Economic Operation and Control of Power System

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Week – 02

Lecture 07

Hello and good morning and welcome you all for this class on the economic dispatch of thermal units. In the previous class we have learnt about very important programming approach, optimization technique, which is dynamic programming, with the help of which we could able to solve the power system optimization problem very easily. We have taken some examples while we explained two different types of dynamic programming techniques, which is forward recursive algorithm and backward recursive algorithm or approach; FRA; forward recursive approach, and backward recursive approach. So we discussed about three different terms which is stages and states as well as policies. So we divide the problem. Let us say you need to move from, you know, a source to a destination.

And then we divide it into different stages. And we keep on finding an optimal state in the case of forward recursive approach moving forward from the source to the destination. So at every stage we will find out that optimal state and then we will move to the next stage. And then we will ultimately we will get the sub policies combining which we will get the overall policy which is the most optimal one.

Whereas in the case of backward recursive algorithm or approach we start from the destination side and moving towards the source side. And we get the optimal states of the future state when we come to the previous stage. So both the techniques have its own advantages, and moreover, the backward recursive approach seems to be more efficient and easy as compared to the forward recursive approach. Now in today's discussion we will learn about very important subject matter which is known as economic dispatch of thermal units. See economic dispatch problem without considering the transmission line effects.

First we will deal about economic dispatch problem without considering transmission line effects. Let us say there are N generators which are connected to the bus bar. I hope you are very much familiar with what do you mean by bus bar. There is a bus to which you connect the lines as well as there could be some loads which are connected to this buses. To this bus bar, there are N generators which are connected.

You can see also in the picture that there is a fuel inputs and there is a boiler. So there is fuel input to each boiler. This is a thermal power system network and the boiler which will convert the fuel could be a coal or could be oil; whatever it could be. And then from this it will convert it into super saturated steam. So fuel input is in terms of chemical energy and output you will get the heat energy.

And that can be transformed into mechanical energy when you apply this to turbines. When these turbines are connected to the respective generators and the generator output would be electrical. So the input is chemical; chemical to heat, and then from heat to mechanical and from mechanical to electrical. This is how a typical thermal power plant energy conversion takes place from one form of energy to another form of energy. And the individual generator output, let us say for the first generator we call it as G1 and its output is P1.

Similarly for generator 2 it is G2 and its output is P2. And similarly for generator nits output is Pn and its output is generatoroutput is Pn where the generator is called as Gn. So all these outputs are connected to one bus here. It looks like this. Similarly there could be different buses also for individual generator as well.

And for this particular bus there is a load which is also connected. So multiple generator outputs which are synchronized together and connected to a common bus and output of all these generators three things are common which is voltage, frequency and phase angle. Then only you can able to synchronize. Now input to each unit is Fi that is cost rate of fuel consumed. So the input is expressed in terms of chemical energy and it has its own cost.

Thermal unit has its own cost where the cost is due to the coal. And output of each unit is Pi and electrical power is generated. Essential operating constraint is that the sum of output power should be equal to the load demand neglecting the losses. That means the power output, P1 + P2 + .

.. + Pn should be equal to P load. This is a constraint. Why this constraint is there? Because if this generation output is not.

.. let us say total P gen summation of i to n, this output is not equal to P load, then important parameter that is frequency gets disturbed. The frequency gets varied and it should be fixed for a safe and constant stable operation of a system which is 50 hertz. So we need to maintain P generation is equal to P load. And economic dispatch problem is essentially the determination of the generation levels such that the total cost of generation Ft becomes minimum for a defined load. For you can achieve summation of P gen is equal to P load with any combination of generations.

But the objective of economic dispatch problem is to find out that particular combination of generators output so that not only you may meet P gen is equal to P load rather you will also achieve an important objective function which is cost minimization. Now economic dispatch with different generating plants. So as we have come across in the first week, there are three different, typically, nuclear power plants, hydropower plants, and thermal power plants. In the case of nuclear power plants they have little scope for the load variation. Because it is very complex to control the nuclear power plant.

So what we do is we give a base load; whatever the load profile is there, load curve is there. So we will give the base load which is constant for the nuclear power plant. This is for nuclear. And hydro power plant you can vary the power output based on the hydro input. And thermal power plant power output can be varied with the help of variation of coal.

Now operating cost of hydro power plants do not change much with respect to the output till the rated capacity. And we have learnt about also the characteristic curve. The information regarding the variation of the fuel cost rates for the fossil fuel based plants are crucial for formulating the ED problem. Ultimately it would be ED problem will be thermal plus hydro. Now optimally utilize the thermal and hydro such that you will meet out the generation, some of the generation is equal to load with the minimum cost of operation.

Now generally the cost of labor, supply and maintenance would be fixed. Now this is a typical characteristics non-linear cost function characteristic of a thermal power plant. You see this is P minimum, this is P maximum. Between P minimum to P maximum the characteristic curve is non-linear. Let's say it can be expressed as $F(P) = aP^2 + bP + c$.

That means the input or fuel input depends upon it's characteristic function and it depends upon the output power. And it has its own non-linear behavior. That means if you increase the power output then there is not linear increase in the fuel input. It is a non-linear increase in the fuel input. So P minimum is the minimum output power level at which it is uneconomical or technically not feasible to operate.

Below this limit it is unviable technically and commercially also to operate a typical thermal power plant. And above P maximum there is a threshold upto which it can generate. There is a thermal threshold for a power plant. Now the problem formulation is the objective function as I already told.

This is the total fuel cost. This includes the fuel cost of first generator, fuel cost of second generator and the fuel cost up to the fuel cost of the nth generator. So total fuel cost of all the generators put together should be expressed as summation of i equal to 1 to N and Fi (Pi). And what is the constraint? The total load, P load is equal to total generation; or the difference between total load and total generation should be 0. What is the inequality constraint? So we have learnt in our optimization preliminary understanding that there will be two types of constraints. One is equality constraint and inequality constraint.

Equality constraint is summation of generation is equal to load. Inequality constraint here is the generation of each thermal power plant generator should be within this range; Pi minimum to Pi maximum. The reason I have already told that means below which it is unviable to operate economically and, above which there is a thermal capacity. Because the conductor size has certain restriction. So beyond which there is a overheating that happens and the generator may get short-circuited or it may get burnt.

So where N is the number of participating generators, this is a constrained optimization problem. We have learnt there are two types of optimization problem. One is unconstrained where we learnt about single variable and multi variable, and constrained, which has equality and inequality constraints. This is a constrained optimization problem. Now how do you find the solution? As we have already learnt, we will find the solution using the Lagrange function.

You see this problem can be easily solved by using Lagrange multiplier method. And what is the Lagrange function? Lagrange function is the total fuel cost, i.e. objective function which you need to minimize, plus the Lagrange multiplier, lambda multiplied by the constraint. Where the constraint is P load plus P loss minus summation of generation is equal to 0 and that should be your constraint, accounts for the equality constraints.

And lambda is a Lagrange multiplier. Then what is the necessary condition? The derivative of Lagrange function with respect to each independent variable is 0. That means partial derivative of Lagrange function with respect to each generator. Doh L by doh P i is equal to doh by doh P i of this total function, which is the objective function. And this is a constraint function multiplied by the Lagrange multiplier. That will give you doh F i by doh P i minus lambda.

If you can derive this, let us say I am doing it for the first generator. What do you get? Doh F by partial derivative of that function is only doh p 1, because for the rest of the generators the partial derivative will come out to be 0, because other functions would be constant. So, doh f by doh p 1 minus the total generation, this is p load is a constant anyway and minus let us say you will get p 1 plus p 2 up to p n and doh p 1 by doh p 1 will give you 1 that multiplied by lambda you will get just lambda. So similarly for the second generator you will get doh f, this is doh f 1, this is doh f 2 by doh p 2 minus lambda.

Then all of them will be is equal to 0. And doh L by doh lambda is equal to phi. This is for the generators. Again you will do the partial differential equation for the Lagrangian function with respect to the lambda and that will come out to be the constraint function. That is it. Thus to have the minimum fuel cost the necessary condition is that the incremental fuel cost of all the generator should be same and is nothing but the Lagrangian multiplier.

So whatever may be the number of generators this holds good. That means doh f 1 by doh p 1 is equal to doh f 2 by doh p 2. Similarly up to doh f n by doh p n is equal to lambda. Of course in this case we are ignoring the losses. We are starting considering that losses are ignored.

Now to summarize we have economic dispatch following set of equations need to be satisfied. Doh f by doh p i is equal to lambda and this is the constraint p i minimum. The generation should be within the range of p i minimum and p i maximum for all the generators and summation of generation is equal to load and this is this note whatever I have mentioned in the note, this is very important. That means within this range of p i minimum to p i maximum the doh f by doh p i is equal to lambda for all the generators. Let us say if one of the generator touches either of the limits if p i is p i minimum then doh f by doh p i is greater than or equal to lambda.

And if p i is equal to p i maximum if it touches the upper threshold then the doh f by doh p i should be less than or equal to lambda. These are the 3 observations that you will make when you solve a problem. This is your function the characteristic function of a typical thermal power plant a i p i square plus b i p i plus c i is equal to f i of p i. Now if you do the partial differential equation then you will get p i is equal to lambda minus b i by 2 a i and recalling the equality constraints summation of generation is equal to load then your p i is nothing but lambda minus b i by 2 a i and that is equal to p load. Then if you solve lambda you will get as p load plus summation of i is equal to 1 to n b i by 2 a i divided by summation of i is equal to 1 to n, 1 by 2 a i.

$$\lambda = \frac{P_{load} + \sum_{i=1}^{N} \frac{b_i}{2a_i}}{\sum_{i=1}^{N} \frac{1}{2a_i}}$$

Now we will take a problem to help understand this approach Lagrangian multiplier based economic dispatch approach. Now the question is determine the economic operating point of for the 3 generating units when delivering a total load of 850 megawatt. Now the total load is given as 850 megawatt and the characteristic function the heat characteristic function of each coal fired stream unit is been given and they have their own unique a b c coefficients. Now the fuel cost is also given which is 1.1 dollar per mega British thermal unit and oil is 1 dollar per MBTU.

Now the you need to have the fuel cost total fuel cost that is the generation characteristics multiplied by their fuel cost. You see here first two are first one is coal fired stream unit and next two are oil fired. For the first characteristic function you multiply by 1.1 and for the second and third you multiply with 1 and then you will get fuel characteristic curve for each plant each unit. Now you obtain the partial differential equation that means you find doh f 1 by doh p 1 for each of them and that should be is equal to your Lagrangian multiplier and what is the constraint p 1 plus p 2 plus p 3 is equal to 850.

Now you solve for lambda that means you from this expression you get to know what is p 1 that is lambda minus 7.92 by this constant. Similarly for you replace for p 2 replace for p 3 then you will get what is lambda. Once you get the lambda then you can easily find out what is p 1 go back and find out with this expression you have got the lambda you can obtain what is p 1 p 2 and p 3 with the lambda that you have already solved. Now the combination of this p 1 p 2 p 3 will be is equal to p load.

That means in order to operate this thermal power plant the combination of the thermal power plant fired by coal as well as oils what is the most economical combination of generations for generation 1 it is 393.2 for generation 2 it is for second generator it is 334.6 for the third generator it is 122.

2 megawatts. And let us take another example. Now determine the economic operating point for the 3 generating units when delivering a total of 850 megawatt the same thing but here there is a inequality constraints which has been added. Earlier there was only equality constraint. Now we are also increasing complexity to the next level we are adding inequality constraint. Now coal price is changed and oil price remains the same. The same approach obtain the fuel characteristics for each unit then obtain the partial differential equation dF 1 by dp 1 and dF 2 by dp 2 and dF 3 by dp 3 and they should be equal to the Lagrangian multiplier which is lambda.

And summation of generation should be equal to 850. Now this will give you lambda as 8.28. Obtain p 1, p 2, p 3 just like the similar example but you see here in the earlier case there was no inequality constraint you need not have to worry whether each generation output for this economical operation have touched their lower or upper threshold or not you need not have to worry that. But in this case there is a inequality constraint which is already been mentioned in the question itself. So check for each unit whether any one of them are violating any constraint or not.

Now the first generation power output is 703.125 megawatt with this limit. The upper limit is 600 megawatt. So that means it is violating. So and for unit 3 also the calculated value suggests it should be 32.157 megawatt but if you can see here in the problem statement it is given as the lower limit is of 50 megawatt.

Now what to do? Let unit 1 be set to its maximum output. That means the Lagrangian multiplier based approach is suggesting increased generation of one as much as possible. So we now the maximum limit is 600. Now you fixed the generation of power the power generation of the first unit to be as 600 megawatt and limit the third generation to 50 megawatt and you will get the immediate value of the second generation. It should be 200 megawatt, because P2 is equal to 850 minus P1 minus P3.

So, you will get 200 megawatt. Since unit 2 is the only unit not at either limit, lambda must equal incremental cost of unit 2. So what is lambda? d f2 by d p2 and then you will get what is lambda because you have already fixed p1 and p3 and they cannot be equal to lambda. If they can be equal to lambda then you should take the previous value and that will violate the inequality constraint of generator 1 and generator 3.

Now you will get lambda to be of 8.626. Now you have to check the partial differential equation, obtain the partial differential equation d f1 by d p1 when p1 is equal to 600. When pn is equal to 600 what is the lambda that you are getting? 8.016 which is less than lambda. Similarly obtain partial differential equation of fuel characteristics of generator 3 with respect to its generation then you will get which is at 50 megawatt then you will get 8.

452 which is also coming out to be less than lambda. But if you remember the note within the limit of generation the partial differential equation of any generator should be equal to lambda. If it touches the upper threshold then whatever lambda you get whatever the value you get that should be less than the lambda and if it touches the lower threshold then it should be greater than lambda. That means d f3 by d p3 as it touches the lower threshold this should have been greater than lambda and that lambda is obtained by that generator which is not violating the any constraint inequality constraint. Here this is agreeing that d f2 by d p2 sorry there is a mistake here this is d f2 this is not d f1 this is d f2 by this is d f1 itself this is d f1 itself. Here d f2 by d p2 that is your generator which is not violating any constraint and that comes out to be 8.

626 then you obtain for d f1 by d p1 and this generator is violating and it is touching the upper threshold. For that you are getting 8.016 which is less than lambda that means this is in agreeable with the minimum constraint maximum constraint d f1 by d p1 is less than lambda this is fine. Whereas d f3 by d p3 which is greater than lambda it should have been greater than lambda but it is coming out to be 8.

452 and this is still less than lambda. Then that means what it is indicating incremental cost for unit 1 is less than lambda indicating that it is at its maximum but the incremental cost for unit 3 is not greater than lambda so it should not be forced to its minimum. Then what we are going to do now is take up fixed generator 1 p1 is equal to 600. Now you optimize for generation 2 and generation 3. So p1 plus p2 plus p3 is equal to 850 and d f obtain d f2 by d p2 and d f3 by d p3 and their value should be equal to lambda now and solving for lambda you will get 850 minus 600 that is what p2 plus p3 should be equal to and replace for p2 and p3 and that is equal to 250 then you will get the lambda. Once you get the lambda you can get what is the value of p2 and what is the value of p3 and that is coming out to be p2 is equal to 187.

1 megawatt and p3 is equal to 62.9 megawatt. Now you again cross check. So d f1 by d p1 at p1 is equal to 600 comes out to be 8.016 and that is less than lambda which is 8.576 and d f2 by d p2 is equal to at p2 is equal to 187.

1 should be equal to d f3 by d p3 at p3 is equal to 62.9 and that is 8.576 which is lambda. So all the constraints are met p1 plus p2 plus p3 is equal to 850 megawatt that constraint is met and individual generator power output should be within their limits thresholds and that is met and you are yet getting the most optimal combination of generator output which is matching summation of generation is equal to load total load. Now if you can cross check with the previous lambda it was 8.

626 that now you are getting lambda less than 8.626 which is 8.576 you got it you will not get any other value lesser than this considering all the constraints. Now let us go to the next level where economic dispatch of thermal unit is considered with the losses network losses. that means the same topology as I shown in the first slide, so total generation or a need to also meet the transmission network losses because in any practical power system there is a generation and there is a load generation is far away from the load demand and there will be transmission line which is present in between, so because of which the line losses takes place. So and we need to include the transmission network losses in order to make it more realistic. Let us say under some cases for a given power output if unit 1 has higher incremental cost compared to the unit 2 for economic dispatch the unit 2 is preferred over unit 1 provided unit 1 and unit 2 are situated closely by close by geographically.

Let us say unit 1 and unit 2 they are situated close by, so if one of the generator is of higher cost we will ignore that we will try to you know extract minimum power from that generator and take maximum power from the second generator whose cost is less. But if the unit 1 and unit 2 are separated far away then there will be a huge transmission loss and transferring power from unit 2 including losses may not be economical. Let us say one generator is very cheap the power output is very cheap but it is situated far away and if you transmit power you need to include the losses, so generation plus losses may not be economical as compared to taking generate power from that generator which apparently little bit costlier but total in considering the losses this first generator output would be more economical. So the economic dispatch problem is slightly more complicated as compared to the case when there is no losses which is inculcated.

So the constraint equation must include the network loss now. The objective function f t remains same as that of the case without losses however the constraint equation must be expanded as now this is the total generation sum of all generation should be equal to p load plus p loss and what is your objective function what is your constraint that is p load plus p losses minus all generation should be equal to 0. So the Lagrangian function is L is equal to f t plus Lagrangian multiplier multiplied by the constraint. This is just same approach obtain partial differential equation dou l by dou p i and that comes out to be dou f i by dou p i minus lambda into 1 minus dou p loss by dou p i earlier it was just dou f i by dou p i minus lambda without considering the loss. Now with inclusion of loss you will get this additional factor 1 minus dou p loss by dou p i. So that means dou f i by dou p i plus lambda into dou p loss by dou p i is equal to lambda and solving you will get lambda is equal to dou f by dou p i minus 1 by dou p loss by dou p i which can be further deduced as lambda is equal to penalty factor into dou f i by dou p i.

And what is the penalty factor this is 1 minus 1 by 1 minus dou p loss by dou p i where dou p loss by dou p i is the incremental loss for bus i that means there is a loss expression that you will see in the problem and this loss expression is a function of this is a function of all the generator or could be of only 1 or 2 generators also p 1 p 2 p 3. Then to this function you obtain partial differential equation dou p loss by dou p 1 similarly dou p loss by dou p 2 and dou p loss by dou p 3 and then you will get penalty factor corresponding to each of the generator. So the minimum cost of operation is achieved when the incremental cost of each unit is multiplied with the penalty factor which is same for all generating units in the system. Now earlier Lagrangian multiplier was not including any penalty factor Now you need to include the penalty factor.

$$\lambda = \frac{\frac{\partial Fi}{\partial P_i}}{\left(1 - \frac{\partial Ploss}{\partial P_i}\right)}$$

That is it. When units are connected to the same bus incremental change in transmission loss with change in generation is same for all the units. So hence all the penalty factors would be same. Therefore if penalty factors are same then naturally the incremental cost will also be same. But if the generator buses are connected in different buses then there will be difference in the penalty factor and that is the reality. That is the reality because so many generators are there, one is let us say Ramagundam you have a thermal power plant and then you have a nuclear power plant in some other place.

So and hydro is in some other place then they have their own network losses individual network losses which are independent and different from each other. So the penalty factor is also different. The summary above equations are collectively called as coordination equations. Solutions of economic load dispatch problem in the above case is a bit complex compared to the case without considering the network losses. There are two basic approach one is the use of network loss formula and the other is the use of optimization tools incorporating power flow equations.

Now the transmission network loss is a function of the impedances and the current flowing in the network I square into Z. The currents can be further considered as functions of the input and load power. And it is more difficult to solve these set of equations. So what is the solution methodology? Step number one pick starting values for each generator that sum to the load then calculate the incremental loss for each generator and the total loss also which is P loss. The incremental losses and total losses will be considered constant until we return to step two.

First assume generation total generation and that is equal to load then you obtain the incremental loss and total loss find out them and then calculate lambda that causes total generation to be equal to the constraint load plus losses. Now the equation are again linear. We are just making it more linear so that we can solve it and compare the generation from step three to the values used at the start of step two. If there is no significant change in any one of the values go to step five otherwise go back to step two.

So we will take an example to understand this. Again there are three characteristic curve and the fuel cost is given the oil price is also given and there is a loss expression which is given in addition. And you can see the loss function is dependent upon individual generator output P1, P2 and P3. Now apply the coordination equation find out dou Fi by dou Pi which is equal to lambda into 1 minus dou P loss by dou Pi for each of the generator. Then you will get these three functions and there is a constraint which is total generation minus loss is equal to total demand load demand.

Now randomly you pick up P1, P2 and P3 values. P1 is equal to 400, P2 is equal to 300, P3 is equal to 150. 400 plus 300 is 700 plus 150 is equal to 850 that is equal to generation. Generation is equal to total generation is equal to total load. Now we do not know the loss that is why we are just making generation is equal to load. Then we will slowly take another iteration and keep on doing the iteration so that we will ultimately reach this.

Obtain the solution where P1 plus P2 plus P3 minus P loss is equal to total demand. Now obtain the incremental loss. What is incremental loss function? This is dou P1 by dou P loss by dou P1 is equal to 2 into the constant multiplied by P1. Similarly obtain dou P loss by dou P2, dou P loss by dou P3. This you have got by obtaining partial differential equation of this function with respect to each generator.

And then you will get these values. Then obtain what is the loss. How do you get the total loss? You see here there is a loss function. You have assumed what is P1, P2, P3. Then you will get what is the total loss. And then now solve for lambda. Solve for lambda to this expression you obtain the lambda because you have got P1, you have got P2, you have got P3 and then obtain lambda from here.

Then once you get the lambda again obtain the generating value generator values. Initially assume something then you find out the incremental loss, you find out the total loss, then find out the lambda. Then after you obtain the lambda you apply to the same expression here.

Then P1 is equal to 0.976 lambda by minus 7.92 divided by this constant. Similarly obtain for P2, P3. Then you would get a different values P1, P2, P3 because this is not yet converged. This will not converge also because initially we have ignored that loss. We have made just generation, sum of the generation is equal to total load. Since these values for P1, P2, P3 are quite different from the starting release we will return to step number 2.

Now you consider this as the values for the next iteration. And obtain lambda and then again P1, P2, P3 the same procedure will follow. You obtain the incremental loss for individual generator. Then obtain the total losses. And then step number 3 is the new incremental loss.

Then you obtain the lambda and then again you come back and find out what is the P1, P2, P3. Ultimately it will take 5 iterations in this problem to have a possible combination of P1, P2, P3 and the losses also you can see between iteration number 4 and iteration number 5 there is no much difference.

It is almost getting converged. So even lambda there is no significant difference 9.52832 to 9.5284. So you can consider, you can keep on doing it to have even more accurate one but the error is almost like 0.001 or something like that. You can converge. You can consider this to be the most optimal combination of the generators which will make summation of generator is equal to total load plus and that is it.

This is the reference book. So in this week we have understood how with the help of economic dispatch, considering economic dispatch problem, how by using Lagrangian multiplier approach we could solve for the first example considering without losses there is no loss, generation is equal to load and first problem we have not even considered the inequality constraint. The second problem we have understood how we can solve by considering inequality constraint also along with the equality constraint. And then the third approach where, third example that we have took where we have also considered the losses into place, we have find out the penalty factor and we could able to obtain the suitable combination of generation which will make generation is equal to load plus losses. Thank you very much.