delta Vd, right. Is there a maximum delta Vd or a or some limit on delta Vd beyond which beyond which this small signal approximation will break down.

So, in order to figure out figure that out we need to understand what is happening with the non-linear terms as well, right. So, so let us see what is going and the current through the diode, note that I have neglected the minus 1 term, I am which means that I am assuming that the diode is sufficiently forward biased. Now, what has happened after I have increased the the perturbation by delta Vdc? The voltage across the diode increased and the current through the diode also subsequently increased and we did this analysis in the previous previous lectures. But what I what I am trying to emphasize here is that is there a is there a maximum delta Vd, right. Is there a maximum delta Vd or a or some limit on delta Vd beyond which beyond which this small signal approximation will break down.

So, in order to figure out figure that out we need to understand what is happening with the non-linear terms as well, right. So, so let us see what is going to happen now. So, so now my new current will be Id plus delta Id, right which will be equal to which will be equal to Is e to the power Vd plus this increment delta Vd over Vt, right. So, if I express this exponential in terms of Taylor series which is evaluated around the quiescent point Vd and Id. So, what should I get? I should get Is e to the power Vd over Vt plus delta Vd first derivative of Id with respect to Vd which will yield Is over Vt e to the power Vd over Vt plus delta Vd squared by factorial 2 times the second derivative, right, the double derivative which is Is over Vt squared e to the power Vd over Vt plus higher derivatives, right.

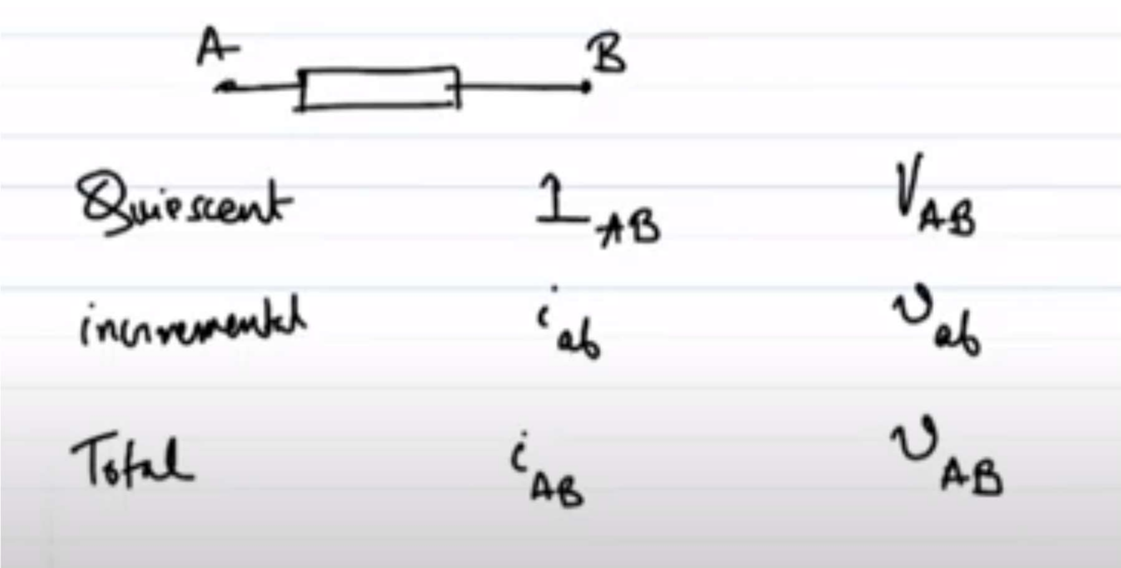
So, we know that we know that these two terms are identical. So, they go off, right. So, what are we left with? We are left with delta Id is equal to delta Vd times this term. So, this term is nothing, but Id over Vt, right plus again delta Vd squared by 2 Vt square times Id, right plus higher order terms, correct. So, the small signal approximation that we have been doing till now was only dealing with the fact that if delta Vd is small enough then I can establish a linear relationship between this delta Id and delta Vd, correct which essentially means that I am neglecting this entire, entire stuff, right.

$$\Delta I_D = \Delta V_D \left( \frac{I_D}{V_T} \right)$$

As long as, as long as these higher order terms are much smaller than the first order term, I should be able to, I should be able to approximate delta Id as delta Vd times a constant, right which means that I should be able to express, I should be able to replace the diode with a register of value Vd over Id as long as this approximation holds. So, what is the limit of this approximation? The limit of this approximation is that this extra term that if I only take the first extra term that is delta Vd squared by 2 Vt squared times Id should be much less than delta V t times Id over Vt, right which essentially means that this goes off, one of these powers go off, one of these powers goes off which essentially means delta Vd has to be much less than twice, much less than twice the thermal voltage, right.

So, for a forward bias diode your small signal approximation of a forward bias diode as a resistor will hold as long as the incremental voltage, as long as the incremental delta Vd is much, much lesser than the thermal voltage, twice that of thermal voltage which is approximately 52 millivolt at room temperature, right. So, note that this is the case for a diode. If more of you are faced with a different non-linearity, right which might not be exponential, which might be polynomial or logarithmic or something else, you will have to redo this analysis at least once to understand under what condition this, under what condition the approximation, the small signal approximation will hold, right.

So, this small signal approximation this is the condition of small signal in a forward biased diode, correct. So, similarly if tomorrow you are faced with a different device which has a completely different set of non-linearity, you will have to do this analysis yourself once to establish under what conditions this your small signal approximation will hold, ok, ok, great. So, now by now you must have, you must have understood that in case of, in order to figure out the currents, total currents and the voltages in a non-linear network, we need to follow three step process. Process step 1, step 1 is find out the quiescent, step 2 replace the quiescent with their incremental equivalent and then find out the incremental currents and voltages and step 3 is to add up the quiescent and the incremental equivalents to get the total currents and voltages, right. So, as it turns out instead of writing this delta V, delta I or total I, total V every time, there are certain notations that are used in literature and we will also follow the same notations and the notations are as follows.




So, let us say I have a, I have an element and let us call this terminals A and B, right. So, if I am trying to denote quiescent voltages or currents pertaining to this element that is if I am talking about quiescent current or voltages, I will denote them as capital I subscript capital B AB or capital B subscript capital AB. If I am talking about incremental currents and voltages, instead of writing delta I AB, delta B AB every time, we will be using small i subscript small ab or small v subscript small ab and when we are trying to find out the total currents and voltages, instead of every time writing as total I, total V, we will follow the notation that is total is small i capital subscript or small v capital subscript, ok. So, please make a note of this that we will be following this type of notation. Sometimes you will see that it is not possible to follow these notations all the time, if for example, instead of AB you have 1, 2, 3, right.

So, in those cases we will be using, we will try to denote, we will be try to differentiate incremental from quiescent by using either delta or total or something, right, but in general we will be following this and these moment languages, ok. So, till now we have been in search of an amplifier, right and we have been we have been doing all these non-linear analysis or setting up a framework to analyze a non-linear network because the ultimate goal is to is to make an amplifier, right. So, let us go back to our to our original problem statement that is I have a source with some resistance source resistance Rs and I have a load RL and we would like to put something in between, right, something in between. So, that so, that I can I can transfer, I can, I can get a power transfer between the source and the load, right and what is this stuff in between that we need to needed to put. We had we established right.

that we needed a one we needed a power source, We needed a power source somewhere, right. So, let us say I required a power source, an additional voltage source, ok and this additional voltage source let me refer it to some common terminal, some ground and the stuff inside the box had to be non-linear. So, this was the bare minimum requirement, correct, but note that I mean the stuff inside the box cannot be floating it should also have it is a two port network, right. So, it should have, it should have a two another port that is coming out and what should I connect it to? In general, in general this networks I can I can say that the voltage source is connected between one port and the and the load is also is connected between another port, right and maybe there is a voltage source there is a DC battery which is connected between one of the nodes. One might argue that this DC battery could have been connected connect it to? In general, in general this networks I can I can say that the voltage source is connected between one port and the and the load is also is connected between another port, right and maybe there is a voltage source there is a DC battery which is connected between one of the nodes.

One might argue that this DC battery could have been connected to the input side, yes absolutely right, right. One might argue that there can be multiple DC batteries one connected to the input side, one connected to the output side 100 percent true, right. Any of those combinations can be correct we are just taking it taking an taking an arbitrary example, ok. So, let me call this VDC, ok. So, since we have we have a two port network we need to denote some terminologies, right.

So, let me call this initial port first port 1 1 dash, other port 2 2 dash and let me call this VDC port as 3 and one might argue that why did not I give VDC a dedicated port why only a terminal and other port is shared. Well, I mean we can as well do that, right without loss of generality we can as well do that let us let us give it a port. So, 3 3 dash, fine. So, what can I from whatever we have done till now, right from whatever we have done till now what should we what should be the way forward? Firstly, we need to know what type of non-linearity that we are dealing with, right. So, it is a non-linear network let us say that this non-linear network is let us say this I 1 that is this is current I 1, this is current I 2 and this is current I DC, ok.

So, let us say the current I 1 is some non-linear function f 1 of V, ok. So, now, we are in trouble because we just now established the some nomenclatures. So, let me follow the nomenclatures and let us say that this is aa dash, this is ba dash, this is ec dash, ok. So, let us say this current I1, right is the function of Vaa dash, Vbb dash, VC prime which in this case is actually equal to Vdc, right. Similarly, let us say I2 is another non-linear function f 2 of Vaa prime, Vbb prime and Vdc and similarly I DC is also a non-linear function f 3 of Vaa dash, Vdd dash, Vdc, ok, great.

Now, among these voltages, right the port voltage, port voltages which one do you think is changing as the input changes and which ones do you think are not changing as the input changes. Clearly, clearly this V DC terms are a constant, right these are not changing, right and the give and in most cases in most circuits the DC voltage sources are DC voltage sources are supposed to hold the voltages constant, right. So, they do not change essentially they act as some sort of parameters and which currents are of our interest clearly the currents I 1 and I 2 are of our interest, right. The I DC is a current that is flowing through the DC voltage source. Even though we are calling it I DC, but note that the value of I DC is not fixed because it is a function of the voltages of all the terminals, right.

So, I DC did not necessarily be fixed. However, at this point of at this juncture of our analysis we are not particularly inclined towards figuring out what is what is going to happen to I DC because our main interest, right our main interest is to figure out what are the consequences of rather what is the relation between I 1 and I 2 because if I can figure out the relation of relationship between I 1 and I 2 in terms of V i, right ultimately then I should be able to figure out what is what is the relationship between the input and the output voltages or in other words I should be able to figure out what is the gain of the system, right. So, essentially our goal is to concentrate on finding out what are the relationship between I mean what can we do about I1 and I2, ok. So, let us see what can be done. So, I1 is f 1 of Vaa dash Vbb dash and V DC, but note that V DC does not change, right.

It is kind of a parameter which does not change. So, I will just drop it from the argument terms, right and similarly I 2 becomes f 2 of Vaa dash and Vbb B prime. So, in other words in other words what are we saying? We are saying that we know that there are DC voltage sources, right available, but for the purpose of we know that there are some DC voltage sources available and this network is non-linear, but for the purpose of our analysis we will assume that, we will assume that this currents that are getting set up this currents I 1 and I 2 that are getting set up between these ports have somehow been set up by the DC voltage sources that are there, right and that is contributing to I 1 and I 2 and we have figured out those values numerically or graphically or using simulation or somehow, right, but note that we are interested in figuring out what if there is an increment, what if there is an increment in these values of Vaa dash, right. How does that increment translate to the output or in other words how does that increment translate to the increment in the voltage at the across RL or rather what is the change in Vbb dash, right and what do we want to relate that to? We want to relate that to the input change the input voltage change Vi, ok. So, in order to do that we will resort to the same old same old same old methodology that is we assume, right.

So, under quiescent condition what is the case? This this is your quiescent condition. Under quiescent conditions you have you have I 1 and I 2 then you add some perturbation to the network. How are you adding the perturbation? We are adding the perturbation by adding a adding an input adding a small voltage source, right What is small again? It depends on what type of non-linearity that we are dealing with, right. So, once we perturb this what is going to happen? This current I 1 is going to change, right. This current I1 will change by delta I1 or let me simply call this small i1, right.

So, this voltage Vaa bar, prime will also change. So, let me just simply write this. So, my I1 will change from I1 to small I1 because my inputs Vaa prime changed. Similarly, the port voltage Vbb prime also changed to VBB prime +Vbb prime. Note that we do not yet know what Vaa prime and Vbb prime are, right.

We do not yet know we do not yet know what these port voltage increments are. These are still unknown, right. These are unknowns. Even this is this I1 is unknown, right. So, we are trying to develop a framework for this two port network, right.

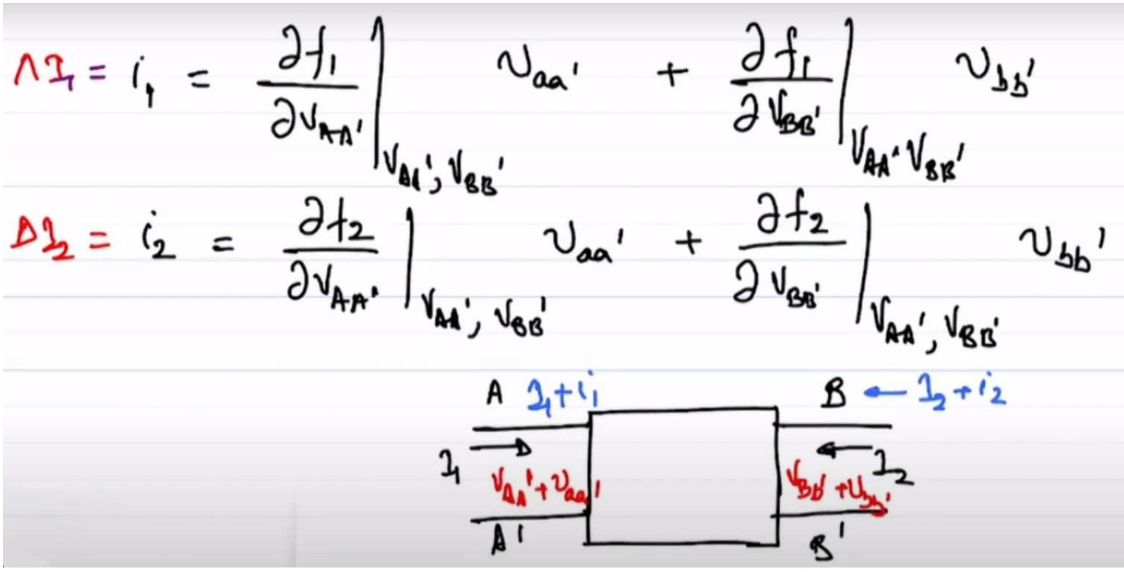
We had developed this framework for the for a normal non-linear network with one non-linear element. Now, we are trying to develop a framework for this two port network, ok. So, what should we do? In this case what should we do? We should simply do the same thing as we did earlier that is we expand this in Taylor series around the quiescent condition. What is the quiescent condition? Our quiescent this is the this is our quiescent network, right.

This is the quiescent network. So, around the quiescent network we need to expand we need to expand around the quiescent conditions we need to expand this non-linear equation. If we expand F 1 what should we get? We should get the original term F 1 of Vaa dash Vbb prime plus the increment Vaa prime and the derivative of F 1 del F 1 del Vaa prime, correct. Around the quiescent Vaa prime Vbb prime plus Vbb prime del F 2 del V b prime around the quiescent point plus higher order terms, correct. So, here I get I 1 plus I small I 1 and we see that these two are identical.

So, they go off, right. So, what are we left with? We are left with the incremental port current I 1 is related to the incremental is related to the incremental port voltages Vaa prime and V b b prime. How? They are related as the derivative of del F 1 del V b around quiescent operating points times the increment Vaa prime plus then del F 1 I think I made a mistake here this should have been del F 1, right. Because ultimately we are dealing with a non-linear we are dealing with a non-linear function F 1 not F 2. So, this should have been del F 1 del V b prime around operating points Vaa prime V b b prime times V b b prime, right. Similarly, if we do the same thing for I 2, right.

So, this I 1 is something that we did for we were this equation that we got here right now that we are staring at in front of our eyes is essentially relating the incremental current at port 1, right. Incremental current at port 1 if there is a incremental change in the port voltages a a prime and b b prime. Similarly, if we follow the same set of arguments for port 2, I should be able to relate the incremental current at port 2 that is this was delta I 1, right. If I have to be if I if we want to write this in terms of deltas, so this is delta I1, this is delta I2 which is being expressed as I2 will be del F the second non-linearity that is the non-linearity associated with port 2 or the current or the currents in port 2, times Vaa prime plus del f 2 /del Vbb prime, ok. So, again to state the obvious what is this what is this set of equations these two equations pointing to these two equations are essentially saying that if I have if I have replaced all the non-linear elements inside this network with their linearized equivalent, right.

If I have replaced all the non-linear elements inside the network with the linearized equivalent and if we had or in other words this is all this is saying is that this is terminal a a prime this is terminal b b prime, we have this current I 1, this current I 2, all this is saying is if this voltage got perturbed by from V a a prime to V a a prime to V a a prime plus V small a a prime and the voltage at the other port got perturbed from V b b prime to V b b prime plus an increment the current I 1, right, would have been perturbed from I 1 to I 1 plus small i 1 and the current I 2 would have been perturbed to I 2 plus small i 2. And the relationship between these small i 1 and i 2 and d a a prime and small d b b prime are being depicted by the two equations above, right. So, in other words in other words the incremental equivalent of these two port network, this incremental equivalent of these two port network if I simply say that this is V a a prime and this is V b b prime, this current is I 1, this current is I 2, I can express I 1 as some constant times V a a prime plus some other constant times V b b prime. Similarly, I can express I 2 as some constant times V a a prime plus some constant times V b b prime. What are those constants? These constants are nothing, but the derivatives the derivatives of the non-linearities, these constants are nothing, but the derivatives of f 1 with respect to the port voltages or f 2 with respect to the port voltages, right.




And note that these derivatives are constant because they are fixed once you have fixed the quiescent voltages, right. Just like in case of a one port network or a single non-linearity the derivatives were a function of the quiescent operating conditions. Similarly, in this case also the derivatives will be a function of the quiescent port voltages and currents, right. So, since these are constants we express them in terms of there is a terminology associated with them. So, we call this is as Y 1 1, we call this as Y 1 2, we call this as Y 2 1, we call this Y 2 2, ok.

And why do we do that? Because this is a standard terminology for an incremental two port network where we are able to relate the currents, the excitations at the between the port, the incremental excitations of the port voltages with the incremental excitations of the port currents, right. And I am sure you must have done some sort of two port network analysis in the past and this is essentially an extension of that, right. And how do we go about doing this analysis of these two port network? Let me write it out once again. So, these terminologies of Y is what is 1, what is 2 has its genesis in the following that is instead of writing those terminologies, those ports as A A dash and B B dash, if we simply say that let us say V A A dash is equal to V 1, right.

So, we call this port 1, right. So, we call this as port 1 and this voltage as V 1 and if we call this port 2 and this voltage as V 2, then V A dash becomes V 1, V B B dash becomes V 2, right. This incremental current through port 1 is I 1 and if we denote the incremental current through port 2 as I 2, right, then what we are essentially saying is I 1, there is some relationship of I 1, some linear relationship I 1 with respect to V 1 and with respect to V 2. Similarly, there is some linear relationship between I 2 and V 1, V 2 and these constants, this proportionally constants, we call this one as Y 1 1, this one as Y 1 2, this one as Y 2 1 and this one as Y 2 2. Again, why this is, why are this called as Y 1 1 and Y 2 2? Because what is the, how do you get Y 1 1? So, what is the definition of Y 1 1 from these equations? This equation, from these equations Y 1 1 is equal to I 1 over V 1 when V 2 is equal to 0, right, right. In other words, so this is, in plain English this means the incremental Y 1 1 is the ratio of the incremental change in I 1 with respect to incremental change in V 1 when V 2 is constant.

So, this part is vital. V 2 is constant, right. So, when V 2 is constant, V 2 is constant means what? Means small v 2 is 0, correct. And since when small v 2 is 0, why the relationship between I 1 and V 1 is simply I 1 equal to some constant, some constant time V 1 and that constant we relate it as Y 1 1, ok. Similarly, what is Y 1 2? Y 1 2 is nothing but Y 1 2 is I 1 by V 2 when V 1, small v 1 is 0 which means the voltage across the port V 1 is held constant, right, right. So, note that this first term, this one relates to, relates to the excitation that I am interested in, in the port number that I am interested in and the second term 2 relates to the excitation, the dependent variable or rather the excitation with which I am taking the derivative. Similarly, Y 2 1 will be I 2 over V 1, right, I 2 over V 1 when V 2 will be equal to 0 and similarly Y 2 2 will be I 2 over V 2 when V 1 equal to 0, right.

So, in other words to make it explicit V 2 is constant when V 1 is constant. So, these are, these are the ways, these are the definitions of Y, Y 1 2, Y 1, Y 1 1, Y 1 2, Y 2 1 and Y 2 2, ok. So, we will stop here and we will start up in the next class. .