Economic Operation and Control of Power System

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Week - 01

Lecture - 04

Very good morning. Welcome you all to the NPTEL course on Economic Operation and Control of Power System. And today we are going to talk about Optimization Preliminaries, means Fundamentals, section 1 of lecture 4. So, importantly when we are talking about power system, most of the mathematical representations as well as equations, they are not purely linear, sometime they are linear, sometime they are non-linear. As well as we need to optimize the overall cost of generation to meet the required load. So, when you talk about optimally utilize the generations to meet the load, that means we need to understand optimization. So, probably the detail of optimization direct application to power engineering, we will talk in due course of time. But before that we need to understand what is optimization and how that can be applied to economic operation and control of power system course. Now, when you talk about optimization, let me go back to your plus 2 fundamentals. So, during plus plus 2 we have been taught that you know a function is you know to optimize any function, we take a function f of x, then we differentiate equal to 0. Then we go for second differentiation and then we substitute those values. If it is positive, then that point is minimum. If it is negative, that point is maximum. That is what actually has been taught to us during school days. But now the scenario is almost same fundamental, but we need to address many more complicated problems today. Now, optimization process aims to minimize or maximize a certain objective to achieve most optimal, but feasible solutions. Now, you need to have a function, which is normally called as objective function. And the objective function could be a linear nonlinear function, most likely it is nonlinear and that need to be either minimized or maximized. Now, we need to have different limits imposed on the optimizing process are called constraints. For example, if I am interested to know the generation output of a particular power plant that is P and that value of P must be less than its maximum generation and more than its minimum generation.

So many times I have told you that if there is a cost characteristic, for example, yeah, so just what I will do, I'll just demonstrate it. Yeah. So if this is going to my cost characteristic and if this is my P minimum and this is my P maximum, so probably within this range you have to change your P. So to me, the P max is the maximum limit and P min is the minimum limit and hence that is my constraint. So P must be, I can rewrite that P must be less than or equal to P maximum and greater than or equal to P minimum.

All right. So this is the equation is known as my constraint equation. Now, similarly, we can also find out the reason bounded by constraint is known as feasible reason, the area within which, for example, if a space is given to you, you can identify a zone in which my optimal point lies. So that is my reason bounded by the constraint is known as feasible reason for the optimizing process. Now, optimal solutions should lie at the border of a feasible reason. This ideally they do occupy a point which is mostly at the extreme point because you optimize it. Now, local optima is a very common term because there are few functions which are keep on oscillating and probably they are not convex functions. Let's say if I do talk about a function, all right, which is keep on oscillating and in that, if this is my point of operation, the gradient at this point is certainly, at that optimal point is going to be zero because it is just a constant point to me and the tangent which is different there. So at this optimal point, the differences of any function equal to zero, there is no doubt about it. And that is also true at this point, that is also true at this point, that is also true at this point. So you find many points where my difference is not equal to zero or the gradient is something different. Now, what happens if I take the starting point of my optimization is x, then probably it will keep on moving, moving, moving and the moment the gradient is zero or the difference is zero, it will say, oh, you are reached your optimal and the program exit. So it will tell you the optimal point is, this is my x dash is the optimal point. However, there is another optimal point which is lying here, all right, so that is not visible to this fellow because the starting point has chosen in such a manner that it lie down at this point and accept, oh, this is my optimal point. So this is some sort of optimization, but it is known as local optimal.

But if you find many local optimal points, if you would have started from here, I could have got this point. If I would have started from here, I could have got this point. So now what is happening, depending upon your starting point, you get an optimization point, but which is not necessarily to be optimal. So that is my local optimal. But if you take all the local optimals of your function and then take the best out of which, which will give me the global optimal. So global optimum is defined as the point whose objective value is best among all the points present in the feasible region. And also the constraint functions and simple variables limit are lumped under the term constraints. So everything can be put in a constraint form. And if the constraints are such that no such reason exists, means if I do say that my P is of range A to B, and I'm not getting any solution within that range, probably I'll say there is no solution exists, which means that there are no values which satisfies all the constraints. So it may so happen. I may tell that, okay, just for an example, the range within which you have to find a point probably do not lead to an optimal solution. And that is possible. And hence we say there is no feasible solution.

There is no reason existing to get it optimized. Now optimal problems, you know, they are different types. One is unconstrained and the second one is constraint. Unconstrained means the whole space is yours and it is a point which is less likely in a practical problem. So practical problems are always constrained, but in textbook for our understanding we solve unconstrained optimization for learning constrained optimization. So before you get into constrained optimization, you need to understand what is unconstrained optimization. Now there are different type of unconstrained optimization problems when a single variable or could be multivariable.

And those multivariable optimization problems and single variable optimization problems could also be having equality constraint and it can also have inequality constraints. Equality constraint means I may say my variable y which is equal to 5 or I may say my variable y which is greater than or equal to 5. So now what happens? It could be one is equality, the other is inequality, okay. And the function could be f of x which is x square plus 3 is a function, okay, which is a nonlinear function. And if I do not have any constraint, only I have to optimize this function, then that is my unconstrained optimization problem.But if I have to do this and put the variable, for example f of x equal to x square plus 3 and x must be less than or equal to 5, so then that optimization problem become a constrained optimization problem of single variable. I can also have f of x which is x1 square plus x2 square plus 25 where x1 is less than or equal to 3 and x2 is greater than or equal to 5. So now it is a multivariable inequality constraint optimization problem, all right. So you can find many problems of this order. To start with as fundamental, what do you understand by unconstrained single variable optimization problem? So it is simple. So you have a single variable which is x, all right, and the function which is f of x. So first of all what you need to do, you have to differentiate and equate to 0. That means if the objective function is f of x, you have to differentiate which is f dash x need to be equal to 0, f dash equal to 0. You can see from equation number 4.1. But now the question is when you make that equation equal to 0, how do I find the minimum and maximum values? That is the question because f dash x equal to 0 does not guarantee that you are at optimal point. But even if you are at optimal point, it does not once again assure you that are you at actually maximum point or minimum point. For example, if I draw a characteristic, all right, now at this point my f dash x equal to 0. At this point also that f dash equal to 0. Now my question is this leads to a minimal point and this leads to a maximum point, all right.

So you need to identify. So first condition f dash equal to 0 means you reach to you are going to reach to an optimal point, but you need to identify whether there may be no more maximum points. Now for that what you need to do, you need to identify whether f dash x, f double dash x up to f n x exist and then f dash x, f double dash x up to f n minus 1 x equal to 0. And the last differentiation that is the nth differentiation must not be equal to 0, then the following conclusion can be made. So if the last differentiation that is the nth differentiation of the variable x, if it is not equal to 0, that means it is either positive or negative. If it is positive, then you are leading to a minimum point and if it is negative, then you are leading to a maximum point. However, if n is odd, if however the n is odd and then x is a point of inflation, okay, and that is very important because for even differentiation you can get maximum minimum, but when n is odd, then the point x is leading to an inflation. Let us consider one simple example:

$f(x) = x^5 - 5x^4 + 5x^3 + 5 \ldots (4.2)$

$f'(x) = 5x^4 - 20x^3 + 15x^2 = 5x^2(x - 1)(x - 3) \ldots (4.3)$

Above equation implies that $f'(x) = 0$ at $x = 0$, $x = 1$ and $x = 3$.

Now, $f''(x) = 20x^3 - 60x^2 + 30x \ldots (4.4)$

At $x = 0$, $f''(x) = 0 \ldots (4.5)$

Therefore, to determine the nature of the point, we have to find $f'''(x)$:

$f'''(x) = 60x^2 - 120x + 30 \ldots (4.6)$

Now, $f'''(x)$ @ x=0; $= 30 \neq 0$

Hence, x = 0 is a point of inflection.
At x = 1, f′′(x) = 20 − 60 + 30 = −10 < 0. Hence, x = 1 is a relative maximum point.
At x = 3, f′′(x) = 540 − 360 + 30 = 210 > 0. Hence, x = 3 is a relative minimum point.

Now similarly, if you extend this problem to a multivariable, it is unconstrained but multivariable.

Instead of one variable, you can have multivariables. I will just tell you what is the importance. You will be surprised to see it is a course on power system, economic operation, control system and we are talking about optimization. My dear friends, when you try to solve a problem, there are thousands of generators and those generators, you need to find the optimal power output of each generator, all right.

And they are P1, P2, P3 up to PN. So it becomes a multivariable problem, okay. So you cannot optimize one generator and say I have achieved optimized solutions. No. You have to optimize not only P1 but you have to also optimize P2, P3 up to PN and hence all the power system optimization problems are constrained multivariable problems, constrained as well as multivariable. So now we are focusing on unconstrained multivariable.

So similar to unconstrained single variable, you can solve the problem multivariable where you have to take the partial differentiation dou f of x upon dou x1 and dou f of x upon dou x2 as well as dou f of x upon dou xn need to be made equal to 0, okay. So it is a partial differentiation concept and you will also try to understand the sufficient condition for minimum or maximum is similar to what we have discussed. Now it says that assuming all the partial derivatives of f of x up to the order k to be existing and continuous. So in the neighborhood of a stationary point x, if my d of f of x, d of f of x equal to 0, all right, but d of f of x is not equal to 0, so this is my rth differentiation and this is my kth differentiation, all right. Similar to the first and second differences, this is the rth differences and the kth differences, then the following holds good.

What it says:

- If k is even, $\mathbf{x}^*$ is a relative minimum if $d^k f(\mathbf{x}^*)$ is positive.
- If k is even, $\mathbf{x}^*$ is a relative maximum if $d^k f(\mathbf{x}^*)$ is negative.
- If $d^k f(\mathbf{x}^*)$ is zero, no general conclusion can be made.
- If k is odd, $\mathbf{x}^*$ is a not an extremum.
- If $d^k f(\mathbf{x}^*)$ takes both positive and negative values, then $\mathbf{x}^*$ is a saddle point.

Now when you talk about constraint optimization, I want your attention here very seriously because all our problems in power system that we are going to discuss are constraint optimization problems. So the one important part is method of Lagrangian multipliers being used to handle constraint optimization I will tell you what is constraint optimization, how it has been handled. Now let us say I do have a function f of x which is known to me and then I do have some constraints, okay. So f of x is my function, objective function and whereas actually constraints, let us say they are actually, you know, kind of actually beta of x, okay. So this is objective function and this is my constraint. Now when you optimize, you need to have a single function which need to be differentiated.

So what you ideally do, this constraint function beta of x being merged with f of x through a multiplier. So you have the objective function f of x plus a multiplier times beta of x, okay, become a objective function, all right. So then you are accommodating those constraint into your objective function through a multiplier and mostly it is known as Lagrangian multiplier, okay. So let us concentrate on a problem of minimizing a function of two variables x1 and x2 with one equality constraint. We will then generalize the method for any number of variables, okay, to understand first actually two variables and one constraint. So what are those variables x1 and x2? So what I am supposed to do, I am supposed to minimize a function:

Minimize $f(x_1, x_2)$ ...(4.10)

Subject to $g(x_1, x_2) = 0$ ...(4.11)

A necessary condition for (x_1^*, x_2^*) to be extremum is that:

$$\left.\frac{\partial f}{\partial x_1}\right|_{(x_1^*,x_2^*)} = \left.\frac{\partial f}{\partial x_2}\right|_{(x_1^*,x_2^*)} = 0 \qquad ...(4.12)$$

Hence, the first differential of $f(x_1, x_2)$ at (x_1^*, x_2^*) is,

$$\partial f = \left.\left(\frac{\partial f}{\partial x_1}\, dx_1 + \frac{\partial f}{\partial x_2}\, dx_2\right)\right|_{(x_1^*,x_2^*)} = 0 \qquad \ldots(4.13)$$

The constraint $g(x_1, x_2)$=0 is always maintained. Hence,

$$\partial g = \left.\left(\frac{\partial g}{\partial x_1}\, dx_1 + \frac{\partial g}{\partial x_2}\, dx_2\right)\right|_{(x_1^*,x_2^*)} = 0 \qquad \ldots(4.14)$$

Assuming $\frac{\partial g}{\partial x_2} \neq 0$, can be expressed as,

$$\left\{\frac{\partial f}{\partial x_1} - \left(\frac{\partial g}{\partial x_1} / \frac{\partial g}{\partial x_2}\right)\frac{\partial f}{\partial x_2}\right\} dx_1 = 0 \qquad \ldots(4.15)$$

Since dx_1can be chosen arbitrarily, one gets following after rearranging

$$\left\{\frac{\partial f}{\partial x_1} - \left(\frac{\partial f}{\partial x_2} / \frac{\partial g}{\partial x_2}\right)\frac{\partial g}{\partial x_1}\right\} = 0 \qquad \ldots(4.16)$$

So, eq 4.16 is a very important expression for two variable constraint optimizing equation. So what we will do, do objective function with respect to x1 variable plus lambda time, the constraint with respect to x2 variable now equal to 0. So finally, one can also rewrite that the equations:

A quantity λ is now defined as,

$$\lambda = -\left.\frac{\frac{\partial f}{\partial x_2}}{\frac{\partial g}{\partial x_2}}\right|_{(x_1^*,x_2^*)} \qquad \ldots(4.17)$$

Using above in (1.25),

$$\left.\left(\frac{\partial g}{\partial x_1} + \lambda \frac{\partial g}{\partial x_1}\right)\right|_{(x_1^*,x_2^*)} = 0 \qquad \ldots(4.18)$$

Again,(1.26) can be written as,

$$\left(\frac{\partial f}{\partial x_1} + \lambda \frac{\partial g}{\partial x_2}\right)\Big|_{(x_1^*, x_2^*)} = 0 \qquad \ldots(4.19)$$

Also,

$$g(x_1^*, x_2^*) = 0 \qquad \ldots(4.20)$$

Equations (1.27) to (1.29) represent the necessary conditions for (x_1^*, x_2^*) to be an extremum. The quantity λ is called the Lagrange multiplier. Following function is called the Lagrange multiplier.

$$L(x_1, x_2, \lambda) = f(x_1, x_2) + \lambda\, g(x_1, x_2) \qquad \ldots(4.21)$$

Now if you talk about a simple unconstrained optimization problem:

Minimize: $f(x_1,x_2)=0.25x_1^2 + x_2^2$

$$\frac{\partial f}{\partial x_1} = 0 \text{ and } \frac{\partial f}{\partial x_2} = 0$$

Gradient of f is defined as: $\begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \nabla f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Optimum:

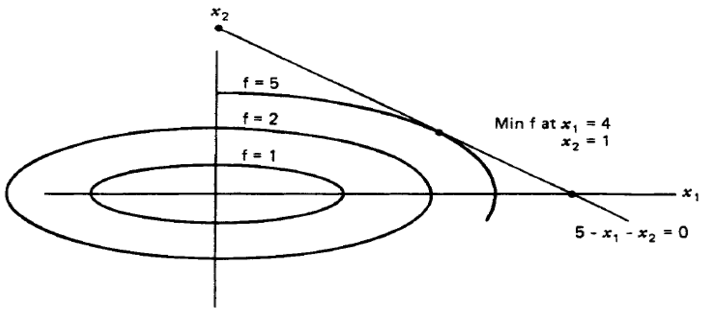
$$x_1 = 0, x_2 = 0$$

Now, Minimize within a linear equality constraints:

Minimize: $f(x_1,x_2) = 0.25x_1^2 + x_2^2$

Subject to the constraint: $\omega(x_1,x_2)=0$

Where: $\omega(x_1,x_2) = 5 - x_1 - x_2$



Therefore, equality constraint simple optimization one numerical example let us solve.

Therefore, in the graphical method we saw how the solutions are at lying at 4 and 1 but now let us solve through Lagrangian multiply. So what has happened what is my objective function:

$L(x_1, x_2, \lambda)=0.25x_1^2 + x_2^2 + \lambda(5 - x_1 - x_2)$

$\frac{\partial L}{\partial x_1} = 0.5x_1 - \lambda=0$

$$\frac{\partial L}{\partial x_2} = 2x_2 - \lambda=0$$

$$\frac{\partial L}{\partial \lambda} = 5 - x_1 - x_2=0$$

$$x_1 = 4$$

$$x_2 = 1$$

$$\lambda = 2$$

So there are three differential values. So probably you could see either use Lagrangian multiplier or even you from this equation linear equality constraint equations you could see that they both are satisfied. So but in power system we will try to solve most of our equations through this equation. I will just try to emphasize little bit here in power system engineering what happens this x1 and x2 are nothing but majorly my variables P1 and P2. P1 and P2 are nothing but the power output of the plant 1 and 2 and it can go as maximum as Pm and what are the constraints? Constraint means for example P1 plus P2 for example equal to 300 megawatt. So I want to meet the total power of 300 megawatt out of two generators. So then you can form this equation that P1 plus P2 minus 300 equal to 0. So that will appear in this zone and the objective function is nothing but my cost characteristics which are nonlinear. So they are ideally AP square plus BP plus C and similarly it could be P1 square BP plus C or sometime it is P2 square plus BP2 plus C. So there are nonlinear equations will come in this zone. So you have cost characteristics of the power plant to generate P amount of power how much money you have to spend. So my objective function will be now the summation of the cost characteristics of cost functions plus lambda time the constraint and that will help me to understand what is P1. So once the objective function are known to me, the constraints are known to me, I can easily calculate the value of power output of each and every power plant and that's why it is important for you to understand what is simple optimization with equality constraint with multi variables. All right then you can have many more complicated problems. This is just a journey for all of you and additional equality constraints which is defined as:

Minimize: $f(x_1, x_2)$

Subject to: $\omega_1(x_1, x_2) = 0$

$$\omega_2\,(x_1, x_2) = 0$$

$$\omega_3\,(x_1, x_2) = 0$$

L=$f(x_1, x_2) + \lambda_1\omega_1(x_1, x_2) + \lambda_2\omega_2(x_1, x_2) + \lambda_3\omega_3(x_1, x_2)$

$$\nabla f + \lambda_1\nabla\omega_1 + \lambda_2\nabla\omega_2 + \lambda_3\nabla\omega_3 = 0$$

Solution:

$$\frac{\partial L}{\partial x_1} = 0, \frac{\partial L}{\partial x_2} = 0$$

$$\frac{\partial L}{\partial \lambda_1} = 0, \frac{\partial L}{\partial \lambda_2} = 0\,, \frac{\partial L}{\partial \lambda_3} = 0$$

So I request all of you to get into mathematical optimization sections or any optimization textbooks, linear optimization, nonlinear optimization you can focus on to know much better on the subjects. So this is how the inequality constraints are being satisfied and then what is the inequality constraint. So inequality constraint means you could see now previously used to say gi x which is equal to 0 but now we are saying that gi x is less than or equal to 0 and then how do you handle it. For example if I say my mVA flow in a particular line is less than or equal to 100 mVA, this is the line i. So this is some sort of my inequality constraint that also can be incorporated in our optimizing problems. So then we talk about good to good conditions and then there are different applications of those equality constraints and this is how the problem when we talk about inequality.

So you could see that this is my objective function and there is an equality constraint as well as there is an inequality constraint. So how do you optimize a function which has the equality constraint as well as inequality constraint. So you could see the functions which is my equality constraint characteristics and then I have the inequality cost function characteristics and then I am probably satisfying those both I have to find my optimal points. So you can take one numerical example. So what you have done for example if this is my objective function and this is my equality constraint, it is the inequality constraint so instead of lambda I have added one more variable that is mu.

$$\mathcal{L} = f(x_1, x_2) + \lambda[\omega(x_1, x_2)] + \mu[g(x_1, x_2)]$$
$$= 0.25x_1^2 + x_2^2 + \lambda(5 - x_1 - x_2) + \mu(x_1 + 0.2x_2 - 3)$$

The first condition gives

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0.5x_1 - \lambda + \mu = 0$$
$$\frac{\partial \mathcal{L}}{\partial x_2} = 2x_2 - \lambda + 0.2\mu = 0$$

The second condition gives

$$5 - x_1 - x_2 = 0$$

The third condition gives

$$x_1 + 0.2x_2 - 3 \leq 0$$

The fourth condition gives

$$\mu(x_1 + 0.2x_2 - 3) = 0$$
$$\mu \geq 0$$

So here the solution, optimal solutions are not easy as similar to your equality constraints. So when you get into inequality so there is a iterative method through which you will get your value of x1, x2, lambda and I strongly recommend all of you to get into wonderful textbooks and the examples of both in Wollenberg and some other optimizing textbook to understand both equality and inequality but nevertheless I tried my best to make you understand how to obtain those solutions when the nonlinear optimization with equality constraints as well as inequality constraints. Thank you very much.