# Economic Operation and Control of Power System

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**Week - 10**

**Lecture – 48**

Hello and good morning everyone. Welcome you all for the NPTEL online course on Economic Operation Control of Power Systems. So in today's class we will discuss our next continuation part of state estimation. Mainly we will discuss about the PMU placement in this class today. So, PMU placement by using integer programming. So the elements of the binary connectivity matrix A for a power system used in the formulation of the optimization problem are defined as A of I, J.

$$\boldsymbol{A}(i,j) = \begin{cases} 1 & if \;\; i = j \\ 1 & if \;\; bus\ i\ and\ j\ are\ connected \\ 0 & otherwise \end{cases}$$

Let us say there are two bus I and J. If this there is a connectivity of line present between bus I and J then it is 1 if buses are connected, if bus are not connected then it is 0. And we will define a binary vector X which is defined as X of I that means this is equal to 1 if a PMU is placed at bus I that means by just looking at this vector binary vector and the entries of this binary vector.

The binary vector $\boldsymbol{x} \in \boldsymbol{\Re^n}$ is defined as,

$$\boldsymbol{x_i} = \begin{cases} 1 & if\ a\ PMU\ is\ placed\ at\ bus\ i \\ 0 & otherwise \end{cases}$$

So we will get to know whether a PMU is placed at a specific bus or PMU is not there.

So the entries of the product Ax therefore represent the number of times a bus is observed by the PMU placement set defined by X. So if you multiply A with X then you will get to know this product matrix Ax defines that how many times a specific bus is being monitored by a PMU. So the objective function, let us formulate the optimization problem now because it is a genuine optimization problem we need to place the PMU where to place PMU that is a question. It is not that we have sufficient money available so that we can place PMU across all the corner of a distribution system it is very expensive it is not possible. So what are the bare minimum locations such that you can place the PMU make the system observable yet the system is very cost effective.

So the objective function V of X for optimization is formulated as in an integer quadratic programming problem and this is the objective function.

$$V(x) = \lambda(N - Ax)^T R(N - Ax) + x^T Qx$$

Where, $\lambda \in \Re$ is a weight, and $\boldsymbol{N} \in \Re^n$ is a vector representing the upper limits of the number of times each bus can be observed

Now let us define part by part lambda is a weight n is a vector representing the upper limits of the number of times each bus can be observed. Now it is not that you know you can make the specific bus observable 100 times or 1000 times it is not possible. So there is a minimum number and there is a maximum number of observability. So the diagonal matrix R has entries Ri, i representing the significance of each bus i, it is very important point.

Not all the buses are very important buses, not all the buses are very significant bus. So there is a weightage being assigned to different buses to indicate that which bus should be given a more priority for a PMU placement. Such that you know if you place a PMU at let us say bus X which may be more prominent bus then you can have more observable compared placing a same period bus number Y. So that is the whole agenda. And then there is a diagonal matrix Q has entries Qii allowing for the representation of varying installation cost of the PMUs of different buses.

So diagonal matrix Q indicates the cost associated with the PMU and diagonal matrix R indicates the weightage of this buses you know. So it could be the weightage could vary from either 0 to 1, this can vary from either 0 to 1 or you can simply mention let us say a specific bus at which if you place the PMU it can help to administer let us say 10 bus. So the number of entry would be 10. The other bus if you place a PMU by placing a PMU at that another bus so you can just administer let us say 5 bus. So the number of entry would be digit 5 there.

It is decided like that. In the generic case as assumed in this study where all the buses are equally significant and the PMU installation cost at all buses is the same Q and R are equal to the identity matrix. So if in a generic case let us say all the buses are equally important and significant and the PMU installation cost are also same then it is a identity matrix. So you will not be able to differentiate or discriminate between any bus in the entry. So the objective function $V(x)$ can be written in expanded form as:

$$V(x) = \lambda N^T RN - 2\lambda N^T RAx + \lambda x^T A^T RAx + x^T Qx$$

$$= \frac{1}{2} x^T (2\lambda A^T RA + 2Q)x + (-2\lambda N^T RA)x + \lambda N^T RN$$

If you just expand the previously defined form so you will get like this and further deduction will yield to expression like this. The optimization problem can therefore be formulated in an integer quadratic programming framework so the objective function is:

$$\text{Minimize} \quad \frac{1}{2} x^T G x + f^T x$$

$$\text{subject to} \quad \mathrm{A}x \geq b$$

Where, $G = (2\lambda A^T RA + 2Q)$, $f = (-2\lambda N^T RA)^T$ and $b = I^{n\times 1}$

So what is the minimization? The minimization here is the objective is to reduce the cost subject to a constraint there will be observability minimum observability. So let us consider some specific case scenarios. Okay you have placed a PMU and it is making system observable and cost is also less but that is not sufficient enough.

You should also take into consideration some unique situations where let us say if there is a loss of a single transmission line there are multiple case studies that we would discuss the first such scenario could be a loss of a long single transmission line. So the PMU placement methodology ensures that the system remains observable even in the case of outage of any single transmission line. So if there is transmission line outage so we had discussed in the security constraint OPF so there is generation outage, line outage and all these things that we have studied. So let us say if there is a line outage so in such kind of a scenario whether the PMU placement helps to administer the buses even in such kind of scenarios that is the question. If the transmission line between buses I and J goes out let us say and elements A I J and A J I in the original A matrix are set to 0 obviously now the entry will be 0.

So denoting the Ith and Jth rows of the modified connectivity matrix A I and A J respectively it is necessary that the following constraint is satisfied to maintain observability under the outage of the line between buses I and J that means now A I A J now the entry would be 0 that specific entry where the line is gone that entry would be 0. So the row corresponding to:

$$\begin{bmatrix} a_i \\ a_j \end{bmatrix} x \geq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

This indicates that there will be at least one measurement being measured I mean one measurement being visualized at any given point of time even in such kind of outage. So let the set of row vectors A that need to be modified due to the branch outages one at a time be denoted by A 1 so the set of additional constraints can then be written as:

$$A_1 x \geq b_1$$

Where, $b_1 = I^{2N_b \times 1}$; $N_b$ is the number of line outages considered

It is identity matrix basically so size of the identity matrix so identity matrix means the entry is 1 so in that case what we are saying is even in such kind of outage where with the new modified matrix A 1 that in that matrix also we could see that you know at least some minimum observability is there. Then let us move on to another case scenario where if there is a loss of a single PMU there could be loss of a transmission and it is quite obvious if there is a fault and the line get outed or if it could be a maintenance issue some reconfiguration of a system due to any reason let us say the line is gone but you know it would ensure that the system have minimum observability.

What if the case if the PMU that we had considered the location of the PMU itself is as gone wrong due to some wall functionality due to some damage something is there. So yet can you make the system observable is a question. So the proposed PMU placement method is designed to maintain complete observability in the case of the outage of any single PMU. So in general a bus is observed by only one PMU by using a direct or a pseudo measurement. A minimum redundancy level of one therefore ensures complete system observability for a single PMU outage.

The constraint to ensure observability under single PMU outage is now this is a second constraint.

$$Ax \geq b_2$$

Where, $b_2 = 2 * I^{n \times 1}$

So here 2 indicates that if there is a PMU that is being lost so at least 2 measurements because it is connected between 2 lines so at least 2 measurement is gone.

So in that that indicates the significance of this number. So this is a constraint that means at least some minimum observability would be again there even if the PMU is gone. So let's formulate the example for the IEEE 14 bus system. So bus standard system that we have considered just for our analysis purpose. So the connectivity matrix is given by this.

It is very simple. Let's take one example and try to understand. So let's say so this is 14 bus system. So 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14. So 14 rows are there and if you observe here so there will be 14 entries again here 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14. So there will be 14 entries 14 by 14.

So 11 obviously I mean if you speak about this is the first entry first bus. So 11 here the first entry that is corresponding to bus 1 obviously the entry would be 1 and if is there any line being connected between bus 1 and 2 it says yes the line is there so the entry would be 1 and bus 1 and 3 so there is no connectivity between bus 1 and bus 3 so the entry would be 0. Bus 1 and 4 there is no direct connectivity again it is 0 whereas bus 1 and 5 yes there is a entry and any other bus the entry would be 0. So on this note we will complete this connectivity matrix A for all the 14 buses and then let's consider a line outage example. If the transmission line between the buses 2 and 3 is removed from the service A32 and A23 will be 0 obviously.

So A32 entry and A23 entry would be 0. So the additional set of constraint is now so take 2 and 3 again redo the analysis. If you see 2 and 3 now this bus this line is being disconnected now this line does not exist. So what will happen now? If you make a new entry for this 2 and 3 so 2 and 3 consists of between 2 and 1 if there is a connectivity yes 1 between 2 and 2 obviously there will be another 1 2 and 3 now it will be 0 earlier it was 1 between 2 and 4 yes there is a connectivity again 1 between 2 and 5 there is a connectivity yes 1 2 and 6 and any other bus it will be 0 0 0 0 0. So I was speaking about the case scenario for the 14 bus system so we will calculate how it is being done.

So we are discussing about bus 2 and 3 let's say this line is gone this line is being taken out the entry would be would go like this. So between bus 2 and 1 there is a connectivity 1 speaking about 2 the bus 2. So between bus 2 and 1 there is a connectivity 1 and between bus 2 and 2 again there is a connectivity 1. Between bus 2 and 3 now the entry would be 0. Bus 2 and 4 yes there is a connectivity 1.

Bus 2 and 5 there is a connectivity 1. And rest of the entries would be 0. Similarly you take bus number 3. So bus number 3 if you speak bus 3 and 1 is there any connectivity? No bus 3 and 2 earlier it was there yes now it is not there so it is 0 again. Bus 3 to 3 of course it would be there.

Bus 3 and 4 again it would be there and rest would be 0 again. So the new entry with the constraint being formulated so it goes like this.

The additional set of constraints is,

$$\begin{bmatrix} 11011000000000 \\ 00110000000000 \end{bmatrix} x \geq \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

This indicates at least one measurement is seen should be seen even in such kind of scenario where you have considered a line being taken out which is connected earlier between bus 2 and bus 3. So then this particular objective optimization problem is being solved to achieve bare minimum operation that means optimal PMU locations is been identified as :

| System configuration | Optimal PMU locations |
|---|---|
| Normal operating conditions | 2, 6, 7, 9 |
| Considering single branch or PMU outages | 2, 4, 5, 6, 7, 8, 9, 11, 13 |

So 2, 4, 5, 6, 7, 8, 9, 11, 13 you understand. So if you have to increase the luxury of a measurement even in such kind of emergency scenarios you have to bare extra price for it nothing comes for free. So if you have to consider there should be at least minimum measurement even during single branch outage or PMU outages then you have to increase the PMU location the placement of PMU location to multiple cases, multiple buses. Now in this case earlier it was just 4 PMUs now it would be increased to 2, 1, 2, 3, 4, 5, 6, 7, 8, 9 it increases to 9 PMUs. So minimum number of PMUs for observability under normal operating conditions in the absence of conventional measurements you can see here IEEE 30 bus again you know we had considered 14 bus system similarly if you extend this to other standard transmission systems standard IEEE systems like 30 bus, 57 and 118.

So you will get to know that during normal condition minimum would be 10, 17 and 32 whereas if you consider the minimum number of PMUs for observability under the outage of a single transmission line or a single PMU then in the absence of conventional measurements then the number of PMUs would increase to 21, 33 and 68 it is almost doubled, it is almost doubled. So similarly optimal locations of PMUs for the IEEE 30 bus test system if you see normal operating conditions these are the location and during considering single branch or outage in exact locations being put up for 30 bus system 57 and 118 just to give you an idea of how the consideration of line outage and PMU outage would increase the number of PMU placement and henceforth increase the number of the total investment. So inclusion of conventional measurements apart from PMUs if you see you know there is a high capital investment you know let us have a existing conventional measurements at some places and let us see how to deal with the PMU location, how to decide the PMU location when we have conventional measurements also parallely being considered. So the case studied here is the one where the power system has more than one island observable by conventional measurements, the optimal PMU placement problem in this case is to find the optimal number and locations of the PMUs to make the system observable as a single island for normal operating conditions as well as for the outage of a single transmission line or a single PMU. And observability analysis is carried out to identify the unobservable branches in the system to ensure observability under single line outages it is sufficient to make the system observable for outage of any of the unobservable branches above.

So that means if there is a conventional measurements being taken considered, now if there is a, if there is any reason that you know the system gets islanded so even in such kind of scenarios the islanded system should continue to provide bare minimum observability of the system independently. So formulation in the presence of conventional measurements, constraint for observability under normal operating conditions SK of X is greater than or equal to 1, greater than or equal to 1 means, greater than or equal to 1 means at least one observability would be there for K is equal to 1 to N island where N island is the number of observable islands in the power system.

$$s_k \boldsymbol{x} \geq 1,\ \forall_k\ k = 1,\ \dots,\ N_{island}$$

Where, $N_{island}$ is the number of observable islands in the power system

$s_k \in \mathfrak{R}^n$ is a vector representing the busses inside the $k^{th}$ island and the connected busses, and its elements are defined as follows:

$$\mathrm{s_k(i)} = \begin{cases} 1 & \text{If bus i belongs to the } \mathrm{k^{th}} \text{ observable island} \\ & \text{or, it is connected to a bus inside the island} \\ 0 & \text{Otherwise} \end{cases}$$

So loss of a single line or PM in the presence of conventional measurements, so single line outage let us consider this.

So take a branch in the branch voltage, branch outage list out of the system the following constraint is added to the optimization process to ensure that observability of the system under the outage of the branch under consideration. So,

$$s'_k \boldsymbol{x} \geq 1,\ \forall_k\ k = 1,\ \dots,\ N_{island}$$

So restore the branch to the system go to step 1 and proceed with the next branch outage until the outage of all the branches in the list one at a time are considered. So basically we are going to consider all the possible bus outages and see to it that bare minimum observability would be there. Then similar to the single line outage let us also consider single PM outage, in this case:

$$s_k \boldsymbol{x} \geq 2,\ \forall_k\ k = 1,\ \dots,\ N_{island}$$

So again IEEE 14 bus system in the presence of conventional measurements, so for observability S 1, 2, 3, 4, 5, 6, 7 into X is equal to this A into X matrix, so you will get at least one there should be at least one observability. So with this as a normal operating conditions the optimal PMU location being 6 and 9 whereas considering single branch of PMU outages the number of PMU locations would increase to 4. So IEEE 14 bus test system in the presence of conventional measurements, so this is the IEEE 14 bus system, you see that there are two PMU locations that we have considered here PMU location, there is a power injection measurement and power flow measurement. The system is assumed to have flow measurements at 70% of its lines and injection measurements at

30% of its buses, all the conventional measurements are randomly distributed in the So this PMU location is very critical, so otherwise what it may happen is let us say I will just take a case study to help you understand that, let us say I am just taking one random bus network, let us say you keep the PMUs, I will just let us say this is PMU location 1 and then there is a PMU location not here exactly, let us take another PMU location to be somewhere here, right. So what it may happen, I mean let us say that if there is a, let us consider this to be also another bus here, so there is a PMU being placed at this bus, bus number 1 and another bus here, if there is a fault, let us say if there is a fault being considered here, so this line is being taken out, in that case now the system is grouped into two, right.

Now there are two islands, let us say this is island 1, this is island 2, now what is happening earlier you had placed two PMUs here itself, though island 1 is very much observable but island 2 is no more observable, so this is a very critical situation. So what you should have done, so instead of keeping the PMU here, let us say if the PMU being placed somewhere here, somewhere, anywhere here, so even if the fault is happening somewhere in between, so both the systems, islanded system would continue to provide bare minimum observability, this is just one example I took but we should understand that the importance of the system being islanded and yet continue to support the observability. So some measurement uncertainty, sources of uncertainty are instrument transfomers, the basic measurement devices, CTs and PTs and cables connecting the instrument transfomers in the digital equipment, analog to digital converters in the associated computational algorithm, typical values of manufacturer specified maximum errors are voltage magnitude is 0.02 percentage of the reading, current magnitude is 0.03 percentage of the reading and phase angle is 0.01 degrees. Assuming the measurements to be independent of each other for measurements other than phase angles there will be only diagonal elements in the error covariance matrix R, for measurements other than the phase angle the diagonal measure elements are determined by the standard uncertainties in the measurements as Rii is equal to Ui square where Ui is the standard uncertainty in the ith measurement, so all the phase angles are referred to the phase angle measured by the PMUs at the slack bus. So hence the diagonal and off diagonal elements of the error covariance matrix corresponding to the phase angle measurements are defined as Rii is equal to:

$$R(i,i) = u_i^2 + u_0^2 \quad \& \quad R(i,j) = u_0^2$$

So the standard uncertainties in the measurement can be obtained from the specified maximum uncertainty by assuming a uniform probability distribution of the entire range of uncertainty, so

$$u_i = \frac{\Delta u_i}{\sqrt{3}}$$

So the major sources of uncertainty pertaining to the PMU measurements are the instrument transformers, cable connecting the instrument transformers to the digital equipment and the A to D converters and the associated computational logic. So the characteristics of instrument transformers near power frequency are practically flat, so the error in measurements due to cables can be related to the length of the cables as well.

The uncertainties due to the instrument transformers and the cables can usually be compensated by utilizing the external calibration facility usually present in a state of the art PMUs and only the uncertainties due to the A to D converters and the computational logic are considered in this work actually. So with this we will conclude, let us continue our discussion with the in the next class with a new topic. Thank you so much. .