**Economic Operation and Control of Power System**

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**Lecture – 42**

Hello and good morning everyone. Welcome you all for the NPTEL online course on Economic Operation and Control of Power System. Today we will discuss a new topic which is Optimal Power Flow. So in the in the economic dispatch problem the total cost of generation is minimized that is objective, minimize the total cost of generation considering only the generation limits and power balance constraints. Summation of generation should be equal to load and the generation should be within the lower and upper limit. The secure and reliable operation of a power system however demands that a large number of operating limits and components as well as operating states be maintained.

So many other things that need to be taken into consideration such as operation limits of lines, the voltage and phase angle limits all these things need to be taken into consideration. Therefore network constraints should also be incorporated to determine most economic generation schedules. These additional limits are different or difficult to incorporate in the simple optimization technique using Lagrange multipliers. Using conventional Lagrange multiplier based economic dispatch problem though we can find out the most optimal generations but considering all these constraints, network constraints that things will be very tedious and it may not be possible using Lagrange multipliers.

So this new modified optimal dispatch problem including network constraints is known as optimal power flow problem. Take an example. So let us say this is a network, the three generators and put together a total load of 315 megawatt and there are certain line impedances which have been given here. So generator characteristics are quadratic and is being given and there are certain limits for each generators. So and there is a network constraint which is also been imposed.

For example here the maximum voltage deviation allowed is 5% in either direction from 1 per unit. That means the voltage limits are 0.95 per unit which is a lower limit and 1.05 per unit which is a upper limit. Lines have maximum power flow capacity of 150 megawatt due to thermal constraints.

Now performing Lagrangian economic solution strategy for total 315 megawatt of loading considering bus 1 as slack bus generations, you get this most optimal generations. So what is happening, but you can see here at bus number 4, there is a voltage violation which is happening. The maximum voltage limit violation is allowed is plus or minus 5%. Here is more than 5%. So lines 1 to 5 and 3 to 4 are overloaded.

This is 150 megawatt is the capacity. You can see here this is 163 megawatt and there is also a line between 3 to 4, this line, this is also overloaded. That means using Lagrangian multiplier you can find economic dispatch problem, but the problem is the network violations may happen. So the objective function could be other than the minimization of the total generating cost. Of course that will be there.

Apart from that there could be so many other objectives that need to be met. The first thing could be minimization of transmission loss in the system. Second could be minimization of total load shedding. You do not want your customer to suffer due to frequent power outages. So you need to satisfy your customer.

So minimizing the total load shedding could be one of the objectives and minimizing the total cost of control actions in case of preventive or corrective control against system insecurity. That means the possible insecurity scenario, we discussed about contingency studies. So you should have appropriate control actions. You should do not want to lose some control actions due to some contingency outages, line outage or generator outages. So that also you want to ensure the system will be more stable in a way.

So the basic structure of optimal power flow, it looks like this.

$$\begin{aligned} &\text{Minimize:} && f(\bar{z}) \\ &\text{Subject to:} && g(\bar{z}) \\ & && \bar{h}^- \leq h(\bar{z}) \leq \bar{h}^+ \\ & && \bar{z}^- \leq \bar{z} \leq \bar{z}^+ \end{aligned}$$

There is a function, minimization function. So this Z represents set of state variables, voltages, phase angles, there are so many other parameters, type changing, transformer positions and all these things. Subject to there is a constraint, equality constraint. And there could be some inequality constraints.

Everything is dependent upon these state variables. And each state variable may have its own limit also. Like voltage is a state variable but voltage has its limits. And phase angle is a state variable and that may have some limits. Frequency could be a state variable and it may have some limits.

You understand? So where f of Z is the objective function to minimize, vector Z is what I have told. It is a state variable and are control variables such as bus voltage, magnitude, phase angles, etc. And H minus and H plus are the vectors containing lower and upper limits of H of Z respectively. And Z minus and Z plus are the vectors containing lower and upper limits of state variables. So OPF has a wide range of state and control variables, some of which are generator voltage, LTC transformer tap position, that means on load, tap changer transformer tap position and phase shifter tap position.

This is also a transformer which shifts the phase. And output of reactive power sources/sinks. There are so many reactive power injection devices. We had discussed in one class regarding various real time case studies having inverters, OLTC, how do you manage the reactive power. And load shedding, this could be one of the state variable and line flows also.

So typical constraints, equality constraints include power balance constraint and inequality constraints include production limits on generators and allowable bus voltage limits and allowable power limit through each transmission line. So typically the following parameters are known. What are known to you? Generator characteristics, that will be given to you. And generation limits, line impedances, network topology and network security limits. So these are the some of the information that you have for a OPF problem.

What are the widely used typical objective functions? Minimization of total fuel cost, minimization of active power loss and security constraint optimal power flow and multi-objective optimal power flow. So applications, finding out the optimum generation and control options to achieve minimum cost and system security together. With short term load forecast, OPF is able to determine preventive dispatch for the system by incorporating system security. In case of emergency, let us say there is an emergency scenario such as an overload, overloaded line or other undesired condition could be fault. OPF can provide corrective dispatch for the system.

How do you manage this kind of emergency scenarios? That OPF would be able to take a decisive action. And OPF is routinely used in planning studies to determine the maximum stress transmission network can withstand. So in the planning stage itself, you know ensure that the system will not lead to further critical situations. You have a better planning such that the system will not undergo severe turbulences in case of outages. OPF is used in economical analysis, economic analysis of the power system as it can determine locational marginal prices as well.

So let us say one of the objective function, minimization of total fuel cost. So the objective function here is similar to the one in the economic dispatch problem that is a minimization of total fuel cost of all the thermal generating units. However, in an OPF formulation, what is the change that one can see? If the wider range of constraints can be included in the optimization problem. Reasonably accurate representation of cost curves of the thermal generating units is needed to construct the objective function. Commonly used cost functions are quadratic, piecewise linear, cubic and piecewise quadratic.

You understand what you mean by piecewise quadratic? Like there is a quadratic function, then you divide it into piecewise linear functions. So you have different sort of characteristic curve, you consider all of them, also consider constraints. Using quadratic cost functions, the total fuel cost is given by:

$$F_T = \sum_{i=1}^{N_G} F_i\,(P_i) = \sum_{i=1}^{N_G} \left(a_i P_i^2 + b_i P_i + c_i\right)$$

Now this is a total fuel cost that you need to minimize. So that means you need to minimize fuel cost of each generation. So with respect to their respective quadratic function.

So minimize this subject to these constraints given by:

- The OPF is formulated as follows:

Minimize: $F_T = \sum_{i=1}^{N_G} \left(a_i P_i^2 + b_i P_i + c_i\right)$

- Subject to, $P_{gi} - P_{di} = V_i \sum_{i=1}^{N} V_j\, Y_{ij} \cos\left(\delta_i - \delta_j - \theta_{ij}\right) \quad \forall i = 1, \dots, N$

$$Q_{gi} - Q_{di} = V_i \sum_{i=1}^{N} V_j\, Y_{ij} \sin\left(\delta_i - \delta_j - \theta_{ij}\right) \quad \forall i = 1, \dots, N$$

$$V_{i,\min} \le V_i \le V_{i,\max} \qquad P_{gi,\min} \le P_{gi} \le P_{gi,\max} \qquad Q_{gi,\min} \le Q_{gi} \le Q_{gi,\max}$$

$$-I_{l,\max} \le I_l \le I_{l,\max} \quad \forall i = 1, \dots, L$$

$\bar{V}_i = V_i \angle \delta_i$ is the voltage at bus $i$

$\bar{Y}_{ij} = Y_{ij} \angle \theta_{ij}$ $ij^{th}$ element of the bus admittance matrix

$I_l$ = Current flow in the $l^{th}$ line

$I_{l,\max}$ = Current limit of line $\qquad$ $N$ = Number of buses

What is that constraint? You see here basically this is generation at bus number i and there will be also some local load that means this is a local demand. So some of the generation at that specific bus subtracted with the demand at that specific bus is the net amount of power that the particular generator is injecting to the network, injecting into the network and that is expressed as this one, Vi, Vj, Yaj cos delta i delta j minus theta ij. It is a very familiar power flow expression. Similarly there is also a quadratic power, reactive power.

This is active power, this is a reactive power. And then there is a limit in voltage and there is a limit in each generation and there is a limit in each generation active as well as reactive power. Further there is also a limit with respect to the current carrying capacity of a specific line, lower limit and upper limit. So Vi represents the voltage at an angle and Yaj represents the admittance at an angle and IL represents the current flow in the lth line and IL max represents current limit of line. So let us say there are n number of buses to which you are calculating this OPF.

So optimization techniques can be applied to achieve minimization of the real power loss in the system. This is another OPF objective, minimization of active power loss. Usually generator real power outputs are not taken as the control variables in the loss minimization process. You do not want to schedule with the objective to minimize the loss. You will schedule in order to maintain generation is equal to load demand.

So you cannot consider the generation of a particular generator to be a control variable to minimize the loss. That cannot happen. So following assumptions are commonly taken while formulating the loss minimization problem. So what are the assumptions? Economic dispatch or cost minimization problem is solved beforehand and the outputs of the generators except the one connected to the slack bus are kept fixed. That means you carry out ED, economic dispatch and then you have a generation available.

Having such kind of generation, how do you minimize the loss? So transformer tap ratios are considered as continuous variables during the optimization after which they are adjusted to the nearest tap position and the iteration is done again. So during the process of this OPF evaluation in a specific iteration, you consider tap position is fixed. Then for the next iteration, it may change, increase or decrease. This is the assumption. So the frequently used control variables for the loss minimization problems are generator bus voltage magnitudes.

How do you manage the generator bus voltage magnitudes? And transformer tap ratios. So the use of reactive power sources are things. Let us say you have DSTATCOM, other renewable source converters as we have already discussed. So these are also the control variables through which you can manage the voltage profile. If you manage the voltage profile, you will eventually reduce the losses properly and phase shifter angles as well.

So there are two basic approaches to loss minimization. First is slack variable approach and second is summation of losses on individual lines. So in the slack variable approach, here the output of the generator connected to the slack bus is minimized. So the disadvantage of this method is that it minimizes only the total loss in the system. In some cases, it may be desired to minimize the loss in some specific area of the power system.

You understand that? So because there is only one slack bus, now if you try to minimize the total power loss using adjusting the power output of the slack bus or considering that there is one variable through which you can control the entire loss of the network, then what it would lead? You may try to minimize the entire loss, but that if there is some specific case studies where area specific loss minimization need to be taken into consideration. So you cannot do that using slack variable approach. It is for the entire network. So what is the second approach? Summation of losses on individual lines. So this approach solves this problem by considering the losses of all the lines.

So more weightage may be assigned to the losses associated with a specific area where loss minimization is of great importance compared to the other areas. That means if there is a big network, you divide the networks into different regions. In the previous class, we were discussing about regions. So different regions, then you have specific control action for that specific regions and then such that you keep particular converters or a capacitor banks in specific region where you have focused towards loss minimization. So in other regions, you may try to ignore or you may try to not emphasize much upon installing the reactive power injecting devices to minimize the losses.

In that way what will happen is you can reduce the overall cost. I mean you are highlighting the problem where exactly you need to solve it and then you invest there. You need not have to invest for the whole system. So this is the advantage that one can think of.

So formulation of the optimization problem. Taking the second approach, the objective function to minimize is given by reducing the total power loss and that includes the summation of power losses happening in each line. So that is what PL is the power loss in line. Using voltage in polar form, PL can be expressed as PL is nothing but IL square by GL. This is GL is the conduct series conductance. This is nothing but I square into R where IL is the current flowing through line L.

Loss can also be written as:

$$
\begin{aligned}
P_l &= \left|\bar{V}_i - \bar{V}_j\right|^2 g_l \\
&= \left[\left(V_i cos\delta_i - V_j cos\delta_j\right)^2 + \left(V_i sin\delta_i - V_j sin\delta_j\right)^2\right] g_l \\
&= \left[V_i^2 + V_j^2 - 2V_iV_j cos(\delta_i - \delta_j)\right] g_l
\end{aligned}
$$

Where, $\bar{V}_i = V_i\angle\delta_i$ and $\bar{V}_j = V_j\angle\delta_j$ are the voltages at the ends of the $l^{th}$ line

So what we are doing is a line is there that is connected between, this is line L that is connected between bus I and bus J. So what we are trying to do is and there is a voltage here VI at an angle delta I. This is the voltage of bus number I and there is a voltage which is VJ at an angle delta J. Now what we are trying to do is we are trying to minimize loss of each line basically.

So that means there is a magnitude V square that means magnitude difference VI minus VJ whole square into GL that is resistance of that line, I mean conductance of that line basically. So ignoring the imaginary parts because we are taking the magnitude ultimately, then by putting this VI is equal to VI at an angle delta I and VJ is equal to VJ at an angle delta J to this expression you will get this and you have to ignore the imaginary term basically and that is nothing but this one. This is further deduced to this expression. So in more general form we can also express like this, like using voltages in rectangular form the objective function becomes VI minus VJ whole square into GL is nothing but EI minus EJ whole square plus FI minus FI FJ whole square into GL. What is EI EJ FI FJ? This is nothing but a real part and imaginary part of the individual buses.

VI is expressed as EI plus JFI and VJ is expressed as EJ plus FJ, JFJ. So the above represents a quadratic objective function that can be easily handled by commonly used quadratic programming methods. We are expressing in terms of this polar form so that polar form to rectangular form so that we can able to solve it using quadratic programming methods. Now the objective function is given by this, summation of L is equal to 1 to NL EI minus EJ square plus FI minus FJ whole square into GL subject to injecting power injected this is nothing but EI into summation of J is equal to 1 to N and now we are splitting in terms of, now if you remember Y can be expressed as G plus JB. This is conductance plus susceptance, admittance is expressed as conductance plus susceptance.

So now because this is voltage real part and imaginary part and there is now that can be split into that multiplied by GL can be split as conductance plus susceptance because G is nothing but what? Y cos alpha let us say and B is equal to Y sin alpha. So that can be expressed like this. Remaining similarly reactive power. QJ minus QDI is nothing but FI into this imaginary part into this part and then real part into this one. So this is you can just take up from the previous expression I will show you that.

You just have to use this expressions. PGI minus PDI is VI, VJ YIJ cos delta I minus delta J minus theta IJ. So using this you can deduce to conductance plus susceptance. Similarly reactive power also. So and subject to this constraints voltage is within the limit of VI minus VI maximum and there is also certain other constraints.

Let us say tap ratio of the ith transformer. TI minimum, TI maximum. Already told in one of the classes that transformer has a specific limits. We discussed plus or minus 16 taps in one of the problems. So upper 16 taps lower 16 taps or 10 taps. So there are certain tap limits are there and then NT is number of tap changing transformers.

How many tap changing transformers are there? P is number of pairs of parallel transformers. Let us say if you have to augment the power transfer to a specific station what you do? You go for paralleling of transformers. You cannot replace one transformer with a bigger transformer. That is not an effective solution. What you will do is let us say in this campus load earlier the power load was 500 megawatt.

Now if power load increases from 500 to 600 you do not replace the 500 MVA transformer with a 600 MVA. With 500 MVA you may add 250 MVA additional transformer in parallel such that you can also plan to meet out the expectation of future demand because demand keeps on increasing. So there will be so many pairs of parallel transformers. So that is also been taken into consideration and TP1, TP2 are the pair of peak parallel transformers. So what are the challenges with OPF? Size of the problem is quite big with thousands of lines and hundreds of control variables.

If you speak about a big transmission network there are so many lines and lot of control variables and nonlinearity and non-convexity the system is highly nonlinear. Power flow equations itself are nonlinear. Then there is a non-convex problem also. And discrete nature of the variables that is position of the transformer taps, status of the switch capacitors or reactors.

So transformer is a integer number, right? Transformer tap position. 2, 3, 4 this is a integer number. So this is a discrete sort of variable present in a continuous, along with the continuous variable. So this column will be sort of mixed integer. We call it as mixed integer linear programming solver or MINLP, mixed integer nonlinear programming. So these are some of the tool boxes which are used to solve the specific problem.

So MINLP is the most realistic because in any system there is also a tap changing transformer and system is also nonlinear. So MINLP optimization problem is most complicated and is also a practical problem that need to be addressed. And even switch capacitor banks also, like how many capacitors that you are connecting in shunt in order to make out the voltage profile, discrete that is also discrete in it. So what are the solution methods that we have? Lambda iteration methods, gradient methods, Newton method, linear programming method, interior point method. The first two methods that means lambda iteration method and gradient method have a slow convergence, right? Especially when you consider a big network into picture.

All of the first three methods suffer from the difficulty in handling inequality constraints because inequality constraint means if there is a line limit that you are considering, voltage limit that you may be considering, that is actual OPF problem. So considering these constraints, lambda iteration gradient, Newton's method will be even more complicated or may not be converging also sometime. Newton method you may try to converge it but the complexity would further increase. Now we have relationship between P, theta, Q, V, we have Jacobians relevant to this. So when you put so many other constraints, accommodating them is not an easy task.

But the last two methods are fully developed optimization technique that are used in production grade software. That means industry preferred choice is linear programming and interior point approaches, interior point methods. So one condition for using linear programming is that the objective function and the constraints have to be linear. This is one constraint.

However, there are no such restrictions for the interior point method. This is even more flexible basically. So we will discuss these techniques. I mean I am planning to have a MATLAB based case studies also so that you will have a feel of how to solve a specific optimization problem. So that will be more easy to understand.

In later classes, we will include that aspect. So with this, I will conclude today's discussion. Thank you very much. .