Economic Operation and Control of Power System

Dr. Gururaj Mirle Vishwanath

Department of Electrical Engineering

IIT Kanpur

Week - 02

Lecture - 09

Hello and good morning everyone. So, welcome you all for the NPTEL online course on Economic Cooperation and Control of Power System. So, in today's class we will try to discover the one of the most fundamental primitive economic dispatch problem solved using linear programming method. So, linear programming is very capable of handling inequality constraints as long as the problem to be solved in such that it can be linearized without loss of accuracy. How the economic dispatch problem can be structured as an linear program is explained below. Let us say first the non-linear input-output characteristics output or cost functions are expressed as a set of linear functions.

So, what with a non-linear cost function as shown in the following figure. Let us take a specific generator which has a non-linear characteristic curve as it looks like this and it has its minimum generation and a maximum generation point thresholds are being given. And now, how can we linearize such that and how much should be the number of segments that one should choose so that the result one who get by solving this using linear programming method will be much more accurate and closer to the realistic solution. Let us say we can approximate this non-linear function as a series of straight line segments as shown in figure below.

Now this is the non-linear curve. Now let us say I will take 3 segments, this is the segment number i1, this is i1, this is segment number i2 and this is segment number i3. And now it was earlier non-linear function, now you will have 3 linear functions. Now you can solve easily this 3 linear functions and let us say the P i variable is replaced with 3 new variables that means the total generation from P i minimum, this is P i minimum to P i maximum is now split into 3 parts.

The three segments for generator i shown will be represented as i_1, i_2, and i_3. The P_i variable is replaced with three new variables: $P_{gen_{i_1}}$, $P_{gen_{i_2}}$ and $P_{gen_{i_3}}$

And each segment will have a slope designated as:

$s_{i_1}, s_{i_2}, s_{i_3}$ (where $s_{i_1} < s_{i_2} < s_{i_3}$);

Then the cost function itself is now represented:

$$F_i\left(P_{gen_i}\right) = F_i\left(P_{gen_i}^{min}\right) + s_{i_1}P_{gen_{i_1}} + s_{i_2}P_{gen_{i_2}} + s_{i_3}P_{gen_{i_3}}$$

where, $0 \le P_{gen_{i_k}} \le P_{gen_{i_k}}^{min}$ for $k = 1,\ 2,\ 3$

what is slope? Slope is nothing but y by x right, slope is nothing but y by x mathematically which is nothing but dF by dP or F by P whatever you can take. This into generation again you will get the cost only dF by dP into generation will give you the cost that means the cost of the ith generator at segment 1. Next this is the cost of the ith generator at segment 2. You can call it as F i 2 let us say for simplicity and this is the cost of this whole function will give the cost of ith generator at segment 3 right. And given the condition the ith generator kth segment the power output should be within the range of 0 to the ith generator minimum power at k plus 1 segment. That means let us say you can easily identify from this slope from this picture also that this is the minimum P gen i 2 minimum. This is also same as P gen i 1 maximum right. So, the generation of the first segment of the ith generator lies between the range of 0 let us say this is 0 point this is starting point between 0 to P gen i 2 minimum or P gen i 1 maximum right. That is what is been mentioned here. So, this is applicable for all the segment. Now and finally, what is the total generation? The total generation of the ith generator:

$$P_{gen_i} = P_{gen_i}^{min} + P_{gen_i} + P_{gen_{i_1}} + P_{gen_{i_2}} + P_{gen_{i_3}}$$

And what is the slope here? The slope is nothing but the y by x the y by x is nothing but the change in fuel cost from i plus 1th generator segment i plus i k k plus 1 segment to k segment of that ith generator. That means let us say the generation which is taking place of the ith generator at k plus 1 instant k plus 1 segment minus the fuel cost taking place for the same generator at kth segment. This is the change in fuel cost divided by the change in generation given as:

$$S_{i_k} = \frac{F_i\left(P_{gen_{i_{k+1}}}\right) - F_i\left(P_{gen_{i_k}}\right)}{\left(P_{gen_{i_{k+1}}}\right) - \left(P_{gen_{i_k}}\right)}$$

Now the cost function is now made up of a linear expression in the three variables $P_{gen_{i_1}}, P_{gen_{i_2}}, P_{gen_{i_3}}$. Because the slopes increase in value the linear program will cause: $P_{gen_{i_k}}$ to be at its limit $P_{gen_{i_k}}^{max}$ before $P_{gen_{i_{(k+1)}}}$ increases beyond 0.

That means for each segment we are considering it to be it has its own 0 limit and a maximum limit right. Now the linear programming solution of the economic dispatch can be written as:

$$Minimize \sum_{i=1}^{N} \left(F_i\left(P_{gen_i}^{min}\right) + s_{i_1} P_{gen_{i_1}} + s_{i_2} P_{gen_{i_2}} + s_{i_3} P_{gen_{i_3}} \right)$$

$0 \le P_{gen_{i_k}} \le P_{gen_{i_k}}^{max}$ For $k = 1,\ 2,\ 3$ ... for all generators $i = 1,\ \ldots, N$

And finally $P_i = P_i^{min} + P_{gen_{i_1}} + P_{gen_{i_2}} + P_{gen_{i_3}}$ for all generators $i = 1,\ \ldots, N$

Subject to $\sum_{i=1}^{N} P_i = P_{load}$

That means the individual generator power output is equal to the power output at different segments of that particular generator plus its minimum. And if you add all such generators which has so many segments of power generation if you add all of them and that should be equal to the total load. That should be equal to the total load. Now together with the linear programming, linear cost function there will be 3 generation equations to capture the value of the generator as a function of the segment powers that is 3 equations of the type given by:

$$P_{gen_i} = P_{gen_i}^{min} + P_{gen_{i_1}} + P_{gen_{i_2}} + P_{gen_{i_3}}$$

. Here we are considering 3 segments. It could be 100 segments and there will be 1 equation to force the total generation to equal the load which is total generation is equal to total load of all the generators given by:

$$\sum_{i=1}^{N} P_{gen_i} = P_{load}$$

Now let us take an example. Let us say there are 3 generators and they are supplying 850 megawatt of load. Now their ABC constants or coefficients have been given. Their minimum generation, maximum generation is also given. In the succeeding table 1, 3, 5, 10, 50 segments for each cost function are used. We have solved using different segments to see what is the impact of variation in the segments on the results that we get. It can be observed that the solution closes in on the same solution as a number of segments increases. It is obvious. As the number of segments increases, then you make a close approximation to the non-linear curve using linear segments and then the number of segments chosen will, the highest number of segments chosen will ensure that you will get a closely accurate result. For the first segment, the total cost was coming out to be 82227.

870. But if you see at the 50th segment, it is coming out to be 8194.357 and this result is also compared with the lambda search method which we are going to discuss and that is in closer to this lambda search method. So, depending on the number of segments used, the

solution will differ from that obtained using the standard method. Increasing the number of segments does not necessarily, this is a very important statement, this does not necessarily bring the solution closer to the exact solution. Just before few seconds, I told that as you increase the number of segments, the solution will be closer to the accurate value.

But it also depends like where is the true solution lying upon. For example, if you say this is the curve, this is a non-linear curve. Now, you can, let us say I will divide it into one segment only. Let us take another example where I will divide it into two segments. Let us take another example where I divided it into three segments.

I will just divide it one, let us say this is second and this is third and it may come like this. So, it, now you see let us say the exact solution of course, the linear approximation, the results that you get is the intersection point between the linear curves with the actual curve. Now that in the case of the first case where there is only one linear approximation, only one linear segment is there, then you have two intersection points. Here you have two intersection points. In the case of second approach where we considered solving the same problem with two segments, two linear approximations, then you have how many? Yes, you have three intersecting points. In the case of three approximation, three linear approximations, then how many points you get? Intersection points 1, 2, 3, 4. Now, let us say your true solution is lying somewhere here. This is the true solution let us say. Now if you compare between two segments and three segments, in the three segments, three intersecting points that means two segments, three intersecting points, you see for the two segment case, two linear approximation case, the result is closer. You see the result is closer compared to that one segment or two intersecting points where the result is far away.

Now, whereas in the case of three segments, in the case of three segments or four intersecting points, you see this solution may be little far away from the, the solution that you get either this solution or this solution point, this is far away from the actual true solution. That may be closer to the case where there was the solution obtained using just two intersecting, two linear segment analysis. You understand it depends upon where your solution lies and if you further increase, it may be even closer or even it may be further. But one thing is for sure, if you have more segments, if you increase the segments to 100 or 150, then there is a very high chance that the true solution wherever it may lie within the plane, you will get a closer value with respect to the true. As the number of segments is increased to 5 and 10 and even 50, the solution comes very close to the exact solution.

Three generators are supplying, let us take another example where three generators are supplying 850 megawatt of load. Now, calculate the most economical power generation plan using linear programming. Now, this is the network topology where you have three buses and you have a generator and load present at different buses. And at each bus, between each bus there is a line and it has its own thermal capacity. That means the maximum power that you can exchange between these two lines, these two segments could be 150 megawatt. This is not P12 maximum, this is P23 maximum. This is 150 megawatt. Now, assume the reference bus to be bus number 1, this is bus number 1 or slack bus you can consider to be as bus number 1 and the MVA system base to be 100 MVA, this is a base MVA. Now, this is your line flow constraint and this line flow constraint should be less than or equal to Pij maximum, the maximum capacity. How do you get this? I hope you are all remembering this equation. Power is nothing but E into V by x into sin of theta m minus theta j. Pij is equal to Ei into Vj divided by xij sin of theta m minus theta j. This is absolute value. That means this if theta i is greater than power flows from bus i to bus j. If theta j is greater than power flows from bus j to bus I . Now, you see here, we are speaking in terms of per unit. That means at per unit let us say voltage are at 1 per unit. That means you will get 1 into 1, 1 by xij. In real sense, the phase angle between any buses is close to 0. The voltage profile will be closer to 1 per unit, whereas the phase angle will be close to 0.

So, we can approximate because sin function for a very small values, sin of theta i minus theta j will be almost equal to just theta i minus theta j. The value you get for evaluating sin of theta i minus theta j will be closer to the value of just theta i minus theta j. Now, what you get here? Ultimately 1 by xij absolute value of theta i minus theta j. Now, this is the per unit value. Now, if you want the actual value, what you will do? The base is nothing but how do you calculate per unit? The per unit of any system is actual by base.

Now, the base we have considered 100 MVA. Now, you need to go at the actual value. That means actual is nothing but per unit into base. So, you will get 100 divided by xij into absolute value of theta i minus theta j. Now, this is the line flow constraint and this should be less than or equal to Pij maximum, that particular threshold. Now, the problem becomes minimizing the cost of all the generators:

$$Minimize \sum_{i=1}^{N} \left(F_i\left(P_{gen_i}^{min}\right) + s_{i_1} P_{gen_{i_1}} + s_{i_2} P_{gen_{i_2}} + s_{i_3} P_{gen_{i_3}} \right)$$

Subject to: $100[B_x]\theta = P_{gen} - P_{load}$

and

$$P_{gen_i} = P_{gen_i}^{min} + P_{gen_{i_1}} + P_{gen_{i_2}} + P_{gen_{i_3}}$$

$$0 \leq P_{gen_{i_k}} \leq P_{gen_{i_k}}^{min} \quad , \qquad P_{gen_i}^{min} \leq P_{gen_i} \leq P_{gen_i}^{max}$$

Alternatively, line flow can be limited by incorporating a slack variable:

$$\frac{100}{x_{12}}(\theta_1 - \theta_2) + s_{12} = 150 \text{ where } 0 \leq s_{12} \leq 300$$

Now, if you solve the solution to the linear program with 10 segments and including line flow limit, you will get this results. What is the conclusion? The linear programming can be built with all line flow constraints built into it, which makes a linear programming simplex matrix quite large. So, if you add so many line constraints into the problem, though it is a linear set of equations, so but it will unnecessarily increase the computation time. So, an alternative is to solve the linear programming without any line flow. First solve without considering any line flow. You just have to solve set of linear equations. And then solve for the optimum and calculate all flows that result and only put line limits into the linear programming for those over the limit. You just calculate, then find out for those buses which is possibly violating the line limits. This process has been called iterative constraint search. So, that is it with respect to the linear programming.