Economic Operation and Control of Power System

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Week - 01

Lecture - 05

Welcome you all to the NPTEL course on Economic Operation and Control of Power System. And today, we will focus on the second part of optimization preliminaries, where we will focus more or less on linear programming approaches. During the previous class, we have seen how different functions can be optimized. They could be single variable, they could be multi-variable, they could be constraint or unconstraint, and the function may be linear or it could be non-linear. And Lagrangian multiplier optimization technique, we have understood and how this can be extended to power system engineering. And today we will focus on a linear optimization, because there are a bit of challenges that if the function, objective function is linear, and then probably when you differentiate it, then the differentiation become a constant.

All right, so there is no opportunity for you to go for further more differentiation, because the function is linear. And optimizing linear functions become a challenge. So we'll understand how linear functions can be optimized with equality as well as inequality constraint and its process. Now first of all, when I talk about linear programming problem, we will try to understand every simple example.

If I'm interested to maximize the production of two different plants, or I have to minimize the cost of two power plants, so then you know, we need to express those in the form of equations. So probably let's say I have to maximize the production cost of two different plants:

Maximize: $3x_1 + 5x_2$

Plant maximum output constraints:

Maximum Production of plant 1, $b_1 = 4$

Maximum Production of plant 2, $b_2 = 6$

Plant must meet additional inequality constraints: $3x_1 + 2x_2 \leq 18$

Mathematical statement of the LP given by:

Maximixe: $3x_1 + 5x_2$
Subject to:

$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$3x_1 + 2x_2 \leq 18$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Now moving to the next case, so what exactly happens when you draw these different points or different constraints? Now first of all, this is my x_2, which is x_2 greater than or equal to 0. And this is a small typo x_1 greater than or equal to 0.

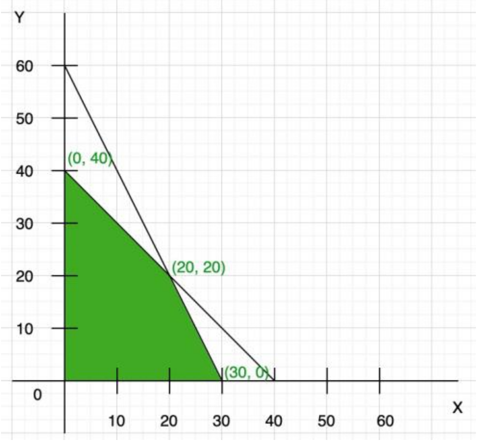
Okay, these are the two lines. And then you have one more constraint, which is x1 less than or equal to 4. And then we can have x2 less than or equal to 6. So you come across this line, and you come across this line, this line, and finally the equation which is 3x1 plus 2x2, which is a linear equation, less than or equal to 18 will give me, all right. So from this, I am getting now the feasible region.

I'm getting different points, 1, 2, 3, 4, 5 points, and the area which is now feasible to me. So I have to find a point within that. Now we can also solve a few more examples, like maximize another function, which is z equal to 8x plus y, x is a variable, y is a variable, similar to x1 and x2. And you can have multiple constraints. And when you have multiple constraints, like x plus y less than or equal to 40, 2x plus y less than or equal to 60 with different constraint, x is positive, y is positive.

You have to draw different characteristics similar to the previous diagram as I have shown. So you can get the feasible region within which the optimization value now being carried out. Now what is the approach of doing it? Now there are two different methods. One is real procedural steps for linear programming. And someone also can solve graphically and get an idea actually what the optimized value are.

So if you look into this to maximize the value z:

Maximixe: $Z = 8x + y$
Subject to:
$x + y \leq 40$,
$2x + y \leq 60$,
$x \geq 0$,
$y \geq 0$



Points	**Z = 8x + y**
(0, 0)	0
(0, 40)	40
(20, 20)	180
(30, 0)	240

So when I plot those points, when it is 0 and 0, I get this point. Then 0 and 40, I get this point. And then I get this point, 30 and 0, I get this point. And this is my 30 and 0, this is my 0 and 40, and this is my 0 and 0.

And then finally I get another point which is 20 and 20. And from the characteristic if you see, I mean, the feasible reason is going to be this one, right, which is satisfying all the points. So once you know the feasible reason, then you can get the maximum value of the function easily from the characteristics.

Now when I talk about a linear program, what is it? It is an objective function with multiple constraints. And the final objective function need to be minimized or maximized.

So we do have different variables.
It talks about minimize the function which is $c^t x$.
Subject to:

$$Ax \leq b$$
$$x \geq 0$$
$$x \epsilon R^n$$

That is how we have seen all the formulas of the previous linear optimization problems.

Now when I talk about the LP objective functions, this exercise as you could see, we have seen that, you know, there are different points here. So this is objective functions which has been plotted with my different variables and constraints. So this is known as actually contours of linear functions, am I right? And you can say the objective functions at different point. And finally, I can get a point either this or this or this or this or this will give me one of the reasonable optimal solution, right, the best one. So now this characteristic, as you could see, if I put this characteristic up, okay, then probably the points can move further actually and which can keep on increasing.

So what I wanted to highlight here, if the objective function is known to me, if the constraints are known to me, even though graphical method, the best value or best solution can be identified within a feasible region satisfying all the points. So indirectly, I do have millions of points within this characteristic, okay, but because we also know the optimal points will lie on the boundaries and hence by checking the boundary points, one can easily see which is my optimal solution or the value of x1 and x2 that leads to the maximum or minimum of the objective functions. So this is one way, but for complicated problems with millions of variables or hundreds of variables or tens of variables, this method is not going to work. This is true for two variables which can be easily done. But if there are five variables, ten variables, how do you solve it? So for that, we need to get into canonical forms with slack variables and where different objective functions with different constraints can be lined up.

So you could see now the variables are not one or two, but there are actually x1, x2, x3, x4, x5. Everything can be placed in a vector form and that need to be solved. So I request everyone to get into the linear optimization textbooks to understand more detail about it, but we are just touching upon to make sure when you get into an objective function which is linear that need to be optimized and you are not able to solve by differentiating easily because if I take this function, a linear function f of x, which is x plus 3, all right, so what is my f dash x? Is 1. So I am gone. So I cannot really get into what is f double dash x equal to 0.

So by differential methods, obtaining the optimal value of linear functions become a challenge and hence we have to go for linear programming approach to handle and avoid the challenges that are being faced through differentiation. Okay, so either you can put graphical methods or we can present them in a canonical form with slack variables to solve this issue and these are all classical structures and the way it can be presented. Now there are basic feasible solutions. So either it could be Ax equal to b is one of my solution. I can say x equal to b as well as x greater than equal to 0 is one of my solution.

I can also say x equal to b and x greater than 0 and x equal to 0, both equality and inequality. Okay, first one is without any constraint. Second one is an inequality constraint or is it equality as well as inequality constraint. So different type of problem formulations can be handled in linear programming optimization techniques. Now what we can do for a complicated problem like, you know, your a value is known to me from this diagram itself, from this representation of canonical form with slack variables.

I can arrive to the point what is a, what is b and then I can have different set of equation for x1, x2, x3 and then and x1, x2, x4 and x1, x2, x4, different variables.

I can determine:

Minimize z= $c^T\ x$

Subject to: A x =b

$$x \geq 0$$

We partition the A matrix and the x and c vectors into basic and non basic parts:

Minimize z= $c_B^T\ x_B +\ c_N^T\ x_N$

Subject to: B x_B + N x_N = b

$$x_B \geq 0$$
$$x_N \geq 0$$

So all those variables x can be stacked into x1, x2, x3, x4, x5 with an coefficient which has been represented by matrix a and the right-hand side 4, 6, 18 they can be of my b. So I can rewrite that equation which is ax equal to b and once I know then I can have different feasible solutions from those. There are n number of set, the set one solution, set two solution, set three solution and four, five, six, seven will get enormous different variety of solutions. Sometime it is possible, sometime it is singular, sometime it is non-singular and so and so. But once you get the values, you know, for example, first case you got x4 equal to 0, x5 equal to 0, second x3 equal to 0, x5 equal to 0, third x3 equal to 0, x4 equal to 0 and then you arrive into different solutions from time to time.

But out of those you can also see to that the because of the values the variables when x1 equal to 0, x2 equal to 0, x3 equal to 0, x4 equal to 0, the different solutions will behave in a different way. That means it is not guaranteed that all the time in all steps you get the solution from time to time. So what we do basically in a simplest method we represent the minimization of the function which is not simply x1 plus 2x2 plus x3 but I can represent that in the form of c transpose of x. Instead of saying x1, x2, x3 I can represent c transpose of x with a constraint, which is axs equal to b as well as x greater than or equal to 0. And then probably if there are many variables, so then I can write minimize the function, the first function, second function you can keep on adding and subject to different constraints that we can solve.

So let us imagine that I have n number of power plant and the cost characteristic of each power plant is linear. I have to optimize the total cost of the system. So then I have to add many linear functions that need to be optimized with constraints such as p max, p min, etc. Now the simplex method which is given as:

$$x_B = B^{-1}\ b - \ B^{-1}\text{N}\ x_N$$

$$\text{z}=c_B^{\tau}\ B^{-1}\text{b} - c_B^{\tau}B^{-1}\text{N}x_N + c_N^{\tau}x_N$$

$\tilde{b} \equiv B^{-1}\text{b}$
$\lambda^{\tau} \equiv c_B^{\tau}B^{-1}$
$\text{Y} \equiv B^{-1}\text{N}$
Where λ is a vector called the "Dual" variable vector

This leads to a reformation of our standard form of LP:

Minimize z= $c_B^{\tau}\ \tilde{b}$-[λ^{τ}N- c_N^{τ}] x_N
Subject to: $x_B = \tilde{b}$ - Y x_N,
$x_B \geq 0$,
$x_N \geq 0$

We will now define $d^{\tau} \equiv \lambda^{\tau}\text{N}- c_N^{\tau}$ as the reduced cost vector, and allows us to write the new standard form as:

Minimize z = $c_B^{\tau}\ \tilde{b}$ - $d^{\tau}\ x_N$
Subject to: $x_B = \tilde{b}$ - Y x_N,
$x_B \geq 0,\ x_N \geq 0$

So the problem here where the lambda is a vector which is ideally the dual variable vector and that also need to be determined.

So if you continue further you could see that the solution can be obtained in a long run and we can define different variables in those linear expressions. Now from one of the important variable in case of linear optimization is pivoting and probably you can add pivoting variables in a particular row to handle such challenges. So simplex mechanism it talks about different conditions like whether the variables are positive or variables are negative and you can understand the different steps actually to solve this linear programming. I am not really getting into detail but I am just touching upon the exterior to make you understand what linear programming is all about due to sort of time and this is what the simplest code that people used to follow. So first of all you get a basic feasible solutions and find out if the current solution is a maximizer and if all djs are less than zero then you stop otherwise you continue to the next step and find out which non-basic variable enters the basics of x and j most positive one is dj and find out which variable x_{Bi} leaves the basis that is minimum of Bi upon Yij, Yij greater than equal to zero and then develop the new basic and get a new basic feasible solution and then repeat go to step number two and keep on repeating till you will end up with a final solution.

So this is how it works actually graphically so you have a initial point and then you move to a first jump all right and then you move to a second jump and then the last jump and then finally you get a solution that you have proved graphically now this can happen your mathematical way also. There are different linear programming features and so to what I wanted to highlight here all are requested to get into the lp tool of MATLAB where any linear problem optimization problem can be solved and you can also use different textbooks to understand more about linear program and this is not necessarily part of this program but at least we believe that the power engineers before they get into different concept of optimization this must carry the basics of linear programming as well as non-linear programming with equality and inequality constraints. So with this note thank you very much. .