Economic Operation and Control of Power System

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Lecture 08

Hello and good morning everyone, welcome you all for the NPTEL online course on Economic Cooperation and Control of Power Systems. Today's class we will try to discuss about economic dispatch problem by using numerical methods. Let me just highlight whatever we had discussed in the last class. Let us say there is a Lagrangian function L is equal to FT plus lambda into the constraint. So this FT is nothing but summation of the fuel cost of all the generators and the constraint function could be equality constraint that is summation of generators minus summation of generators minus P load and this should be as close as possible to 0. Now for each generator you obtain the incremental fuel cost and ultimately the Lagrangian function would lead to DF by DPi is equal to DF by GP1 is equal to DF1 by DP1 is equal to DF2 by DP2 which is equal to DF3 by DP3.

Similarly up to DFN by DPN is equal to the Lagrangian multiplier that means incremental fuel cost of each generator is same and that is Lagrangian multiplier and then we could able to solve couple three problems where in the first problem we have considered just equality constraint there was no inequality constraint constraint considered and then we could able to obtain the incremental fuel cost and then we could solve the lambda and then the second problem we had considered the inequality constraint as well. The inequality constraint could be the generation of any generator has some limits P minimum to P maximum. So once this limits have been imposed then we need to ensure at any given combination of generator output which will come out to be a optimal lambda yet you should ensure that, you know, it will not touch any of this thresholds increase or lower than the minimum or it should not go beyond the maximum. If it touches then we will assign that threshold to that specific generator.

Suppose a specific combination comes out to be for a worked out problem we could understand that for generator 1 and generator 2 and generator 3, let's say for the first generator if it increases if it is increases beyond the maximum capacity then we have assigned if it increases the maximum capacity then we have assigned P1 is equal to P max and for the second generator it was within the limits. This was within the limits of P minimum to P max and let us say for the third generator if it is less than the P minimum we have assigned P3 is equal to P minimum and then the one important thing that we have noticed is we should ensure that if any one of the generator is violating the limits then we should also check whether if it is touching P maximum then incremental fuel cost of that particular generator should be less than lambda. And let's say if it is if a particular generator is touching the minimum threshold P minimum then the incremental fuel cost of that specific generator is greater than lambda and if it is within the limit then the incremental fuel cost should be equal to lambda. One of the student was asking why it is so. This is very important to know why it is so.

So we will try to understand this with the help of some graph. Let us say this is dF by dF i by dP i with respect to P. Let us say this is the incremental fuel cost of one generator. There is a incremental fuel cost of another generator, incremental fuel cost of the third generator 1, 2. Then if you see here this is a incremental fuel cost of each of the generators and then they have their limits.

For example, before that I will explain you with the help of the fuel characteristics curve. This is for the first generator, this is for the second generator, this is for the third unit. So lambda means what this is the demand, whatever is the load demand. So you need to find out the lambda such that the demand is met. Now if let us say the specific demand is too high then it is touching two of the, it requires the generation from the two of the generators that is generator 1 and generator 2.

And let us say you are getting this as a lambda. The combination of generator 1 and generator 2 is sufficient enough to meet out the load demand Pd. And then you see there is a generation at generator characteristics, fuel characteristics of a third generator and it has its own lambda. This is P minimum, this is P minimum, this is P maximum. So the lambda corresponding to this is df3 by dP3, this is the P minimum lambda, lambda minimum for that particular generator.

Now you can see here the optimal lambda that we could able to find out for the specific demand is this lambda that is as shown is lambda and you can see here that lambda is Lambda is less than lambda 3 minimum that means the minimum generation of this particular generator will be greater than indirectly that lambda. You got this point. The other way, the lambda is less than the lambda 3 or lambda 3 minimum is greater than lambda. Let us say you have a generator combination, the characteristics curve goes something like this. This is a fuel characteristics, this is for generator 1 and let us say this is for generator 2 and this is again for generator 3, something like this.

Then for a specific combination you are getting lambda as something like this. This is the lambda. Then what is happening? You need to operate this, this is P maximum and this lambda is here. So what it suggests? The lambda for this specific generator that is this is P1 and this is for P2 or the generator 2. Let me write like this.

This is for unit 1 and this is for unit 2, this is for unit 3, the third generator. So this particular unit 1 generator has its lambda minimum and it has its own specific lambda maximum. You can see here the actual lambda that you could able to find out to meet out the constraint, summation of generation is equal to total demand is this lambda. And you can see here when a particular generator unit 1 is touching its upper limit, the maximum generation, the maximum land of that particular generator, let us denote this as DF1 by

DP1 maximum is less than the actual lambda. That means for any combination of generators, if a particular generator is touching its upper threshold, then you have find out the lambda and that particular generator lambda will be always less than the actual lambda.

And if it is touching the lowest threshold, then that particular lambda will be greater than the overall lambda of the operation. I hope this is clear, this is very important to understand. So let us move on to the actual topic of today. The economic dispatch with piecewise linear cost functions. Let us say you have multiple generators and let us assume that all units are running.

We can start with all of them at P minimum. Let us consider so many generators you need to meet out the low demand. Let us start with the minimum and then begin to rise the output of the unit with the lowest incremental cost segment. That means there are so many generators, they have their individual incremental fuel rate characteristics DF by DP. As you can see here, this is DF by DP is a fuel characteristics and it has its own minimum and maximum value.

And the incremental fuel cost rate for this function is constant because if you derive it of a linear function is a constant. Then like this you have for another generator, this is let us say for generator 1 and you have for another generator it could be like this, the incremental fuel cost rate. That means the lambda of this generator 2 is at a higher value compared to the lambda of the generator 1. It has its own P minimum. This is P minimum for the another generator, it has its own P maximum.

Then you would choose to start with for a specific given load, you will select the generator 1 in the first go because it has the lowest lambda. And then you would complete I mean whatever may be the load demand, you would first completely utilize the maximum capacity of that first generator which has the lowest lambda. And still if the load demand is not met, then you would pick up that immediate generator which has the most close by or next lambda as the next higher lambda. Then similarly you keep on choosing those specific generators with the increase in the load. You increase the lambda, you take the next generator which has the next highest lambda and then total you meet out the load demand.

So this is what is piecewise linear cost functions. Eventually we will reach a point where a unit's output is being raised and the total of the all units power output should be equal to the load demand. At this point we assign the last unit being adjusted to have a generation that is partially loaded for one segment. Let us say like this, let us say you have 5 generators with the help of 4 generators, you have total 5 generators, 5 generators, With the help of 4 generators you could able to obtain certain generation, but still you have not met the total demand. Now you have only last generator which is left out.

With that last generator you need not have to load completely. Even partial loading also you could meet out the total demand. This is what I was expressing. Let us say this is a fuel rate characteristics in practical thermal power plant also. So the fuel characteristics need not be exactly linear as I have already discussed.

So it has its own, between two points it is linear, then again between that point to next point it has its own linear function. So it is a combination of multiple linear functions. So now the incremental fuel characteristics you can obtain, it is discontinuous. And if there are two units with exactly the same incremental cost, they can be loaded equally. Let us say you have two units with the same incremental cost, you can share the load.

At table giving each segment of each unit its megawatt contribution, the right hand megawatt minus the left hand megawatt can be created. Ultimately you can obtain a table starting from, you can start there are multiple entries into this. You can, the first entry could be of that particular generator which has the most minimum lambda. Next generator, next entry would be of the next generator which has the next lambda which has its own minimum lambda, where lambda minimum, lambda minimum 2, then lambda minimum 3, like this you know formulate a table and with that table you can simply pick up the individual segment. And let us say you have a particular generator, it has you know the characteristics curve which has multiple linear values, right like this.

Then each generator has its own linear functions and it has different lambdas as you can see here. So, you would enter those ultimately you will take out the piecewise segment and you would make it as a ascending order. Let us say for example, this is for generator 1, right. This is for generator 1 or unit 1. Let us say you have similar characteristics for another generator and it goes like this.

Let us say for the third generator you have like this. Then the first entry will be, first entry of the table will be, let us number it for your easy understanding. Let us say this is 1 and this is 2 and this is 3. In this order you would enter actually in the table 4 and then like this 5, 6, 7, 8, 9. Got it? Next, base point and participation factors.

Here it is assumed that the economic dispatch problem has to be solved repeatedly by moving the generators from one economically optimum schedule to another as the load changes by a reasonably small amount. Let us say you have already defined a set of combination of generators which is coming out to be the most economical combination to meet out the specific load demand. Now, if there is a small change in the load, with respect to the small change in the load, what would be the optimal share of individual generators to meet out the change in the load demand? That can be obtained by using participation factors, this approach. Now, start from a given schedule, there is already a most optimal schedule that is your reference point, that is the base point. And next, assume a load change and investigate how much each generating unit needs to be moved, i.e. "participate" in the load change. In order that the new load be served at the most economical operating point. And assume that both the first and the second derivatives in the cost versus power output function are available, that is Fi double dash and Fi double dash exist. The incremental cost curve of the ith unit is given in the figure. That means now, earlier we were choosing dF by dP, by incremental fuel cost of each generator is obtained and that should be equal to lambda.

Now, you got the optimal schedule. With now, there is a change in the load demand. Now, what you need to find out? This is F dash, this is F dash. Now, you need to find out F double dash. That means, with respect to change in the load delta P, what is the change in the Lagrangian multiplier, delta lambda. The change in delta load, load demand delta P leads to how much change in the delta lambda.

That means P plus Pd, earlier was Pd, let us say the load demand, Pd plus delta P, the change in load demand should be equal to delta lambda plus lambda, where delta lambda plus lambda will be the most optimal new lambda. That will be the most optimal new lambda value. This is your lambda new. That means you can see here, there is in this characteristic curve, you can see here, earlier this was the load Pi naught. Now, the load is changed from Pi naught to some value.

Now, this is the change in load. For that, there is a change in lambda. As the unit load is changed by an amount delta Pi, the system incremental cost moves from lambda naught to lambda naught plus delta lambda for a small change in power output on this single unit. Now, you need to find out change in lambda leads to how much change in, change in power demand leads to how much change in delta lambda. Right? Now that is nothing but Fi double dash, which is delta lambda by delta Pi or delta Pi is equal to delta lambda by Fi double dash, Fi double dash. Similarly, you have for individual generators.

How much is the additional contribution from the individual generator? Right? Now, the total change in generation is equal to change in system demand and that is the total change in generation is the sum of the individual unit changes. That means put together the individual contribution of the generations will be equal to the total change in the load. That is it. Now, let Pd be the total demand on the generators, where Pd is equal to P load plus P loss. Then to delta Pd is equal to delta P1 plus delta P2 up to delta Pn.

Now, what is delta Pi? Delta Pi is nothing but 1 by Fi double dash. Delta Pi is nothing but just a minute. Now, delta Pi you put in this formula, you need to find out delta Pi by delta Pd. Right? This is nothing but delta lambda by Fi double dash. Right? Delta P1 by delta Pd divided by delta lambda by total, you need to add individually.

Delta lambda Fi, this is delta P summation. That means 1 delta lambda by Fi double dash. Let us say you are finding delta P1 by delta Pd, you get delta lambda by F1 of double dash divided by delta lambda by F1 double dash plus delta lambda by F2 double dash plus delta lambda by F3 double dash. Similarly, you will get up to delta lambda by Fn double dash. So, you take delta lambda common, so ultimately you will get 1 by Fi double dash divided by summation of 1 by Fi double dash.

$$\left(\frac{\Delta P_i}{\Delta P_D}\right) = \frac{\left(\frac{1}{F_i''}\right)}{\sum_i \left(\frac{1}{F_i''}\right)}$$

Very simple mathematics. Now, let us take an example. While supplying an 850 megawatt load, 3 units generate 393.

2, 334.6 and 122.2 megawatt respectively. So, this is the reference value. It is been optimally scheduled. Now, this is the generator, most optimal power output. And there the heat characteristics and fuel cost is been given. Now, it is saying use the participation factor method to calculate the dispatch for a total load of 900 megawatt.

Now, load is changed from 850 megawatt to 900 megawatt. Then what is the change in demand from 850 to 90? 50 megawatt, right. Now you have the heat characteristics multiplied by the fuel cost, you will get cost function. And you obtain the incremental cost function for individual generator. You will get this value and that should be equal to the lambda.

Now by using this expression delta Pi by delta Pd is equal to 1 by Fi double dash divided by summation of 1 by Fi double dash, right. So using this function, using this expression you obtain for individual generator. Delta P1 by delta Pd, similarly delta P2 by delta Pd and delta P3 by delta Pd. You will get this factors. Actually this is the sharing value of the individual generator with respect to the change in demand.

That means for the change in 50 megawatt of load 47 percent is the share contributed by the generator 1. right ? And similarly for the generator 2 it is 38 percentage share. From the generator 3 it is 15 percentage, put together we will get 100 percentage, right. Now it is simple now. Earlier whatever was the generation with that you need to multiply, you need to add the share.

That is it. The share is 47 percentage into 50 for the first generator and everything put together you would get the most optimal generator outputs for the new change in the demand, right. Now let us discuss about composite generation production cost function. Suppose there were n generators units to be scheduled against a generator T that has a fuel schedule constraint as shown in the figure. That means let us say this is the combination that n number of generators and it has individual generator has its own fuel characteristics and now they are put together serving a load demand of P load. Now you need to find out the optimal scheduling when there is a new generator coming up and it has its own fuel constraint, right.

It has its own fuel constraint and none of the other generators, the earlier generator combination were not having any fuel constraint. Now how to do it, how to solve this problem? Let us, we need to you know simplify this where the total generators up to 1 from 1 to n whatever the number of generators were there that need to be represented by

1 single generator effectively. So that the problem will be more simple that there are multiple machines. Now you are replacing with that 1 single machine, 1 single unit which can have the characteristic curve which is equivalent to that of the overall characteristics put together of the individual generators, n number of generators. And now it is the dealing between the composite generators and the new generator which has some constraint need to be addressed, right.

Now how to obtain the composite characteristic curve that we need to identify first that is a basic problem. Now there a composite curve for units 1 to n can be developed. Let us say the Fs is the fuel characteristics of that composite overall characteristic curve and that is the summation of the fuel characteristics of the individual generator. The total power generation Ps is equal to the total generation P1 to Pn, right.

Now fuel cost should be equal to lambda, right. It is same. If one of the limit units hits a limit, its output is held constant, right. A simple procedure to allow one to generate Fs of Ps consists of adjusting lambda from lambda minimum to lambda maximum in specified increments, right, where lambda minimum is equal to minimum of all the lambdas. That means what we are doing is individual generator has its own lambda minimum and lambda maximum that I have discussed in the first part of the discussion, right. It has individual generator has its own lambda minimum and lambda maximum.

Now you obtain lambda minimum of all the generators from 1 to n. Among them find out the most minimum lambda among all those lambda minimums and obtain the lambda maximum of all those generators and obtain the most maximum. That means now lambda, actual lambda will vary from the most minimal lambda to the most maximum lambda. You got it? Now at each increment, delta wise you increment, you go from lambda, most minimum lambda to the most maximum lambda and at each increment calculate the total fuel consumption and the total power output for all the units. These points represent points on the overall fuel characteristic curve of the composite generator, right. Now I will explain you with a case study also, then you will be able to understand.

Now the points may be used directly by assuming that Fs of Ps consist of a straight line segments between the points or a smooth curve may be fit to the points using a least squares fitting program. Let us say ultimately for the fuel cost characteristics, this is the composite fuel cost characteristics and you will be getting let us say, it is like connecting different dots, right. You are getting, let us say you are getting the dots like this. That means this you are obtaining with by choosing the three n number of generators, you are taking the minimum of one of them and then you are moving forward from lambda minimum to the lambda maximum and ultimately you are reaching the maximum value like this, let us say. This is the most minimum lambda, most minimal value and this is the most maximum value, okay.

And then now you need to connect the dots. You need to obtain the ultimate characteristic, the total characteristic. The accurate one may be looking like this, something like this, but you need to make it more smoother. So roughly you would approximate to fit the curve as close as possible. This may not be accurate, but that would be helpful for us for the further analysis. You are obtaining the, by using curve fitting problem, you are obtaining the most smooth curve.

However, such smooth curves may have undesirable properties such as non-convexity. Example, the first derivative is not monotonically increasing. So I hope you understand what you mean by convex and non-convex. It is very simple to understand.

Let us say you have a function like this and you draw a line segment. So between these two points, intersecting points, all the points, the curve should lie below this intersection and that is convex, right. This is a convex problem. But whereas in the other case, let us say the curve is something like this. Here what is happening? Let us say you have an intersection between two points and you are drawing a straight line.

At some part of the curve, the curve lies between the, below the line. At some point, some point of the curve lies above the line. So this is your non-convex. So when you have this, you should ensure that, you know, when you obtain the smooth curve, you should not have a non-convex characteristic ultimately. Then it will make problem more simple, right. Now whatever I have explained, I am taking a flowchart to make you understand in a better way.

I am taking help of the flowchart. You can see here, the procedure is explained here. First start the lambda. First take the some lambda and then obtain and let us assume that that is the lambda minimum and then you obtain, calculate the Pi alpha such that dF by dP is equal to lambda I of alpha.

Here alpha represents the iteration value. This is the iteration curve. Alpha represents the alpha iteration, right. And I is the individual generator, right. Like they are from 1 to n, for all the generators, you obtain the incremental fuel characteristics dF by dP. If unit I hits a limit, now you have already chosen the lambda.

The lambda is already fixed. You have taken some lambda. For that lambda, first you obtain dF by dP and that should be equal to your lambda. And you obtain for all the generators, dF1 by dP1, dF2 by dP2. Similarly, dFn by dPn.

You have chosen the lambda. Of course that need not be the optimal lambda. And you obtain the incremental fuel characteristics. With this, you will get a value of generation. What do you make dF by dP? dF by dP, this is nothing but let us say your fuel characteristic function is A plus BP plus CP square.

dF by dP is B plus 2CP, right. And this is equal to lambda. Now you obtain P because lambda you already got, you have assumed. You obtain the value of P, right. And you obtain the value of P for that particular iteration for all the generators, right. And then you sum up the total generators value, whatever you are getting.

And that is a composite total power output. The individual generator, your ultimately the total generators, you are representing with one generator. Now that is a composite power output of the composite generator at that specific iteration, right. And then you obtain the fuel characteristic. Then you increase the lambda again, okay. From lambda, for the next iteration, lambda alpha is, lambda is nothing but the previous lambda plus some increment in lambda.

That incremental lambda you can choose up. That could be any value. It could be lowest as, as lowest as possible 0.001, whatever it could be, right. And then you check whether lambda alpha plus one is greater than the most maximum lambda. Then if it is that, if that is the case, then you have reached the most, you have touched upon all the generators, right. If that is not the case, then you again go and calculate the generators, power output and now you get all those points, right.

And then you get all those points, like I already told, you get all those points, then you need to obtain the, by using curve fitting curve you need to connect the dots. Let us try to understand with the help of example. Let us say there are three generators. It has its own fuel characteristics, heat characteristics and you get the fuel cost also.

Now let us write a MATLAB program or any software to obtain the fuel characteristics. Now we have done this for each of the generators, the incremental fuel characteristics looks like this. This is DF1 by DP1, okay. This is, is it visible? DF2 by DP2 and then the DF3 by DP3. You can see here the incremental fuel characteristics is linear and that is different for the different units because they have different cost functions. Now which is the most minimal lambda? The most minimal lambda you can obtain for the first generator, that is this value.

You can check that, that is coming out to be exactly 8.3886. The most maximal lambda you obtain for the third generator, that is coming out to be 14.

847. Now a program was written such that lambda varies from 8.3886 to 14.847, right. At each increment the three units are dispatched to the same lambda and then outputs and generating costs are added. I have a table to help you understand now. Yeah, then ultimately you would get this points and you connect those dots. In this case it is more smoother because they are almost like a linear increase. Now how we got this? You see here, now lambda it is increased in a step by step manner to reach the maximum value, right.

And for each lambda increase you obtain the Ps, the total generation and the total fuel cost corresponding to that. And then this is the actual value that you get and with the curve fitting formula you obtain a curve and for in that curve actually for that specific lambda you are getting 4138. Now there is small error but it is around 1 percentage.

This can be acceptable. It should not be like you know error is some 10 percentage associated. Like that you obtain for all the from lambda minimum to lambda maximum. And then once you get this curve then you get this a plus bp plus cp square. This is a composite characteristic function, right. a plus bp plus cp square. That means this characteristic curve represents approximately closer to the summation of the individual characteristic curve of the generators. So we will discuss about lambda iteration method in the next class. Thank you very much. .