**Economic Operation and Control of Power System**

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**Week - 02**

**Lecture 06**

Hello, good morning everyone. Welcome you all for the NPTEL online course on Economic Cooperation and Control of Power Systems. This is Dr. Gururaj Mirle Vishwanath. I am currently working as an Professor in the Department of Electrical Engineering at IIT, Kanpur. So, in the previous week, you might have understood the various aspects with respect to the power system, the fundamental aspects relevant to the generation and various types of storage units.

Generation may also include the thermal power plant, the description about hydro power plant, description about nuclear power plant, and description about renewable power plants which include solar as well as wind energy. So, various types of generators associated with the wind energy such as DFIG, PMSG all of these aspects have been covered. Apart from that, we have also covered various aspects with respect to the optimization techniques; the preliminary aspects with respect to the optimization technique, the mathematical understanding to start the optimization associated with the power system. Now in today's class we will discuss about the dynamic programming which is a very important means of optimizing any power system problem.

So, let us try to understand briefly about the history of dynamic programming. So, Richard Bellman in the 1950s developed the concept of dynamic programming. Dynamic programming gives solutions for combinatorial optimization problems. It is nothing but selecting a combination of values with a set of variables to minimize or maximize a particular objective. So, there are so many variables among which, which one you need to optimize, which variable is optimal solution for a particular objective function to meet.

For example, objective function could be loss minimization, or objective function could be maximizing the profit. So, we need to identify the suitable approach to find a solution for this. And it gives exact solution and guarantees optimal solution that is not an heuristic or inexact method. So, the advantage in dynamic programming is it gives more optimal solution as compared to the other heuristic techniques. And many discrete and optimization problems use dynamic program to find solutions.

Some of the typical application of dynamic programming are the scheduling. So, in day to day this is a challenge for any power system operator. There are multiple generators.

Let's say there are 10 generators. Among them which generators, which combination of generators would give you a best optimal solution in terms of cost minimization, if that is objective function to start with.

Or there are so many other applications which including packaging and inventory management, string editing and economic dispatch, that's what I told, the optimal solution of dispatching a particular generator for a particular time. So, that we can reduce the overall cost of operation and, unit commitment. So, we will go in deep about all these aspects in further classes. So, let us try to understand the basic concepts associated with the dynamic programming. So, these three notations or definitions need to be clearly understood to start understanding the dynamic programming.

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Let us say there is a problem and it is divided into different stages. And each stage is made up of several available policies or alternative actions. Policies are those which is a solution which is a path to reach the particular stage. Now, each stage we need to have a proper identified optimal state. Now, what is state? In each stage there are number of states present.

One such state may belong to the optimal path in a given stage. Let us say you need to reach from Delhi to Bombay. In between there could be many stops. So, which stop you take so that you optimally reach moving forward to the Bombay. So, that in every distance, let's say at every distance there is a n number of states.

And you will choose the most optimal state moving forward to reach the ultimate destination. In a policy, a policy is chosen at each stage to move from one state to another state. That is a path traversed from one state to another state that is called as policy. The policies are the available paths at each stage. Need not be that there is only one policy to reach a particular state at a particular stage.

So, there could be different routes to arrive at a particular state starting from the initial point. Now, let us take a small example to understand. Let us say you need to reach from A to D. And there are different stages. Let's say we divide it into three stages.

This is stage number 1. Let's say this is stage number 2. And this is stage number 3. To reach from A to B we have three different policies. Let me denote this as P1, this is P2, this is P3.

And to reach from B to C there are again different policies P4, P5, P6. From C to D there are two policies P7, P8. Now from A to B by using greedy method approach it looks like P2 is the most optimal route, the most optimal policy, because the cost associated to reach from state A to state B is 2. It could be 2 dollars or 2 rupees that is a different story. Now from B to C it looks like P4 is the best one and from C to D it looks like P8 is the best one.

So, ultimately you will get sum total is 2 plus 1 plus 5 which comes out to be 8. So, this is the most optimal route advised for a person to travel from point A to point D or state A to state D. But one may get confused or one may get distracted or one may come to an sub optimal solution if he just blindly follow greedy approach. For example, let us say again there is another problem that I am taking for your understanding.

This is stage 1. Again we will divide it stage 2 and then there is stage 3. Now from stage 1 to stage 2 in stage 1 you have how many states? There is only 1 state which is A. At stage 2 you have 3 different states which is E, G and B. And at stage 3 you have 3 different states which is F, H and C. And moving from stage 3 to the final destination you have 3 different policies as well.

So, anyway now let us say from...

moving from state A to state... stage 1 to stage 2, now you have 3 different policies.

Let us... let me denote this as P1, P2, P3. From simple common sense you will immediately say that sir moving from state A to state E would be the best option. Because that cost only 1 dollar or 1 rupee. Then if you follow that path moving forward then it will take you to a very.

.. you know suboptimal solution in the sense you will get into the destination at which is point D via A, E, F and D and this comes out to be 23 dollars. Now approach 2, but let us say you take another path another policy path where you move, instead of going from A to E you choose to go from A to B. Though it looks like A to B is the most uneconomical path from A to B there, but from B to D it is very cheap. It takes just 4; 4 units. So, totally it comes out to be 9 rupees or 9 dollars.

So that is what you cannot immediately come to a optimal solution path just by following the greedy approach. So, now let us try to understand how we can solve this problem. Now Bellman's principle of optimality suggests that an optimal policy has the property that whatever the initial state and initial decisions are the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. The solution methods are based on the calculus of variations. And the principles are introduced by presenting examples of one dimensional problems.

I will take an example to help you understand. Now consider the cost of transportation from one city to another via several routes. Now it is a very complex problem. There are so many paths available; so many nodes in between as well, stopping points, you could say. So from that the starting point is A, and the destination is F.

Now you can choose any path; the objective is to minimize the cost. The optimal sequence is called the optimal policy. Any sub sequence is called a sub policy. And the optimal policy contains only optimal sub policies.

Overall there is a optimal policy. You divide it across different stages. And the overall optimal policy includes optimal subpolicies. Route is made in sequence. So there are two various approach or various methods followed to arrive at the solution. The first method is forward recursion algorithm.

And second method is known as backward recursion algorithm. So we will take first the forward recursion algorithm to understand how this problem can be solved. So in this method one starts with finding the optimal path for the first stage and moves forward. At every stage I will find out which is the best possible optimal path. And then that means I am finding optimal subpolicy at every stage.

And ultimately if you put together you get the overall optimal policy. So let us say starting at.

.. the minimum cost path to F is ABCDEF. And starting at B the least cost path to F is BCDEF. And similarly starting at C the least cost path could be CDEF to reach F. And starting at D the least cost path to F is DEF. And starting at E the least cost path to F is EF.

So overall is ABCDEF. And that comes out to be 1 plus 7 plus 2 plus 2 plus 3 which is 15. Now we will try to understand mathematically. And then by step by step how we have come to this solution. Now the stages are first divide up the field of paths into different stages $I$, $II$, $III$, $IV$, $V$. And at the terminals at each stage, at the end of each stage, there is a set of nodes, or stops.

And at each... at stage $III$ the stops are for example, at stage $III$ in the given problem, what are the stops available for you at stage $III$? K, D, L, M This is at the end of stage $III$. You have 4 different states K, D, L, M. And a set of cost can be found for crossing a stage. Let us say $X_2$ is a state, $X_3$ is another state. Now the cost associated while you travel from state $X_2$ to state $X_3$ is called as $V_{III}\left(X_2, X_3\right)$.

And a cost is dependent on the starting and terminating nodes of a stage. That means the cost associated to move from point C or state C to state K is fixed. That is 4. If you see the previous picture the cost associated to move from C to K is 4.

Similarly, from I to D it is 14. Now, the minimum cost for traversing from stage $I$ to stage i; that is, $I$ to small i, and arrive at some particular node or stop $X_i$ is defined as $f_I$ $(X_i)$. The minimum cost, this is the minimum cost, this function is $f_I$ $(X_i)$. Now the minimum cost from stage 1 to stage 2 for nodes G, B, H, are.

.. Now can you say stage $I$ to stage $II$ what are the nodes here? G, B, H. So, the only way is starting from point A. There are no multiple states available here. There is only one state, that is A.

So, minimum cost to arrive at G starting from A is 6. You can also take another way. If you are interested to find the most uneconomical path, you can reach at G going from A to B, from B to C, and then from C to G. There is another way also. How much it cost 1 plus 7 plus 13; 21. Or you can choose another path moving from A to H, H to J, J to B then B to C and then C to G.

So many ways possible. But which is the most economical one if I am interested to move at point G... to reach at point G starting from point A? It is 6.

that is the least pricey path. Now, similarly to reach point B from A then that corresponding minimum cost is 1. Similarly to reach point H starting from A the minimum path is 5.

Now the minimum cost from stage *I* to stage *III*... now... till now we are speaking from stage *I* to stage *II*. That is from point.

.. starting from point A to either G or B or H. Now, moving forward, it is not our ultimate destination. You need to move to the stage *III*. Now stage *III* has different nodes. So what are the nodes available in stage *III* or the states available in stage *III*? You have C, I, J. Now let's say at every stage I am finding the most minimal cost associated to arrive at that particular state.

Now let me come to a conclusion that I am identifying a path to reach to point C or state C, starting from point A. And I am finding all the possible paths and among which I am finding that one possible path which is the most economical. Like that, I am doing for all different states in the corresponding stage. So let's say point or state C; what is the total cost, the minimal cost to reach point C? It is the minimal cost till the previous stage, that is stage *II*, the most minimal cost associated to reach till the second stage and the traveling cost from that point to point C, traveling from stage *II* to stage *III*. That means, the minimal cost from stage *I* to stage *II* and the cost associated from stage *I* to stage *III* among which I am finding the most minimal one.

Now, how it will be find out? Let's say from A to G what is the cost? It is 6. Starting point is A and to reach G you took 6 rupees. From G to C it is fixed. G to C it is 13. Similarly, because this is the most minimal cost to reach A to G, that I have explained, now that is the most minimal cost, that is 6.

From G to C the most minimal is 13. Similarly to reach at point B the most minimal is 1 rupee and from B to C it is 7. So 1 plus 7. And to reach point H from point A it took 5 rupees. And from H to C it is infinite because there is no direct path available.

You got it. So, among these 3 possible paths to reach point C, whether from A to G, from G to C or A to B and from B to C or A to H and H to C, which is the most minimal one? It is this one, that is via A, B and C. Similarly you find all the cost associated at different nodes moving forward. For example, I have given the table here. At each stage the minimum cost should be recorded for all the terminus nodes. At each stage whatever the states are, there you have to find out the minimum cost associated with that.

And use the minimum cost of the terminus of the previous stage whatever the minimum cost was there at the previous stage you take that, and identify the minimum cost path for each of the terminating nodes of the current stage. Now you see here, from A to G it is 6, A to B it is 1 and A to H is 5 and next stage.

.. this was stage number 1; at stage *II* you need ... the state is C. To reach state C the minimum path is A to B. And to arrive at state *I* the minimum path is A to B again. And to reach a state J the minimum path is A to H. So to reach at point C, I, J the overall cost associated with the passing through path AB and AB and AH is 8, 6 and 11 respectively. Similarly to point to stage 3; Now you see here, which is the most economical one till you reach the point the previous stage is It is AB, right? AB is the most economical one.

Now AB is 8 rupees, and you are interested to reach K via C. Then it costs around 12 rupees. And to reach D ... you see here, you need to reach D, and you have chosen that the path is ABC is the most minimal one.

From point C it takes 1 + 7 + 2 which comes out to be 10. And to reach point L you need to take the path AB and I. And that comes out to be 12. Similarly to reach point M, you take the path AHJ and it comes out to be 16. Among all these paths which is the most economical one? It could be ABCD, right? It is ABCD.

And moving to the next stage, you have 2 states E and N. You have already identified ABCD is the best one. Now following through ABCD path you need to reach point E or point N. Now 2 is the most economical path compared to 7. So you are freezing ABCDE as the best path.

And ultimately A B C D E if you have reached then the next step is just F. So A B C D E F is the path that you follow. Now there is another approach which is backward recursion algorithm. In this method one starts with finding the optimal path for the last stage. It is going starting from the backward, the last point.

It is like starting from the destination you will reach the source starting point. So in this method one starts with finding the optimal path for the last stage and moves backwards. The information regarding the best policy in the remaining stages is conveyed backwards, which saves the computational effort of finding the optimal path for the remaining stages.

So it will have... you know... it is most followed approach because it will reduce the computational effort. Again you divide it into different stages and there are different states; the philosophy remains the same. Now let Xn be the immediate destination at stage n. At stage n you identify which is the immediate, most immediate destination.

and therefore the route selected is going from A to J. Let's say you are interested to reach from A to J. And the routes selected in going from A to J is given by A to $X_1$ and $X_1$ to $X_2$ and $X_2$ to $X_3$ then $X_3$ to $X_4$, where $X_4$ is your destination which is J. Let

$f_n(s, x_n)$ be the total cost for the overall policy for the remaining stages; not only for the particular stage. Whatever may be the overall cost associated to reach the ultimate destination that you call it as $f_n(s, x_n)$, where S is the current state and Xn is the next, the ultimate destination.

And then stage $n$ is about to be started and $X_n$ is the immediate destination. $X_n$ is considered to be the immediate destination; $S$ is the current state. Let $x^*$ minimizes

$f_n(s, x_n)$. That means to move from current state to the next immediate destination let us say there is a particular point which will give the most economical path. And the corresponding minimum value is designated by $f_n$ $(S, x^*)$. Now the total cost for the overall policy for the remaining stages can be decomposed as, The overall cost

$f_n(s, x_n)$ = immediate (transition) cost at stage n + minimum future cost (stage n + 1 onwards) So starting from let's say one point to stage *I* to stage *IV* and from stage *IV* to stage *V*, now you see what is the minimum cost associated from stage *III* to stage *IV*.

And from whatever cost minimum, that gives to move from stage *IV* to stage *V*, the ultimate stage. So, $f_n(s, x_n) = c_{sxn} + f_{n+1}^*(x_n)$

this is the expression $f_n(s, x_n)$ is equal to $c_{sxn}$ which is the cost of moving from state s to $x_n$ , and the total cost, the most minimal cost associated moving from state $x_n$ to the ultimate destination. So I will take you an example to help you understand in a better way. Consider the case for n=4, where the traveler has one more stage to go that is X4=J, this is the final destination.

S can be either H or I. now I am in this point. This is my current stage and how many states do I have? H or I. Now the policies are described as below. Now the overall cost associated to move from state H to state J is just 3. and that is the most minimal cost because you do not have any other way to go and you are freezing the point which will take you to the minimal path that is J. And then starting from I the only cost is 4 and the destination point is J And that is the most minimal point because this is the first starting point of calculation.

Then go backward. Now consider for N=3. When the traveler has two more states to go following table is valid. Now you are at this stage. There are two more steps now from here to here and from here to here.

There are two more states to go. Now let us formulate the table. Now what are the current states? E, F and G. At this stage. And what is the cost associated moving from state S to state X3, that means to the next state? What are the next states associated with the next stage? H and I. Now moving forward you are interested to find out the total cost associated moving from stage E to stage H. And that includes the transition cost from stage E to stage H, and whatever the minimal cost associated to move from stage H to the ultimate destination; that means from E to H the cost associated, with respect to H if you are starting from E.

Now what is the total cost? That is E to H. It is 1. And from H onwards, which is the most minimal one? It is 3. So, the total cost is 1+3=4. And this will make a person sitting at point E to reach to the point J.

The overall cost included is 4. Let us say you are interested to move to stage I starting from point E. Then what is the cost? E to I it is 4. And from I to J the most minimal is 4. So, this is from this point onwards the most minimal cost right.

So, that is 8. Similarly starting from point F how much is the cost? From F to H it is 6. From H to J it is 3. Total is 9. Similarly you do it for F to I, G to H, G to I. Right? And among this which is the most of most minimal one? Now starting from point E you have two possibilities. Either you go to H or you go to I.

And what is the cost, most minimal cost? It says going from E to H is the most minimal one. And moving forward to reach the ultimate destination also. Right? And what is that most optimal point at that particular stage? It is H. If you are starting from E then you can blindly understand that you need not have to go to I.

The best way to go to J is via through H. Right? And similarly from F to H it is 9 and F to I it is 7. So, the best way is to go to J via I, which gives 7. Similarly from G to H or G to I which one is the best? It is G to H, and you can freeze H. Right? Now going backward, now n=2. Right? Now what are the different nodes at the previou s stage? B, C, D.

The next immediate stage is having different states, which is E, F, G. Now just follow the similar approach. From B to E it costs 7. From E onwards we have already chosen, we have already identified which is the most economical path. Now you need not have to calculate whether should I go from E to I and I to J. That is not required because in the previous stage itself we have found out H is the most optimal solution.

Now what is that cost associated for moving forward? It is 4. Right? Then you will get 7+4, which is 11. And B to F; This is anyway there, 4. Plus if you come to point F then which is the most minimal approach? You need not have to go to H.

You should go to I and then you can go to J. This is what we have identified in the previous step. I is the best. if you come at point F that particular stage of evaluation is telling you that you should not go to H at any way. You should go to point I and similarly from B to G it is 6.

And from G onwards you should go to H. Right? And that will give you 12. Correct? 6+3+3, this is 12. Now among all these, whether to go from B to E or B to F or B to G, this will tell either you can go from B to E or you can go from B to F. Because B to E gives you 11, B to F gives you 11 rupees, B to G will ask you to pay 12 rupees. So, better to go to E or F. Similarly you do it for C and D for each state; that means from C to E, C to F, C to G, and similarly from D to E, D to F and D to G. Now you have got a most optimal or most minimal cost at this stage, which is, starting from B it is 11, starting from C it is 7, starting from D it is 8.

And which is the frozen point? It is E or F if you are starting from B, or E if you are starting from C, and either E or F starting from D. Now there is the last step. Now you are starting from A and what are the next states? It is B, C, D.

Now A to B it is 2. From B onwards what is the most minimal cost path? E or F. But the cost is same, which is 11. Now, from A to B it is 2 plus 11; which is 13. Similarly, from A to C, this cost 4 rupees. From C onwards the most economical is E; then the cost is 7.

So, it comes out to be 4 plus 7, which is 11. Similarly from A to D it is 3. From D onwards how much is the cost? The most minimal cost is 8. So, 3 plus 8 is 11. Among all these paths, B, C or D which is the most minimal one? It is either C or D because it is coming out to be 13 here.

So, you either freeze C or D. Now you have got all the possible optimal states at every stage. Now you obtain the overall policy. You got the optimal subpolicies. You find out the overall optimal policy. Starting from A it is not advised to go to B; it is advised to go to C.

Now you have chosen to move to C. You can choose to move to C or D, not B. If you choose to go to C then it says if you go to C, you have to choose E. You should not choose F or G. If you choose C; that you have to go to the previous state to understand.

If you go to C then the only option for you is E. Then if you go to E then you have to go to H not I. That means, if you choose A to C then go to E. If you go to E then choose H.

If you go to H then choose J.

this is one optimal route. The second optimal route is if you choose instead of going to C you choose to go to D. If you go to D then either you can choose E or F. If you choose to go to E then if once you reach E now the destination is fixed. You go from E to J via H. Now if you choose to go to D again but you are not interested to go to E rather you are interested to go to F still that is fine.

But you should not choose to go to G. If you go to F then the best way would be not to go to H. You go to I or J. Any of this path, 1, 2, 3, any of this path would give you the most optimal solution in terms of minimizing the overall cost. Now what are the different applications of dynamic programming? The economic dispatch of thermal power plants which I have already told, the solution of hydro thermal economics scheduling problems because if you have thermal power plant and hydro, I told it is more complicated they have their own set of constraints, having hydro and thermal how do you optimize by using dynamic programming? And then for some practical solutions you read the reference book Wood and Wollenberg. And that ends today's session. Thank you very much.