

## Economic Operation and Control of Power System

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Lecture – 47

Good morning and welcome you all for the NPTEL online course on Economic Operation and Control of Power Systems. We will continue our discussion with respect to the state estimation. So as discrete Fourier transform we were discussing DFT is used when limited number of samples needed over a window and DFT of a discrete signal  $x$  of  $n$  over a window of  $n$  samples there is a window let us consider this window consists of  $n$  samples and that can be expressed as:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

Where,  $0 \leq k \leq N - 1$

Similarly, IDFT is given by –

$$Nx(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}$$

Where,  $0 \leq n \leq N - 1$

Fourier Series  $\alpha$  coefficient is linked with DFT as:  $\alpha_k = \frac{X(k)}{N}$

Now phasor estimation using discrete Fourier transform.

Harmonic term in Fourier series –

$$x^k(t) = \sqrt{a_k^2 + b_k^2} \cos(2\pi k f_0 t - \theta_k)$$

Phasor value of above term –

$$X^k = \frac{1}{\sqrt{2}} \sqrt{a_k^2 + b_k^2} e^{-j\theta_k} = \frac{1}{\sqrt{2}} (a_k - jb_k)$$

Fundamental phasor –

$$X^1 = \frac{1}{\sqrt{2}} (a_1 - jb_1)$$

From DFT –

$$\alpha_1 = \frac{X(1)}{N}, \quad a_1 = 2 * \text{real}(\alpha_1), \quad b_1 = -2 * \text{imag}(\alpha_1)$$

Put the values of  $a_1$  and  $b_1$  in  $X^1$  to get the phasor value.

Now let us move on to PMU phasor measurement unit. Most PMUs have binary output modules for transmitting binary signals such as trip signals to open a circuit breaker. You see especially when you keep this PMUs even at the distribution side where you have a distribution transformer so the transformer tap changing is also in terms of binary numbers either the number is going in a increasing order or in a decreasing order it is a binary number or the position of circuit breaker either it is 1 or 0 capacitor bank the positions all these things some of the data's would be of binary number. Henceforth we also use MINLP multi integer non-linear programming based approach of finding optimization so because the power system is non-linear and it consists of integers as well. So some vendors have PMUs integrated within protection relays or digital fault recorders also.

So if you see a typical structure of a PMU so we get current and voltage measurements by using CT and PT and there is a GPS which will help us to synchronize the sampling and synchronize the phasor measurements happening at different PMUs which are geographically located at different distances and then once this data is PMUs data has been generated that is the phasor measurements that can be communicated to different control units using SCADA and also it can be synchronized with other PMUs also. Eventually we will be able to make out some appropriate actions and control actions based on this PMU measurements. So let us go in detail so this is the overall flowchart measurement from instrument transformers as already mentioned by using CT and PT and then we will go for filters because there could be some glitches in the data or some unwanted things that need to be eliminated so anti-aliasing filter will be used and then we will go for A to D converter because anyway we have to go for sampling and then by using GPS we will lock this sampling so that every PMU data is been synchronized and then there is a digital filter because before after A to D conversion also there will be some filtering need to be done and then at the end of A to D converter then you will get the frequency estimator, frequency being estimated and then after the digital filter you will get the phasor estimation that can be also used for the further control action. So basically direct measurement of bus voltage and angle and direct measurement of frequency and

rate of change of frequency these are the outcome of a PMU. In real time we need to have the measurements of the states which are voltage and phase angles and frequency and rate of change of frequency will also help us to determine the inertia present in the system and helps in executing dynamic processing algorithm such as dynamic state estimation and measurements are sampled with minimum 24 samples per cycle rate.

Let us say if there is a signal of 20 millisecond typically 50 hertz is our frequency Indian grid frequency which constitutes to 20 millisecond so every sample is taken in the range of 20 millisecond by 24 so nearly 5 by 6 millisecond. So this is our 0.8 millisecond so this at every 0.8 millisecond you will be obtaining one data. So helps in getting snapshot information of the power system.

Ultimately we need to have snapshot image of the entire power system so that we will get to know what is the different the voltage profile at different buses and phase angle at different buses so that we can able to understand whether the voltage has violated, voltage limits have been violated or not or the phase angles have been violated or not and we can able to calculate the load flow such that the thermal capacity of the lines should not be violated as we have discussed in our security constraint OPF. We need to understand whether system is stable, healthy and be able to you know perform in a better way. PMU measurement helps us in achieving that. So relative information across all the buses can be established and helps in taking quick actions. So some mathematics so in relevant to phasor calculation so this is a signal let us say this is a magnitude and this is a semi-straddle signal with some frequency angular frequency omega and there is also a phase angle phi.

So then there is a real component and there is also imaginary component into it because there is a phase angle. The N data samples of the input x of n is the sampling is happening and the N samples from 0 to N minus 1.

$$x(t) = X_m \cos(\omega t + \varphi)$$

$$x(t) = \text{Re}\{X_m e^{j(\omega t + \varphi)}\} = \text{Re}\left[\{e^{j(\omega t)}\} X_m e^{j\varphi}\right]$$

$$x(t) \leftrightarrow X = (X_m/\sqrt{2})e^{j\varphi} = (X_m/\sqrt{2})[\cos \varphi + j \sin \varphi]$$

The N data samples of the input  $x_n: \{n = 0, 1, 2, \dots, N - 1\}$

$$x_n = X_m \cos(n\theta + \varphi), \quad \theta = 2\pi/N$$

$$X_c^{N-1} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n \cos(n\theta) = \frac{X_m}{\sqrt{2}} \cos(\varphi)$$

$$X_s^{N-1} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n \sin(n\theta) = -\frac{X_m}{\sqrt{2}} \sin(\varphi)$$

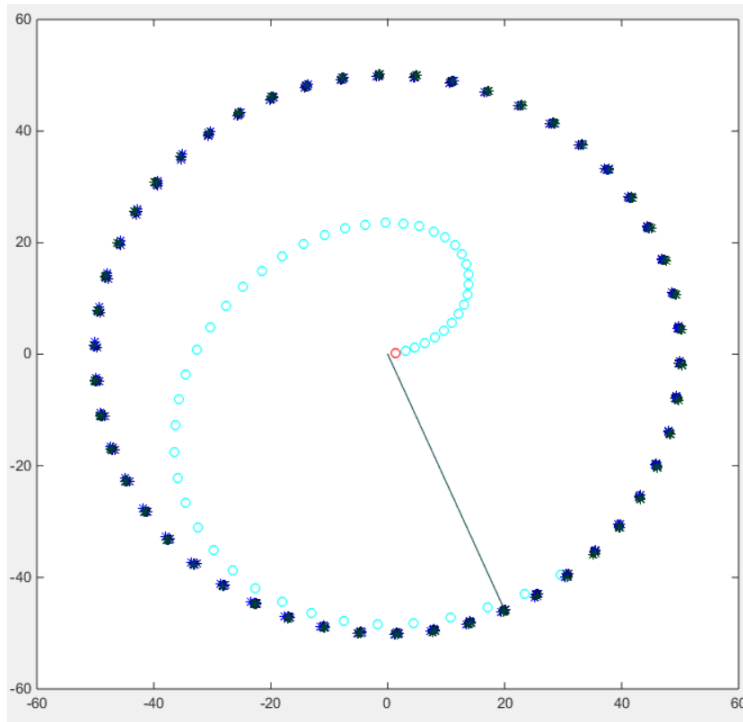
$$X^{N-1} = X_c^{N-1} - jX_s^{N-1} = \frac{X_m}{\sqrt{2}} [\cos(\varphi) + j \sin(\varphi)] = \frac{X_m}{\sqrt{2}} e^{j\varphi}$$

So basically we will be getting the magnitude as well as the phase angle. So let us take an example estimate the phasor from the given sample. So here is a set of samples that we have got this is a sample number and we have the magnitude of that particular sample. You see here sample and the magnitude and we are obtaining at a specific sampling rate where in this specific case we are considering one sample per cycle.

So that way you can see the 0th harmonic or DC component is existing in this specific sampling of the signal and there is a fundamental  $x$  of  $t$ , the magnitude is being given frequency is given. So overall cosine and sine component is also been given to us. So and then there is no other frequency you see second harmonic, third harmonic, fourth, fifth, sixth none of the harmonic component is present. So if you want to you know represent this signal so that is nothing but it consists of a DC of magnitude 1 and there is a fundamental frequency component  $\cos \omega_0 t$   $\sin \omega_0 t$  there is no other harmonic. So there are two means of obtaining phasor measurements one is non recursive approach and there is a recursive approach as well.

So let us discuss about non recursive approach. So data from  $n$  is equal to 0 to  $n$  minus 1  $x$  of  $n$  minus 1 is given by root 2 by  $n$  there is a cosine and sine component and you see here let us say this is a sinusoidal signal and we are making a windows this is window 1 you see here one cycle of window where number of samples are present. So that is what we told in the beginning there is a minimum number of samples which is required otherwise you will not be able to you know get the exact information of a specific signal. So that number is at least 24 samples per cycle. So actually what we are doing is this window is sliding it is moving on and we are getting new sample of data at every you know as the window progresses.

So we have plotted as well in using MATLAB you can see here:



it is a rotating phasor basically it is a rotating phasor the magnitude and the phase angle is rotating basically. So non recursive phasor update as the data window advances by one sample the phasor will rotate in the counter clockwise direction by angle  $\theta$ . Since the phasor calculation are performed fresh for each window without using any data from the earlier estimates the algorithm is known as a non recursive algorithm. Basically it happens like this let us take a simple example of let us say you know 5 data's 1, 2, 3, 4, 5. There are 5 sample data is present.

Now let us say there is a new data which is shipped in and this data let us say this magnitude is some 6. Then in non recursive phasor update it so happens that the first data is being replay. Now first data is eliminated because this window consists of specific size then what we do is the new window consists of the data which looks like this 2, 3, 4, 5 and 6. This is non recursive based phasor update non recursive phasor update. The first sample which could be of magnitude 1 is been replaced and is been moved and now we get the new data entered.

So non recursive algorithms are numerically stable but non recursive algorithm is computationally intensive because you are doing computation at every window so it is computationally quite intensive. Old sample data is discarded and fresh data is acquired every time to estimate the phasor value. So phasor will keep on rotating with a constant magnitude and sampling angle  $\theta$ . So this is just a mathematical representation of how it is done. Now we are discussing about recursive.

Now this is all about non recursive. In case of recursive it so happens that the difference between non recursive and recursive is let us say in case of recursive the same example let us take 1, 2, 3, 4, 5. Now what happens is if there is a new data which is been chipped in which is let us say 6 now this data is been replaced in the beginning rather than at the end. Now it looks like this 6, 2, 3, 4, 5. In non recursive we could see actually the number of data the same number of data but instead of keeping at the last now in recursive it is been pushed in the beginning itself.

So this is just a mathematical indication of how it is been done. Let us say there is this is the phasor of  $N$  plus  $R$  that means there is a new sample, new latest sample entry data entry which is  $R$ . Now this phasor we need to find out obtain. Now this is nothing but the previous phasor  $X$  of  $N$  plus  $R$  minus  $1$  plus root  $2$  by  $N$   $X$  of  $N$  plus  $R$  this is a new sample we are considering this and we are eliminating the previous one minus  $XR$ . So because this was already present in phasor so that is been eliminated.

Basically we are replacing this  $1$  with the new entry which is  $6$ .  $1$  is been taken out and  $6$  is been added. So we again plotted this recursive based phasor update. So it looks like this. Unlike you know non recursive this is not a rotating phasor.

This is a magnitude response is magnitude basically and this is phasor. So some features of recursive phasor update  $N$  samples span exactly one period of the fundamental frequency. The phasor  $X$  cap  $N$  defined by above equation differs from the non recursive estimate by an angle angular retardation of  $\theta$ . Here only angle is been seen so the henceforth this makes phasor stationary unlike in the previous case and not to rotate with sampling angle  $\theta$ . In general the recursive algorithm is numerically unstable.

So an error in the estimate from one window propagates in estimates with subsequent windows. So error propagates here. This error is always present in all the phasor estimates from then on. Because of the great computational efficiency of the recursive algorithm it is usually the algorithm of choice in many applications. So phasor calculation the accuracy required by the C 37.118 this is standard is less than one percentage total vector error. This is a standard means of you know validating the capacity or efficiency of your phasor measuring unit PMU. Phasor calculation in case of DC offsets  $I$  of  $T$  is equal to  $A \cos \omega t$  plus  $B \sin \omega t$  minus there is a DC component  $C$  exponential minus  $T$  by  $T$  which is decaying DC component. So basically this is nothing but  $A$  minus  $C$  for  $T$  is equal to  $0$  minus. So consider  $M$  samples at a rate of  $N$  samples per cycle then this signal can be represented like this which consists of you know so many samples.

This is the matrix into this coefficients  $A$ ,  $B$  and  $C$ . So the above equation can be solved for  $A$ ,  $B$  and  $C$  and then by adding  $C$  of  $R_n$  which is a DC component to each sample of the current the DC offset can be removed from the waveform. Basically we are trying to

estimate. One is the actual value of sampling sample data that we are receiving and we are also estimating because as we have discussed in the state estimation we need to also estimate the data such that we will not be able to you know consider any error into the picture because whatever the measurement that we are making that will be eventually used for taking some appropriate control action. So any discrepancy in the data will also lead to inappropriate control action which may lead to instability in the power system operation.

So henceforth state estimation plays a very vital role to understand which data is a genuine data and which data is you know there is some error into it. If it is that error is within the considerable range we can accept it otherwise there could be so many reasons why this error is existing in this specific sample or the measurement. So least square solution as we have already discussed just a recap. Objective to find X which is the state variable which satisfies Ax is equal to B. The least square solutions of Ax is equal to B are the solutions of the following matrix equation.

Basically A is as we have discussed is a weightage matrix. We give weightage for individual measurements and based on that we will be able to identify which measurement could be error or which measurement is not error consists of true results. So  $A^T$  into A X is equal to  $A^T$  into B just multiplying by A transpose and then by using least square solution we get X cap is equal to or estimated value of stator state variables is  $A^T A^{-1}$  into A transpose into B. So PMU steady state compliance requirement just in detail explanation about this specific standard. In the IEEE synchro phasor standard CE 37.118.1 the accuracy of the phasor computation of a PMU is specified using a real quantity called total vector error which is given by square root of there is estimated quantity and there is an actual quantity and similarly we have for real and imaginary both estimated and actual measurement.

$$\text{Data from } n=0 \text{ to } N-1: X^{N-1} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_n [\cos(n\theta) - j \sin(n\theta)]$$

$$\text{Data from } n=1 \text{ to } N: X^N = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x_{n+1} [\cos(n\theta) - j \sin(n\theta)]$$

So by using this we will be able to find out total vector error. So PMU steady state compliance suggest that permissible synchronizing error of PMU is equal to 1 microsecond only this much of error can be allowed that constitutes to 0.018 degree for 50 hertz. So that means a phasor measurement can at the max consists of 0.018 degree deviation from the actual value of the measurement. So permissible maximum T V is that is 1 percentage and steady state compliance requirements for class P this is a protection class and there is a measurement class. So two class of PMUs are there and steady state compliance requirements are different for different class. The reason being the protection class here accuracy is not so important as compared to the speed in which you calculate

this phasors whereas in the case of class M which is basically helpful for our regular measurements in steady state operation. Here measurement accuracy is very important as compared to the speed of the measurement.

So there are two different classes as such. So class P PMU does not have anti-aliasing filter because if you keep on adding filters there could be some delay in measurement. So a calibration device used to verify performance in accordance with the standard. Documentation shall be provided by any vendor claiming compliance with this standard that shall include what is a performance class whether it is class P or class M and measurements that meet the class of performance. Test results demonstrating the performance and equipment settings that were used in testing and environmental conditions during the testing. Error analysis if the verification system is based on an error analysis.

So a PMU verified for a particular performance class shall meet all performance requirements specified for that class at all required reporting rates. So test conditions which applies to all class of PMUs that is protection class and measurement class. All compliance tests are to be performed with all parameters set to standard reference conditions. So reference conditions for all tests are as follows. One is voltage at nominal, current at nominal, frequency at nominal, voltage current phase and frequency constant, signal total harmonic distortion, THD should also be less than 0.2% of the fundamental, all interfering signals that is they should be less than 0.2% of the fundamental. So measurements at reporting rates lower than 10 frames per second shall not be subject to dynamic performance requirements. All testing to certify compliance shall be performed at standard laboratory test conditions that include the following. Temperature should be at 23 degree Celsius, pressure minus 3 degree Celsius and humidity of less than 90%.

So whatever measurements that you are doing, subject it to specific environmental conditions. Otherwise it may not operate exactly in a real time or physical actual system. So frequency and amplitude modulation test to check if PMU is designed to reflect small signal or low frequency oscillations. The modulation frequency shall be varied in steps of 0.2 hertz or smaller over the specified range and the total voltage error, frequency error, rate of change of frequency error shall be measured for compliance at the given reporting rate. Not only TVE, there is also frequency error required expectation, change of frequency error. So this also need to be considered. And frequency ramp test. Measurement performance during system frequency change shall be tested with linear ramp of the system frequency applied and step change in magnitude and phase angle test to simulate transient scenario. Basically the PMU is subjected to all practical challenges in terms of rate of change of frequency, sudden jump in frequency and also change in magnitude.



To understand during transient condition if there is sudden fall in a voltage especially during fault or if there is sudden change in load pattern because especially when renewables are pitching in and electric vehicles are present, so what would happen is there will be sudden change in voltage and frequency in the system as the active and reactive power may vary because of involvement in such a involving such a dynamic resources such as renewables as well as electric vehicles. So the PMU that you design whether it is a transmission level or the distribution system level, this need to cater the real time challenges and henceforth that can be used for realistic applications. So measurement reporting latency compliance. This is another important consideration which has to be included. So latency which is nothing but communication delay is the delay in measurement reporting is a critical factor for measurements used in real time applications particularly controls because the data you may be receiving may be correct but if you are receiving the data at a very wrong time then it will lead to wrong action.

So right data, right measurement at the right time will be able to take out action at a right time. So this is very important. In addition to measurement latency there are many factors contributing to reporting delay such as communication, coding and transmission distance. So this will also, this may also add on to this latency issues. The application using the data shall take into account all delays to determine system performance.

For P class it is 2 and for M class maximum measurement reporting latency could be 5 because in P class as I already told the protection class the latency could be less whereas for measurement class the latency could be high because the application is different for both of them. So a PMU in a substation could easily have access to line currents in addition to the bus voltage. Sampling both voltages and currents of the same sampling instance would mean that all phases would be on the same reference. So measuring line currents can extend the voltage measurements to buses where no PMUs installed. Some places where you have current measurement and you can try to obtain the voltage by just using the current measurement because you know the line impedances and the number of PMUs can also be reduced so that we can achieve the most economical operation.

So power system observability, this is very important because the more power system is observable the more the system accuracy would be because we are going to have a better control action but the problem is you cannot have you know unlimited number of measuring devices across the distribution or transmission corridor as it increases the economic viability of the system, of the operation. So optimal PMU placement is very important. That means what is that minimum number of PMUs and where do you place them such that the system is still visible and yet you operate at a very economical range. So a power system is called observable when its states can be uniquely determined from the given measurements. The entire power system is visible only when you can say that all the states are visible to me or known to me.

So by the minimal states of a power system we usually mean the voltage magnitude and phase angle at all its buses. The main objective of the observability analysis is to detect the unobservable lines in the system basically and hence identification of the various observable islands. So there could be one PMU which may be taking care of the measurement of multiple bus voltage and phase angles and making the other bus voltages where actually the PMU is not present still it is visible to us. So what are the techniques for observability analysis? There is the two approach one is numerical and topological. Numerical use of matrix algebra or various factorization algorithms and topological which is done using the help of graph theory.

So placement of PMUs for complete observability one is exhaustive search which gives global solution but suitable only for a small number of candidate PMU locations and integer programming which can be used for large power systems and use of intelligent techniques for optimization which is not so suitable for very large systems. So rules for observability analysis using PMU. When a PMU is placed at a bus it can measure the phase angle of the voltage at that bus as well as the phase angles of the voltage at the buses at the other end of all the incident lines. It is assumed that the PMU has sufficient number of channels to measure the current phases through all the branches incident to the bus. In case of zero injection buses if the phase angles of all but one incident bus are known the remaining one can be determined by using KCL.

As we know some of the currents at a specific node algebraic sum of the currents at a specific node is zero as per KCL. So if one incident bus phase angle is not known that can be known by using this KCL. However it leads to the propagation of measurement and parameter errors. There could be some error propagation if you are okay to compromise with that we can you know measure this phase angle of that incident bus. So just an example to show you how the optimal placement of PMU is done.

See you can see here in this specific topology or the network we are going for only three PMUs 1, 2, 3 at bus number 6, 9 and bus number 2. So what is really happening is you see here there is a PMU being placed at bus number 6. Now this PMU can able to take care of the measurements of bus 12, bus 13, bus 11 as well as bus 5. And now the other buses are left out.

Now there is a PMU being placed at bus number 2. Now this PMU can able to take care of the measurements at bus number 1, bus number 5 again and then bus number 4, bus number 3. And then there is another PMU being placed at bus number 9. Now this PMU will be able to take care of bus number 10, bus number 14 and bus number 7. So by just keeping these three PMUs we can administer the voltage, phase angle, frequency and rate of change of frequency of all the buses in the network such that with minimum PMUs we will be able to take care of the entire power system network. So with this we will conclude for today. Thank you very much. .