Economic Operation and Control of Power System

Dr. Gururaj Mirle Vishwanath

Department of Electrical Engineering

IIT Kanpur

Week - 10

Lecture – 46

Hello and welcome you all for the NPTEL online course on Economic Operations Control of Power We will continue our discussion with respect to state estimation. So first of all why state estimation understanding is required for PMU? So that we will try to understand here. So PMU Phasor Measuring or Phasor Measurement Unit. PMU data are very essential for the most accurate state estimation. In PMU the power system signals received from voltage and current sensors are sampled and converted into phasors. First of all we will be receiving signals from voltage and current sensors through CTs and PTs and whatever voltage and current sensor based signals that we receive that will make it as different samples and these samples are later converted into phasors and these phasors are used in phasor measurement unit.

Now phasors are complex number representation of the sample signals which are commonly used in the design of and inputs to control and protection systems for bulk power transmissions of power transmission grids. So basically the phasor measurement unit will give us the phasor of a particular signal by which we can able to make our power system a more controllable, more accurate because this information that we are receiving is a time sample data that we are going to discuss in the next point. So the phasors are time stamped from a timing pulse derived from the GPS and then streamed into the wide area communication networks as fast as one phasor per cycle of the power system frequency. So the sampling rate is quite high so that the phasor that we obtain from the out of the sampled voltage and current signals their frequency of sampling is quite high so that we will be able to accurately determine what are the different states of a power system network.

Once we determine the power system states we will be able to make appropriate decisions and controlling can be more accurate. The more the system is observable and if observability accuracy is quite high then equally the controllability So the phasor angle differences provide useful information concerning system stress or modes of oscillatory disturbances in the power system. Eventually we will be able to make system more stable as we reduce the system oscillations. So state estimation using PMU data. A PMU samples the AC voltage in current waveforms and computes the phasors by using an algorithm such as the discrete Fourier transform.

As I already told through voltage and current sensors we will be determining the phasors as we sample out the voltage and current sensors receive data and this phasor conversion happens through a specific algorithm which is working based on the discrete Fourier transform DFT. So we will define DFT first in order to understand PMU and the state estimation we need to understand the Fourier analysis aspect of a particular signal. So first of all we will discuss the important properties before going to methods for estimating the phasors and frequency of the AC signals. To make the discussion somewhat self-sufficient we will start with an introduction to the Fourier series and Fourier transforms. Now the Fourier series and Fourier transform.

The Fourier series of a periodic signal is given by:

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos k\omega_0 t + b_k \sin k\omega_0 t)$$

Where $\omega_0 = 2\pi f_0$ is the fundamental frequency in rad/s, and

f_0 is the fundamental frequency in Hz.

Since $T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$,

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left\{a_k \cos \frac{2\pi k}{T_0} t + b_k \sin \frac{2\pi k}{T_0} t\right\}$$

This integration of a periodic signal over the length of the complete period is the same for any starting point of integration.

This is an important note. The coefficients a_k's and b_k's of the Fourier series described above are given by:

$$a_0 = \frac{2}{T_0} \int_{T_0} x(t)\, dt$$

$a_k = \frac{2}{T_0} \int_{T_0} x(t) \cos k\omega_0 t\, dt$ and $b_k = \frac{2}{T_0} \int_{T_0} x(t) \sin k\omega_0 t\, dt$

For real $x(t)$, the coefficients a_k's and b_k's are real. Let, $a_k = C_k \cos \theta_k$, and $b_k = -C_k \sin \theta_k$

Then, $x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (C_k \cos k\omega_0 t \cos \theta_k - C_k \sin k\omega_0 t \sin \theta_k)$

$= C_0 + \sum_{k=1}^{\infty} C_k \cos(k\omega_0 t + \theta_k)$

Where, $C_0 = \frac{a_0}{2}$, $\theta_k = \tan^{-1}\left(\frac{-b_k}{a_k}\right)$, $C_k = \sqrt{a_k^2 + b_k^2}$

This compact form shows that a Fourier series can be expressed as a sum of sinusoids of frequencies 0(DC), ω_0, $2\omega_0$, …, whose amplitudes are C_0, C_1, C_2, …, and phases are 0, θ_1, θ_2, …

So every signal has a magnitude, frequency and a phase. A plot of ck's, the coefficients versus k's is called the amplitude spectrum and theta k's versus k's is called the phase spectrum of the signal. The two plots together are called the frequency spectra of the signal. One can reconstruct the signal given the frequency spectra. Using Euler's formula, the following can be written as:

$$\cos k\omega_0 t = \frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2}$$

$$j \sin k\omega_0 t = \frac{e^{jk\omega_0 t} - e^{-jk\omega_0 t}}{2}$$

So with this expression we will be able to map this cos and sin terms in terms of the exponential terms. Hence $x(t)$ is equal to:

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left\{ a_k \left(\frac{e^{jk\omega_0 t} + e^{-jk\omega_0 t}}{2} \right) - jb_k \left(\frac{e^{jk\omega_0 t} - e^{-jk\omega_0 t}}{2} \right) \right\}$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty} \left\{ \left(\frac{a_k - jb_k}{2} \right) e^{jk\omega_0 t} + \left(\frac{a_k + jb_k}{2} \right) e^{-jk\omega_0 t} \right\}$$

We now define the following $D_0 = \frac{a_0}{2}$, $D_k = \frac{a_k - jb_k}{2}$, $D_{-k} = \frac{a_k + jb_k}{2}$

Using above expressions,

$$x(t) = D_0 + \sum_{k=1}^{\infty} \left(D_k e^{jk\omega_0 t} + D_{-k} e^{-jk\omega_0 t} \right)$$

$$= \sum_{-\infty}^{\infty} D_k e^{jk\omega_0 t}$$

Later on we will see how d k s can be further evaluated numerically after we discuss discrete Fourier transforms.

$$D_k = \frac{2}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} \, dt$$

In a periodic signal any non-periodic signal can also be represented as a periodic signal with period t naught tends to infinity.

After representing the signal in the exponential form, the coefficients for an aperiodic signal can be written as,

$$D_k = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t}\, dt$$

$$= \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0} \int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t}\, dt$$

Let us now define, $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$

Then, $D_k = \frac{X(k\omega_0)}{T_0}$

Since we did not put any restriction on omega naught we can take it arbitrarily very small and the multiplier k can be used to vary the term k omega naught in small steps.

So why we are making it small steps? So we can express in terms of more sampling rate actually. So replacing omega naught by delta omega for notational convenience the above equation can be written as x of t:

$$x(t) = \sum_{-\infty}^{\infty} \frac{X(k\Delta\omega)}{T_0} e^{jk\Delta\omega\, t}$$

$$\omega_0 = \Delta\omega = \frac{2\pi}{T_0}$$

We can also write x of t is equal to:

$$x(t) = \frac{1}{2\pi} \sum_{-\infty}^{\infty} X(k\Delta\omega)\, \Delta\omega e^{jk\Delta\omega\, t}$$

$$\approx \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega\, t} d\omega$$

The integral on the right hand side of the above equation is called the Fourier integral and the function x of omega is called the direct Fourier transform of x of t you see there is a time domain signal which is transformed to frequency domain signal by using this Fourier transform so this is a typical expression and x of t is the inverse Fourier transform of x of omega if there is x of omega then x of t is its inverse Fourier transform. To summarize Fourier transform:

$$F\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega$$

In polar form, $X(\omega) = |X(\omega)| \angle X(\omega)$

Now this is Fourier transform this is Fourier transform and this is inverse Fourier transform. The Fourier transform x of omega being a complex quantity is its amplitude and phase spectrum can be obtained for varying values of omega in polar form x of omega is magnitude of x of omega into angle of x of omega. Now there are certain conditions that need to be satisfied before we say any specific signal can be transformed into a Fourier signal. So Fourier transform of a signal exists if it satisfies the following conditions these are the very necessary conditions and these are called as Dirichlet conditions after this scientist Dirichlet x of t should be absolutely integral this is the first requisite.

x(t) should be absolutely integrable, i.e., $\int_{-\infty}^{\infty} |x(t)|\, dt < \infty$

So there should be countable discontinuities within a specific time interval that is what it says and x of t must contain only finite number of maxima and minima within any finite interval. So if you count the number of maxima and minima within a specific time interval that should have some number and it should not be infinity. So let us take up some examples find the Fourier transform of a impulse function delta of t right.

Example: Find the Fourier transform of $\delta(t)$.

$F\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t} dt = e^{-j\omega 0} = 1$

Or, $\delta(t) \Leftrightarrow 1$

Example: Find the inverse Fourier transform of $\delta(\omega)$.

$$F^{-1}\{\delta(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)e^{j\omega\, t}\, d\omega = \frac{1}{2\pi}$$

Or, $\frac{1}{2\pi} \Leftrightarrow \delta(\omega)$

Example: Find the inverse Fourier transform of $\delta(\omega - \omega_0)$.

$$F^{-1}\{\delta(\omega - \omega_0)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega - \omega_0)e^{j\omega\, t}\, d\omega = \frac{e^{j\omega_0 t}}{2\pi}$$

Or, $e^{j\omega_0 t} \Leftrightarrow 2\pi\delta(\omega - \omega_0)$ also, $e^{-j\omega_0 t} \Leftrightarrow 2\pi\delta(\omega + \omega_0)$

Example: Find the Fourier transform of $cos\,(\omega_0 t)$.

$$cos\,(\omega_0 t) = \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)/2$$

Using previous example, $cos\,(\omega_0 t) \Leftrightarrow \pi\{\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\}$

Example: Find the Fourier series of an impulse train

$$\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$$

Using exponential Fourier series representation,

$$\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} D_k\, e^{jk\omega_0 t};\ \omega_0 = \frac{2\pi}{T_0}$$

Where, $D_k = \lim_{T_0\to\infty} \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\, dt$

Choosing the interval of integration from $-T_0/2$ to $T_0/2$, and noticing that within this interval, $\delta_{T_0}(t) = \delta(t)$,

$$D_k = \lim_{T_0\to\infty} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \delta(t)e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}$$

Using above,

$$\delta_{T_0}(t) = \frac{1}{T_0}\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t} = \frac{1}{T_0}\{1 + 2(\cos\omega_0 t + \cos 2\omega_0 t + \cdots)\}$$

Expressing the signal $x(t)$ with period T_0 in terms of exponential Fourier series,

$x(t) = \sum_{k=-\infty}^{\infty} D_k e^{jk\omega_0 t};\ \omega_0 = \frac{2\pi}{T_0}$

Taking Fourier transform of both sides of above

$$X(\omega) = 2\pi\sum_{k=-\infty}^{\infty} D_k\delta(\omega - k\omega_0)$$

So working with discrete time signals, a PMU works with discrete or digitized values of the voltage and current signals. To compute the phasors AC voltage and current signals are sampled at appropriate rates and converted to digital values through an ADC before applying DFT like algorithms to compute the phasors. The following section discuss the essentials of sampling techniques. Now a sampling, a signal whose spectrum is band limited to B Hertz that is x of omega is equal to 0 or magnitude of omega is greater than 2 pi B this is a band width, band limit B let's say some number.

So can be reconstructed without any error from its samples taken uniformly at a rate fs is greater than twice the bandwidth. This should be the sampling frequency. The minimum sampling frequency should be twice the bandwidth then only you can reconstruct the whole signal. In other words the minimum sampling frequency is fs equal to twice the bandwidth of the specific signal and sampling of x of t at fs Hertz is done by multiplying x of t with an impulse train delta t of t consisting of unit impulses every t seconds where t is equal to 1 by the sampling frequency 1 by fs. Now sampled is represented by:

$$\bar{x}(t) = x(t)\delta_T(t) = \sum_k x(kT)\delta(t - kT)$$

The sample signal contains impulses at every t seconds and can be represented by:

$$\delta_T(t) = \frac{1}{T}\{1 + 2(\cos\omega_s t + \cos 2\omega_s t + \cdots)\}$$

Now let's say there is a signal and that is run through impulse train. Now we found out the impulse train Fourier transform and then you will get the sampling rate actually. So sampled data of each signal, the signal which is been sampled over a period that can be obtained.

Now, the frequency of the impulse train in rad/s is,

$$\omega_0 = \omega_s = \frac{2\pi}{T} = 2\pi f_s$$

Using the previous three equations,

$$\bar{x}(t) = x(t)\delta_T(t) = \frac{1}{T}\{x(t) + 2x(t)\cos\omega_0 t + 2x(t)\cos 2\omega_0 t + \cdots\}$$

So with this we will conclude for today and then we will continue in detail about PMUs and how it is our understanding about Fourier transform is helpful to obtain the PMU data. Thank you very much.