Economic Operation and Control of Power System

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Week - 05

Lecture – 24

So, good morning and welcome you all for the NPTEL online course on Economic Operation and Control of Power Systems. So, today we continue our study with respect to hydrothermal scheduling. So, some of the constraints also need to be considered that I am going to discuss now while we plan for hydrothermal or even thermal scheduling as well. Like fuel scheduling is also one of the important aspects. The major elements in the fuel supply and delivery system like there are the first thing is raw fuel suppliers, it could be coal, oil and gas companies and there will be long term contracts with minimum and maximum limits on fuel per time period. And then prices may change subject to the renegotiation provisions in contract because over a period of time before the contract beginning you might have fixed some price like we have discussed about take and pay policy.

So there is an agreement which is been which has been made for bare minimum usage for specific duration you have to use it at any way. And even if you do not use it you have to you know bare the minimum cost because he has hold his stock for your purpose. So, he is not delivering to someone else. So but over a period of time due to global market scenario the price may increase, oil price, gas price that may increase and because of which the renegotiation has to be done.

And transportation, railroads like not only the fuel suppliers have their constraint even transportation like from supplier to the end delivery there would be various means of transportation like railroads, river barges, gas pipelines companies and they also play a crucial role and represent problems in scheduling of fuel deliveries. Let us say there is some strike in a transportation company. So you will not be able to deliver though the supply is there and demand needs supply. So there is no possibility where you can supply. So this is another constraint and fuel scheduling.

Let us say the inventory is also one of the important aspects. Coal piles like you store in terms of coal piles, oil storage tanks, underground gas storage facilities at the premises where you are generating. Now keeping proper inventory levels to forestall fuel shortages. You should have proper inventory storage facility such that even in case of such kind of issues with the transportation or with the supply chain somehow or the other you should able to cater to the need and the load levels may also exceed beyond the forecast. You should have sufficient forecast, sufficient inventory available with you and suppliers or shapers unable to deliver due to any reasons.

And also generation stations need to be you know maintained well so that they can able to participate in unit commitment. Now fuel scheduling problem it has the total time period is broken into discrete time increments. Any fuel scheduling problem is solved like this and the constraint functions there could be some other constraint functions which will span for two or more time steps like there are discrete time steps, intervals we say. So for each there could be some constraints which may also seen for multiple time steps like ramp rate, ramp rate you cannot suddenly increase or decrease. It may this constraint may foreseen for three four time steps together.

So how do you solve the solution techniques? We can use linear programming as well as dynamic programming. So procedure that minimizes the linear objective function. Variables are also subject to linear constraints. Any non-linear function or objective or constraint function, constraint equations must be approximated by linearization about an operating point or by piecewise linear function. So non-linear function is divided into multiple pieces and then you make it as a linear function look like a linear function and then you carry out this linear optimization, linear programming approach.

So let us try to solve the operation scheduling with a linear program, considering the fuel constraint. Now this is what the data shows show up that there are two coal burning generating units. I am not speaking about hydro as of now. Both must remain online for a three week period. The combined output is to supply the following route.

Week 1 is 1200 megawatt, week 2 is 1500 and week 3 is 800 megawatt. One coal supplier is under contract to supply 40,000 tons per week to be split between the two plants. There are two plants and there is a contract take and pay agreement has been signed where this coal supplier is supposed to deliver 40,000 tons per week. And there are existing inventories at the start of the three week period. So there is also some inventory available with these generating units.

Now what is the objective to find the operation schedule for each plant for each week and the coal delivery amounts to be made each week. What is the amount of coal delivery that need to be done and what is the operation schedule and there is further some more data. This is the heat value 23 MBTU per ton, cost is 30 dollar per ton and then inventories that I have already told for three weeks the plant 1 has already a stock available of 70,000 tons similarly with the plant 2. And each plant has a maximum coal storage of 2 Lakh tons. It is not that you know you can have infinite amount of storage.

So they have the maximum storage up to 2 Lactons. The generator 1 has this minimum and maximum generation capacity 150 megawatt and 600 megawatt and this is what the linear approximation look like. So H1 of P1 is 380 plus 8.267 P1 and the Q here represents the fuel consumed basically fuel consumed and this is a function of the generation output P1 and this you from this expression you just divided by 23 MBTU per ton you will get this expression. And then you multiply this expression I will just tell H1 of P1 divided by 23 will give you this and then Q1 of P1 into 30 because 30 is the cost that will give you this expression.

And then similarly you do it for plant 2 which has a limit of 40, 400, and 1000 megawatt. Now you get Q2 of P2 and F2 of P2. Now assume that now we are going to the solution process assume that the units are operated at a constant rate during each week. Coal deliveries are made at the beginning of each week. The delivery should happen before the beginning of starting of the week.

The problem is set up for one week time interval. Each the interval considered is one week. Then plant 1 and plant 2 I am just obtaining Q1 of P1, F1 of P1, Q2 of P2, F2 of P2 for the entire week. That means you have already got for this is per hour. So the data that we have calculated this is per hour.

Q1 of P1, F1 of P1 these data are per hour. Now for per week this has to be multiplied Q1 of P1, F1 of P1, Q2 of P2, F2 of P2 should be multiplied by 168 which is something but 24 hours into 7 days. So you get this data. Now what is objective function? The objective function is to minimize. What to minimize? The total fuel consumption for the two units for all the three weeks.

This is fuel consumption for the generator 1 for the first week. Fuel consumption for the generator 2 for the first week. So this is for week 1 and this is for week 2 and then this is for week 3 for all generators 1 and 2 for 1, 2 put together. I am just formulating some equality constraints. So power delivery P1 of 1, P2 of 1 try to be very familiar with the terminologies here.

P here 1 represents the subscript 1 and 2 represents the generator 1 and 2. The big number which is there in within the bracket represents the week here. So P1 of 1 plus P2 of 1 is equal to 1200 megawatt. Here this is 2, this is not 1, this is 2. So P2 of 2 plus P2 of 2, P1 of 2 plus P2 of 2 is equal to 1500 megawatt.

Similarly P1 of 3 plus P2 of 3 is equal to 800 megawatt. This is the generation should be equal to the load. And then correspondingly the coal deliverable, coal deliveries you can see here. This is the fuel consumption or the fuel requirement of the generator 1 for the week 1 and for coal input for generator 2 for week 1. Put together it should be 40000.

Similarly for the week 2, week 3 this is the same number. And similarly there is a volume of coal. You see here. See this is the volume of generator 1 for the week 1. At the end of week 1 before going to week 2 what is the total volume of coal available for the generator 1.

That is what we are calculating here. You see here the total amount of volume that you have at beginning of week 2 is whatever the volume you had before the beginning of the week that week plus whatever the incoming inventory to the inventory stock minus what is the consumed whatever you had plus whatever is the incoming input coal and whatever is the consumed rest whatever is left out that is the source for the next week. Similarly that you do it for generator 2 for week 1 and generator 1 and 2 for week 1 and 2, 3 respectively. Ultimately you will get for week 4 also because this is for the next week. Though we are not interested but ultimately we will have some coal left out for the next week schedule.

So this is what is being placed here. Now what we have done is there are so many constraints and there are so many weeks and all of them put together in one single chart. So you could get the complete constraint list and the generation, load demand, variables all these things. For example, let us try to walk through this table. Let us say this is in the left hand side you can see there is constraints.

There is total 12 constraints put together. You can see here what we have done is these are the you know different variables like D1 of 1 as I have already told the coal input to generator 1 for week 1 and coal input generator 2 for week 1. These are all D represents the coal input for all the generators. And there is another thing which is the generation P1 of 1, P2 of 1 these are all generators power output. P1 of 3, P2 of 3 that depends upon the load.

And similarly you have the volume of total volume of coal that is left out for the as input to the next week. So this also is there. Now you can see here what it indicates first constraint P1 of 1 this is P1 of 1 plus P2 of 1 should be equal to what? This is 1200 megawatt. This is the load demand. Now we are whatever is there we are just putting in a same table nothing else.

And next you see the second constraint what it indicates it is D1 of 1 plus D2 of 1 should be equal to 40000 delivery. Similarly you see the third one third constraint D1 of 1 I will just write it down D1 of 1 minus 60.4 P1 of 1 and what else is there minus V1 of 2. That is equal to the third one which is 277.4 minus V1 of 1. How do you get this number? You can see here this one Q1 of P1 because based on this expression ultimately you need to replace for Q1 of 1. So minus Q1 of 1 means minus 2775.4 minus 60.4 P1. So that if you just replace to that expression then you will get what is nothing but V1 of 2.

That is the thing. So similarly you do it for all the constraints and similarly in the downside also you can see here. So there is also variable minimum and variable maximum given that means for D1 of 1 the minimum is 0 maximum is 40000 and for P1 of 1 minimum is 150 maximum is 600. Like that for each variable there is a minimum maximum given also. Now this is a complete table and once you have this kind of table you can solve this operation schedule by simply using a MATLAB program. So you need not have to work out the readymade function is already there in the MATLAB.

You can solve and have a feel like how this looks like and you can also try to solve it on your pen and paper to have that feel that how this program is working. Fuel scheduling by linear programming. This linear programming can be solved in MATLAB. This is just a function that you need to type in the MATLAB command window.

Fuel scheduling by LP

This Linear Programming problem can be solved in MATLAB

- $x = linprog\,(f, A, b, A_{eq}, b_{eq}, lb, ub, options)$

  minimizes with the optimization options specified by input arguments.

Where, f = Objective function (Minimize this)

$$Ax \leq b$$
$$A_{eq}x = b_{eq}$$
$$lb \leq x \leq ub$$

Linear programming of these are the different objective functions and the constraints.

So this minimizes the optimization options specified by input arguments. These are the input arguments. So where F is what is F indicates the objective function minimize the fuel cost and Ax here you can see here A and B represents the inequality constraints. That means the total consumption should be less than or equal to 40000 or whatever. And then A equivalent of x is equal to B equivalent indicates the load demand P1 plus P2 put together for a specific week it could be whatever amount of load.

And then there is also limits with respect to the generation. It should not the generation should be less than 150 it should be greater than 150 and less than 600 for example. So all these constraints that is what is been put up as the input argument. Then it will solve and give you the ultimate dispatch.

Next we will move on to dynamic programming. Hydrothermal scheduling with dynamic programming. A single hydro plant is operated in conjunction with a thermal system with storage. Now we discussed only about thermal. Now we discuss about thermal plus hydro by using dynamic programming approach.

It is serving a single series of loads PL. Time intervals are denoted by J where J runs between 1 and J max. So the time intervals are J and the load is termed as PL. So there are so many notations that we need to understand:

r_j =net inflow rate

V_j =storage volume at the end of period j

q_j =flow rate through the turbine

P_{Hj} =power output

s_j =spillage rate

P_{Sj} =steam-plant output

$P_{load,j}$ =load level

F_j =fuel cost rate

- the rate of flow through the hydro-unit during interval j is

$$q_j = \frac{(V_{j-1} - V_j)}{n_j} + r_j$$

Let us say QJ is zero. Then there will not be any power output because there is the input for the turbine and SJ is spillage rate because when water is flowing, high flow is there, there could be some water which is outflowing above the reservoir.

I have discussed this in the beginning. PSJ is steam plant output. Similarly, there is a power output from the steam plant. And PLoad J is load level. FJ is fuel cost rate.

This fuel cost rate for the thermal power plant. Now the rate of flow through the hydro unit during interval J is QJ is the rate of flow through the hydro unit. VJ-1-VJ divided by NJ plus RJ. That means what it is indicating. So this is, now we are interested to calculate the outflow at the end of the Jth interval. Now what is happening? The whatever is the volume was present before this interval.

Whatever is the volume of water at the end of this interval. So you will get the net total amount of volume. NJ indicates total number of hour. So per hour we are finding out what is the outflow. So this indicates per hour what is the total accumulation of water even though there is inflow and outflow within the reservoir.

There is a net accumulation of water plus this indicates the inflow. That means let us say this, let us try to understand by keeping one constant and varying other. Let us say your total volume, the net volume is zero. For example, I am saying this term is zero.

This entire term is zero. That means at this interval, this specific interval there is no net accumulation of water that is taking place. That means what is happening? QJ is equal to RJ. That means if there is no net accumulation what is happening? Whatever is the inflow that is outflow. Before the starting of the interval whatever was the water capacity still the same amount of volume of water is present. And let us try to understand that if this is kept constant, this is kept constant.

This I have already discussed. This is condition 1. Then let us try to understand that this is kept constant. RJ is kept constant. VJ minus 1 minus VJ by NJ is constant.

This is case 1. Now let us say the volume is not constant. This is not constant. This is increasing. The volume is increasing and the QJ is kept constant. If QJ is kept constant means what? I will just take another colour here.

If say QJ is constant. That means what? That is RJ. RJ is there is net inflow that is anyway is there. If I have based on RJ, if QJ has to be kept constant, this that means this plus this should be increasing. If RJ is increasing, then simultaneously what is happening? The outflow is kept constant. We are not increasing the outflow.

So simultaneously the volume also increases. The volume also would increase. Or you can say here QJ is kept constant. We are speaking about QJ is kept constant means this total volume VJ minus V1 minus VJ by NJ should be decreasing because something is increasing, this should be decreasing. But whether this can practically happen, that is the question.

Output is kept constant. Volume, total volume, net volume is decreasing where the incoming is happening. Whether this is possible? There could be one possibility but it depends if there is evaporation or spillage, whatever where it is not leading to any power output. You understand? But this is very extreme case but mathematically we can analyse and there could be some loss of water which is being catered with the inflow.

So this is another thing. But this scenario is very rare to occur. This is very rare scenario. And what is the other possibility? Now we kept VJ minus VJ1 constant. Next we kept QJ as constant. Another thing is RJ is kept constant. Then what is happening? There is a whatever was the rate of inflow, inflow rate of flow of water will be always constant that is what we have considered here.

If RJ is kept constant then whatever is the QJ based on the proportionally the volume will also is related. So anyway, so both initial and final volumes are 10,000 acre feet. This is the condition. The storage volume limits are 6000 and 18,000 acre feet.

► During any period, the discharge rate through the hydro-unit is

$$q_j = \frac{(V_{j-1} - V_j)}{4} + 1000$$

► The discharge rate must be nonnegative and not greater than 2260 acre-ft/h.

► States: The storage volume steps at 6,000, 8,000, 10,000, ..., 18,000 AF.

► Stages: Number of intervals.

► Forward DP:

$TC_k(j)$:The total cost from the start of the scheduling period to the end of the period j for the reservoir storage state V_k

The natural inflow is 1000 acre feet per hour. Now the scheduling problem to be examined is for a 24 hour day with individual period taken as 4 hour each. So we are dividing it into 6 interval and for each interval the total load is 600 megawatt and inflow is 1000 acre feet per hour.

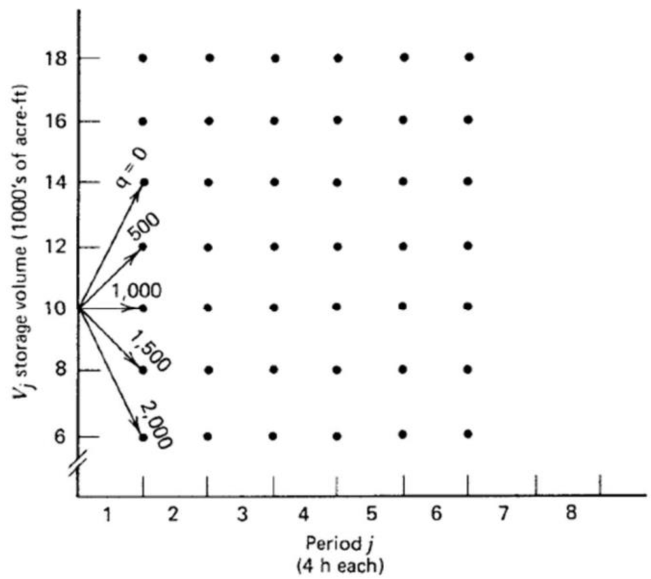

FIGURE 5.24 Initial trajectories for DP example.

This is constant you can see here in this problem. Now during any period the discharge rate through the hydro unit is VJ minus 1 minus VJ because NJ is 4 here, number of hours is 4 plus this is the inflow 1000 acre feet per hour.

So this is what the discharge rate of the hydro unit. Now the discharge rate must be non-negative and not greater than 2260 acre feet per hour. You can see this you can get this number from this expression. There is a Q here given. So you just put 200 to this and you will get what is the total. And then and now because dynamic programming I have already discussed there is states, stages.

So what are the states here? The storage volume steps at 6000, 8000, 10,000, 18,000 acre feet. These are the steps, different states. And stages is number of intervals. And we are trying to solve using forward dynamic programming.

So there is another term which is TCKJ. This indicates the total cost from the start of the scheduling period to the end of the period J for the reservoir storage state VK. Because this we need to minimize, total cost we need to minimize. For this example, we will limit our efforts to a 4 hour time steps. Right? Now there are, that is why you can see 1, 2, 3, 4, 5, 6.

In the X axis we have period or the intervals, stages. And the Y axis you have storage volume. So this we are starting from 6, 8, 10, 12, 14, 16, 18. So for this example, we will limit our efforts to 4 hour time steps and storage volume steps that are 2000 acre feet apart. You can see this difference of 2000 acre feet.

$j = 1$		$P_L(1) = 600\ MW$		$\{i\} = 10$
V_k	q	P_H	P_S	$TC_k(j)$ $
14	0	0	600	15040
12	500	24	576	14523
10	1000	74	526	13453
8	1500	124	476	12392
6	2000	174	426	11342

For the J is equal to 1 for the first interval. Right? Let us try to understand how we get this cost here for the first interval. So there are these possibilities. You are starting from 10,000 because that is the data given here. Both initial and final volumes are 10,000 acre feet.You are starting from 10,000 acre feet. Right? So starting from 10,000 acre feet, we are here. Okay. So this, let us take the first case.

14,000 acre feet. So you are reaching up to 14,000 acre feet, starting from 10,000. So what is the Q for the first condition? So Q is nothing but VJ minus VJ minus 1. Sorry, VJ minus 1, what is that expression? VJ minus 1 minus VJ by NJ plus RJ. Q is equal to VJ minus 1 by RJ plus by NJ plus RJ.

VJ minus 1 is 10,000 minus 14,000 divided by 4. What is the inflow rate? 1000. 1000. Right? What you will get? This is 0. This is 0. That is what you have got here. If outflow is 0, then what is the power output? 0.

That means for J is equal to 1 interval, the load demand is 600 megawatt. If hydro is not able to generate any power, then what is happening? Total power is being supplied by the thermal power plant. And what is the cost associated with thermal power plant? How do you calculate that? It is very simple. Go to this expression. Fuel cost characteristics is already being given.

To this expression, you just have to put PS is equal to, for the first case, 600 megawatt. Then you would get this number. 15040. This will also help you understand some aspect here. Let us say if there is an inflow, but outflow is 0, then there is a volume increase.

There is a, earlier it is starting from 10,000 because water cannot go anywhere. So 10,000 is the starting, before the starting of the interval, 10,000 was the amount of volume in terms of acre feet which was stored in the reservoir. Now at a constant rate of 1000 acre feet per hour, the water is being flown inside. And that is the reason at the end of that interval, 4 hour interval, you could see the water has raised from 10,000 to 14,000.

1000 into 4, 14,000, 4000. 10,000 plus 4, total amount of volume of water is 14,000. Water is not gone anywhere. That is being stored. That is the reason there is no net discharge from that reservoir and the hydro is not generating any power. Similarly, if you go to the next, you see here, that means 12.

If 12 is the case, then what is happening? Then your Q would be 500. Just put to this expression, 10,000 minus 12,000 by 4, this comes out to be minus 2000. What is it? 10,000 minus 12,000 by 4.

How much it is? Minus 500. Minus 500 plus 1000. You will get 500. This is what you get. So if 500 is the volume of water, based on that you can find out what is H, pH because there is already a linear expression given to you. You see here, there is a relation already established. So you know what is Q. You can see what is the pH. Now once pH is known, what is thermal power output? It is just PL minus pH apart from whatever hydro is generating.

So you would get this number, 576 megawatt. Based on that you put that to the thermal fuel characteristics, you would get the total cost from the thermal power plant. So ultimately what we are trying to do is, over a period of the intervals, we need to find out what is that optimum utilization of hydro that would result in the minimum cost of generation from thermal power plant because thermal and hydro are running parallely now. Now you can put up the thing here. You can see from this specific time interval, jth interval, you could see I can keep myself because it is in my control now.

How much water I want to discharge is as a owner, as a controller, it is in my control. I can keep as it is, sorry, as it is means 10,000. Earlier it was 10,000, now also it is 10,000. That means what whatever is there, inflow that is going as outflow, inflow is 1000, outflow is also 1000. Correct? Now, if that is the case, then hydro is generating 74 that is

134553, whether that is an economical one need not be. So then you have to see, okay, I will not discharge, let me store and I am getting some amount of power and cost from the thermal.

Then let us say I will discharge completely water, whatever, the minimum is 6000 that is what is given here 6000 to 18000. That is why we are considering these cases here. So 6000 we are seeing at the end of that interval. That means whatever is 1000 is coming, everything I am, see I am starting from 10,000. Now there is a 6000 that means there is an inflow also which I am discharging 4000, the 1000, inflow is 1000, that is also I am discharging and whatever I have stored earlier that is also I am using.

After four interval I am discharging that also partially whatever. So then that is leading to reduced cost. That means it is indicating if you keep on discharging hydro, that thermal power cost will naturally decrease because the power that the thermal power plant is generating is decreasing. But this is for, this is a picture of only first interval, jth interval. But if you go on to the next interval, the dynamics may change and you have to finally find out that path of volume of water usage for a hydro power plant that will result in reduction in total power cost.

So this is how it goes. For j is equal to 2, you would come out with this kind of analysis. I will not go in detail. So somebody can work out and find out using dynamic programming what is the cost associated with the jth interval, accumulating the total cost. You remember how to solve for dynamic programming using forward dynamic approach. And then for j is equal to 6, at the end of j is equal to 6, the picture looks like something like this.

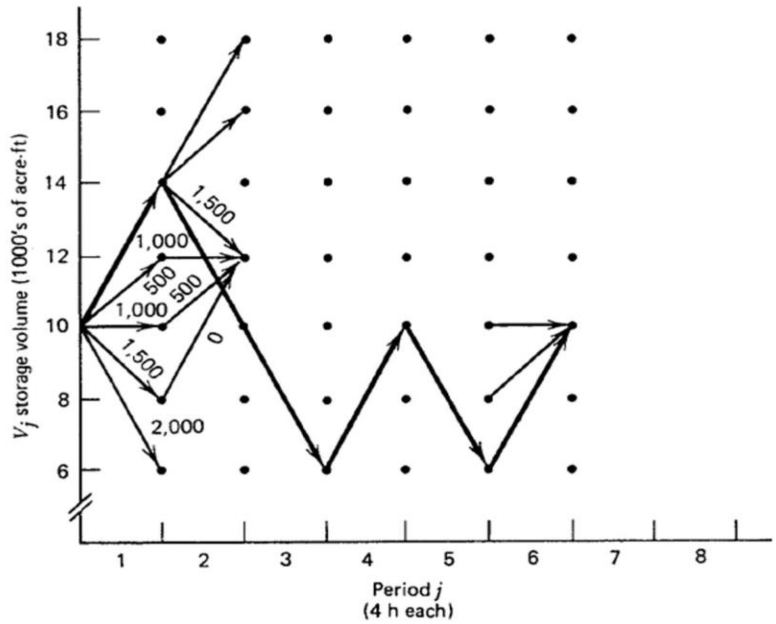

FIGURE 5.26 Final trajectory for hydrothermal scheduling example.

So from 10 to 14 and then 6, then this. So this is the path that it is showing. So you start with 10, go to 14 and then it is not saying starting from 10 go to 6. You understand that is not giving you the complete overall cost. So from 10 to go to 14, then 14, this is the total path, right? So from 10 to 14 and then you asking it to come to 6, then 10, then 6, something like this.

So this will give the complete picture. This is what we are ultimately interested in. So with this we will complete, we have completed hydrothermal scheduling. So next class we will discuss further topics. Thank you very much.