Economic Operation and Control of Power System

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Lecture – 23

Hello and good morning everyone, welcome you all for the NPTEL online course on Economic Operation and Control of Power System. In today's lecture we will continue our discussion with the hydro thermal scheduling. Some of the basic constraints that we need to consider. So let us say economic operation with limited energy supply. So in the previous cases of our economic dispatch we have not considered the constraint with respect to the fuel input from the fuel side source. We assume that fuel is anyway available and we used to you know dispatch our available sources.

Now that is case number 1. There is no limitation on fuel supply where we used to carry out the economic dispatch using only the present condition as the data, right. Now case 2 is energy resources available at a particular plant is a limiting factor in operations. So the economic dispatch calculation calculations must account for what has happened before and what will happen in the future.

That means what is the fuel constraint as such. The fuel constraint include the limited fuel supplies, fixed cost fuels, take or pay contracts and surplus fuels and all these extremities. So we will just try to explore these aspects. Now there is a term called take or pay fuel supply contract. What it means? Now the utility agrees to use a contracted minimum amount of fuel during a period that is called as take.

That means now there is agreement between the fuel supplier and you know the owner of a generation company. So they would have a they would have an agreement among themselves that this is the bare minimum amount of fuel that I am going to purchase from you for this amount this duration that you have to abide with that rule. And he also agrees to pay the minimum charge in order to purchase at a bulk price. That means this much amount of fuel I will purchase at this much minimum price. So then there is agreement that is take or pay agreement.

Now if the utility fails to use the minimum amount, it agrees to pay the minimum charge for the minimum amount. Let us say if I do not I have already made a con you know agreement or contract. Let us say some x amount of fuel that I am going to purchase. Even though if I do not purchase from you whatever x amount of fuel cost you to that minimum amount of price that is already been decided in an agreement that I have to pay. So the better thing is at least use that bare minimum amount.

Anyway you are paying the money. So you have to use that. Now while this unit's cumulative fuel consumption is below the minimum, the system excluding this unit should be scheduled to minimize the total fuel cost. Let us say there are in a set of there is a set of generators N plus let us say M where N number of generators are having no constraints on the fuel intake, no constraint. Where M number of generators have there is a constraint with the fuel intake, there is a constraint.

Now what it says is this unit's cumulative fuel consumption is below the minimum. The system excluding this unit should be scheduled to minimize the total fuel cost. That means you exclude this constraint, constraint generators where the fuel constraint is there and then you have the minimum cost associated, carried out the what is the bare minimum cost that the rest of the generators can operate actually. Subject to the constraint that the total fuel consumption for the period for this particular unit is equal to the specified amount. So that constraint for this generator whatever bare minimum amount that you need to bare that you have to meet out.

Now once the specified minimum amount of fuel has been used, the unit should be scheduled normally. So this amount of fuel that you need to bare then you dispatch for the rest of the generators. As a simplification let the maximum fuel consumption be equal to the minimum amount. To start with let the maximum fuel constraint fuel consumption be equal to the bare minimum amount that you have already made a agreement or contract. Now let us consider a special case where the minimum amount of same thing where the minimum amount of fuel consumption is also the maximum amount.

The system is shown below. Now that is what I told. From 1 to n there are generators where there is no fuel constraint and there is only one generator which has a fuel constraint. So now you need to optimize this total generation of n number of generations plus a specific generator there is a fuel constraint that means n plus 1 generators. The objective function is to minimize the cost, overall cost such that you will meet out summation of generation is equal to load demand.

Now consider a system with n classical thermal units and one turbine generator fuel under a take or pay agreement. Now this is under take or pay agreement that means there is agreement made. Now consider the operation of the system over j max time intervals. The time intervals is j max. Now let us try to define our Lagrangian function with some constraints.

Now the objective function is from j is equal to 1 to j max. This is the interval. You need to minimize. What you need to minimize? The total fuel cost. What is nj suggest? The number of hours in interval j.

$$min \sum_{j=1}^{J_{max}} \left(n_j \sum_{i=1}^{N} F_{ij} \right) + \sum_{j=1}^{J_{max}} n_j F_{Tj}$$

$$\varphi = \sum_{j=1}^{J_{max}} n_j q_{Tj} - q_{total} = 0 \ , \quad \psi_j = P_{load,j} - \sum_{i=1}^{N} P_{ij} - P_{Tj} = 0$$

This is the number of hours in the interval j. So this is the interval j is equal to 1 to max. Let us say if there are from 0 to 24 hours, let us take that you have 6 intervals. That means there will be 4 hours per interval.

4 into 6 is 24. This nj include 4 if j max is 6. So for this total number of generators, their fuel cost that need to be minimized for each interval plus this aspect is corresponding to the, this part of the objective function is with respect to the generators where there is no fuel constraint. Now here j is equal to 1 to j max nj fTj. I am just separating out both of them. Subject to there is a constraint with respect to the total fuel consumption. You see here, this is with respect to fTj only. That is why I am mentioning here. You need to pay more attention with respect to the terminologies here. I am saying qTj. qTj is that specific generator where there is a take or pay agreement is been formulated.

For the total duration nj, j is equal to 1 to j max nj, that means 24 hours. The total fuel consumption should not exceed a specific amount. Let us say 40, I am just have a agreement, 40 volumes of fuel, something, some unit. So I should not increase that amount of fuel consumption and there is also a bare minimum amount that I am also taking from them for which I am giving bare minimum amount of price. And this constraint function is anyway very much known to all of us where total load is equal to summation of generation of constraint generator and the non-constraint generator put together is equal to 0.

Just you can see here Pij, Pij indicates unit i power output at time j, qTj is fuel input for unit t at time j and qPload is total load at time j, at jth time and Fij is cost for unit i for interval j, FTj is cost for unit of unit t for interval j and nj is number of hours in interval j. And now ignoring for the moment that the generator limits, that the generator limits the term j is equal to 1 to j max, nj FTj is a constant, because the total fuel to be used at plant T is fixed. For time being, let us assume that this term is constant because anyway total amount of fuel that you use is constant, is fixed. Now the total fuel cost of that fuel is constant and is dropped from the Lagrangian function, it is not a variable anyway. Now what you get? Now summation of j is equal to 1 to j max, nj this is the cost function of the total generators that need to be minimized and there is a Lagrangian multiplier here.

Lagrangian multiplier with respect to the equality constraint load demand is equal to generation. Always the Lagrangian multiplier is with respect to this constraint, summation of generation is equal to load. Now there is a new constraint which is coming up, that means this one. The total amount of fuel consumed should not exceed the total as per the agreement, total consumption as per the agreement put together all that utilization in the entire time interval. So j is equal to 1 to j max, nj QTj minus Q total and this is been taken care by this gradient gamma.

Now this constraint is met with respect to lambda j and this constraint is been taken care by gamma. Now there are two variables lambda and gamma. Now the independent variables are the powers Pij and Ptj for any given time period j is equal to k, now you obtain partial differential equation. Lagrangian function with respect to Ik, so there are two generators, one is constraint generator and unconstraint generator. Dou L by dou Pik is equal to 0, that means now you do the partial differential equation for this expression.

So what you would get, nk into Df Ik by Pik, that means j is equal to k means for that specific interval, j is equal to k for the ith generator, nk into Df Ik by Dp Ik minus lambda. Now this is constant, this is eliminated and this is also constant with respect to Pik, this is a constant. So this is eliminated, only this will be left out. So minus lambda k, this is equal to 0. This is applicable for all the generators 1 to n.

And now you differentiate this Lagrangian function with respect to Ptk, this is nothing but now gamma factor will not be there. So gamma into nk because j is equal to k now, gamma into nk Qt, you replace j with k, Qtk by d Ptk minus here, you see here now this is a constant, now this is a constant. These two are constants now. Minus lambda k into Ptk or lambda k into d Ptk by d Ptk by d Ptk and then this is 1. Now you will get minus lambda k only.

Now you get these two expressions. Now gamma is referred to as a pseudo price or a shadow price. This is not actual price, just try to help you to minimize the overall cost while considering the constraint. Now it revalues the fuel price of a limited fuel supply for economic dispatching, means without gamma that means there is no fuel constraint, there is only lambda available. Now having gamma in place where the fuel constraint is being put into picture, now you need to reevaluate the gamma or total cost of the generation. Now discrete load patterns have been considered.

Solving the fuel limited dispatch requires dividing the pattern into intervals. So these are the intervals. Let us try to understand gamma search method which is helpful for solving the limited fuel dispatch problem. Now to start with, let us assume one specific gamma. Now there are two variables, lambda and gamma.

It is like if you remember dual function, we optimized one and then solved another one, then again optimized the second one like that. Now let us keep one as constant. Let us try to solve the gamma, lambda. Now gamma is taken some random number. Now let us enter to the loop for all the J intervals.

Using all the J intervals, 0 to whatever, 6 interval or 8 interval whatever, for all these interval, for load P load J, calculate the economic dispatch. Now you have generators for the load at that specific interval J, you find out the economic dispatch for all the generators. What do you get? These are the functions, nJ dFij by dPj is equal to lambda. Now you get with this because gamma is already fixed, now you get a lambda here.

Lambda you get for all the J intervals. That means for one specific J interval, there is one lambda. Let us say J is equal to 1, you have one lambda. And then J is equal to 2, you have another lambda, lambda 1, lambda 2 like that. So as many number of intervals are there, that many number of lambda you get. And for each lambda, again there is another constraint, another equation that you got from the previous expression, gamma nJ, dou Q to eJ by dou PTJ is equal to lambda J.

Now you anyway got this lambda, you have fixed the gamma, but this is not the optimal lambda, optimal gamma. With this gamma you find out lambda and again by using this lambda, you obtain gamma again, for each interval. And then you check for this constraint. You have lambda and gamma for this specific interval, for all the intervals, J intervals. This is the iteration 1, first iteration we are in now.

Now you check with this constraint whether the total fuel consumption for all the intervals is not exceeding or not. Now this is the tolerance limit. If this is true, that means whatever error that you are getting is less than the tolerance, then you can assume that lambda and gamma are free freeze now. This is the optimal value. If this is false, that means you again carry out the iteration considering the latest lambda gamma values.

You take the gamma and then find out the load dispatch, then there is a lambda you get and then you get a gamma, then again you find out and this is the iteration you carry out. Finally at the end of the iteration you get, because here for the taken gamma, lambda is taking care of the constraint summation of generation is equal to PG is equal to PLoad. Now gamma is taking care of this. Now ultimately, let us say this gamma is taking care of this, for this gamma you find out the gamma and whether this gamma is taking care of this function or not, that is what we are checking.

Ultimately both the constraints need to be met. As soon as these constraints are met, then obviously the fuel cost is also met. The objective function of minimum fuel cost is also achieved. You understood? So we are finding the optimal value of the gradients basically, lambda and gamma. And then let us take an example to understand this. Now find the optimal dispatch for a gas-fed steam plant.

HTPT is equal to, this is the characteristics curve. Now HT means T indicates there is a constraint and there is another plant where there is no fuel constraint. The fuel constraint for this specific gas-fed steam plant is the plant must burn, here you can see here, 40 into 10 to the power of 6 feet cube of gas, the volume. This is the amount of volume that you need to burn. That is the agreement is already formed. And there is also, because this is a gas-fed plant and there is a capacity, minimum capacity, minimum and maximum generation limits of this particular unit that is 50 and 400.

And there is also fuel rate given, 2 dollar per ccf. And 1 dollar per ccf is equal to 10 to the power of feet cube, some number is there. Now the another plant where there is no fuel constraint and the fuel cost characteristics of that is 120, some quadratic expression, dollar per hour. And the minimum and maximum generation is 50 and 100. The load pattern is given, 0 to 4 hour, that means there are 6 intervals.

And each interval has 4 hours. So let us put together 24 hour duration. From 0 to 4 hour load need to be limited to 400 and then so on and so forth. So this list is been given to you. Now you carry out lambda gamma iteration and before carrying out lambda gamma iteration, let us try to understand this problem without considering gamma. What is the amount of price that you need to pay? Then we will understand or appreciate the importance of lambda gamma iteration.

Now ignoring the gas constraint, that is what I am saying. Now let us forget about the gas constraint as if the constraint does not exist for all the generators put together. Now the optimum economic schedule is, this is what you get. That means there are only 2 generators and there is no gamma factor in the Lagrangian function. There is a Lagrangian function now just constitutes the total fuel cost that need to be minimized plus lambda into the objective function, sorry the constraint, equality constraint. What is this equality constraint? Summation of load minus generation, i is equal to 1 to n, Pi.

i is equal to 1 to n, Pi, n plus 1 let us say because n earlier our understanding is n means only those generators where there is no constraint. Now we are just scheduling including everything. So i is equal to 1 to n Pi. This is your Lagrangian function.

Now there are 2 units as such in this given problem. There are only 2 units as such. Now you know the simple thing, df by dp is equal to lambda. That is it. Very basic Lagrangian multiplier based economic dispatch problem that we have already very much familiar about it. Then by solving this you get this kind of economic dispatch schedule.

For the first hour, in fact we did the mathematics before this class and for the first hour it was coming out to be 391 or something point something for the PS generator and 8 point something some odd number for the PT. This is the generation we got. But there is a minimum that it need to operate. So PT is put at 50 and the rest for PS 350.

So similarly we did for all the interval. So you got this optimal schedule. Now operating cost of the composite unit for entire 24 period. Now you need to find out the total fuel cost. Now this is the generation for each generator.

Now find out the total cost for PS. Composite unit means it is about PS. We are speaking about PS. Now you got the generation for each interval 350, 500, 500, 450, 150, 250. Now what you have to do is simply go to this expression.

This expression FS of PS. There is already fuel cost inbuilt inside this. Wherever PS is there you put that number. Then you keep adding for all the intervals. That means let us say 120 plus 5.1 into 350 plus 0.0012, 350 square plus again with another optimal schedule. What is the next one? 500. So next again 500 up to the last interval, sixth interval.

Now this cost is coming out to be 52128.3 dollars. Now the total cost include the cost of this plus cost of this generation. The composite generator is this much and the total gas consumed. Now for this generation that we have calculated 50, 150, 300, 50, 50, 50. It was found out that the total gas consumed would be 21.8 into 10 to the power of 6 feet cube. At a cost of 80,000 per 40 that means 2 dollar per CCF. That is the fuel rate. But it has to consume 40 into 10 to the power of 6.

That means 40 into 2 it will be 80 dollar now. That is what it has to bear. You use it or not you have to utilize the total, this is the constraint actually. Now you see here, though for this much amount of generation your actual fuel consumption required is 21.8 into 10 to the power of 6. But since there is a constraint already been imposed upon you, though you are using this much only you are forced to pay money for 40 into 10 to the power of 6.

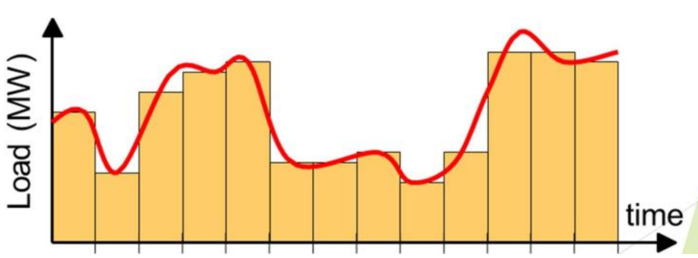
That means that will come out to be this number 132128 rupees. Let us say there is no constraint. Let us say there is no constraint for the same problem and then you have to pay price per only whatever amount of fuel that you consume. That means 21.

8 into 10 to the power of 6. Let us try to calculate how much it comes out to be 21.8 into 2, 2 dollar. So plus 52128 or 52.18. How much it comes out to be? 20 this is 42, 43.6 plus 52.18. It comes out to be nearly 96 or something. Let us say it is 95 point something or 96 odd. Actual price you should have paid is 96 thousand dollars. But since you are forced to utilize 40,000, 40 units, what is that? 40 into 10 to the power of 6 feet cube.

Then how much you are paying is 40 into 2 plus 52.128. This is coming out to be 132.128 kilo dollars or 1000 dollars or 132128 dollars. You got this difference? There is no fuel cost means this is the best one and there is nothing yet more to be optimized. But there is a fuel cost now this is the amount that you are getting. That means this indicates just by lambda having just lambda as a constraint you cannot have a most optimal one. Now let us consider gamma also into picture and then we will see whether can we have a lesser cost compared to this one or not.

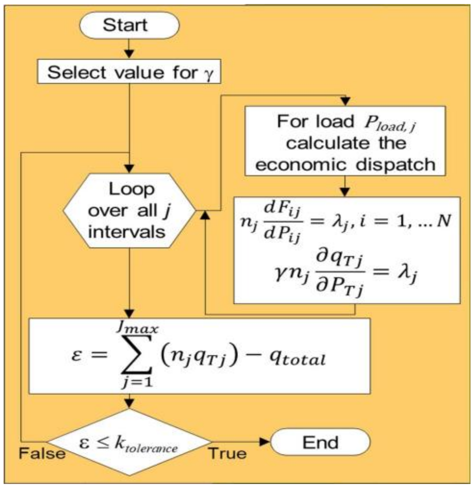
Then we solve the same problem using lambda gamma iteration with whatever flowchart that I have already explained to you. Consider now the gas constraint using the gamma search method the gamma ranges from 0.5 to be started with a gamma of 0.5. Now gamma ranges from 0.5 to 0.875 with a final value it converged at 0.8742 with the lambda gamma iteration. Then now the economic optimum economic schedule is coming out to be like this for each interval. Ps is this, Pt, Ps, Pt and all this.

You can see here everywhere there is minimum and maximum constraint is being met. So it is more than 50 and less than 400 or 500. Now the operating cost for the composite unit is not 52000 it is coming out to be 34937. And anyway you have to utilize this much 40,000. 40 into 2 is 80000, 80 plus 34 this is coming out to be 114,937. You got it. Now having gamma in place you can minimize the fuel cost of the other composite generator where there is no constraint. You understand? By optimally you know total amount of fuel that you need to utilize. How you utilize in each interval optimally such that the rest of the generator their fuel consumption can be minimized and overall cost of the entire operation can be minimized. This is what we are going to also do in the hydrothermal scheduling. Now there is a gas, now actually there is no price for hydro, but we artificially create some lambda, the pseudo cost such that we optimally utilize the volume of water in such a way that the thermal generation cost can be minimized.



Now you understood this problem right? Now this is anyway familiar to you but I will just try to recall. In fact, we have discussed this aspect in some class prior to this class. So there is a composite generation aspect, composite generation production cost functions. The composite production cost curves are a useful technique to mix fuel constraint and non-fuel constraint generation.

There is a couple of constraint and non-constraint generation. Then you have to have a combination and you have to obtain a composite generation where one generator represents the equivalent of some of the generators put together. So this we have already discussed. So if one of the units hits a limit its output is held constant. A simple procedure for generation is considering lambda adjustment.



You keep on increasing lambda such that the generation and the load demand is met. You can see here, set a lambda, minimum lambda and then calculate for each of the generator what is the generator for this specific lambda and obtain df by dp and if unit i hits a limit then set that generator at a higher limit such that summation of generation put together as Ps and fuel cost you obtain and then you increase the lambda such that you know total ultimately you reach to a minimum lambda to the maximum lambda. There is a lambda minimum if you remember, lambda minimum lambda maximum for each generator. The most minimum lambda minimum to the most maximum lambda maximum from the minimum, most minimum to most maximum you just increase it and obtain a piecewise linear or smooth piecewise linear characteristics such that whatever you get with the curve fitting formula you match with the actual one.

So we have also discussed this. This is for different characteristics. This is lambda minimum lambda maximum for the first unit, lambda minimum lambda maximum for the second unit and lambda minimum lambda maximum for the third unit. Then you move from this value to this value.

We have already discussed this. Then it starts from 8.3886 to 14.810 and we will try to you know this is the actual one and this is the fitted one. So you can see here it is almost matching. So this is important. I just wanted to recap this so that we can utilize this aspect for the next problem that we are going to solve in the next class. So with this we will conclude today's discussion and we will take up one or more problems in the next class to understand hydrothermal scheduling in a better way. With this I will conclude. Thank you very much.