Economic Operation and Control of Power System

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Hello and good morning everyone. Welcome you all for the NPTEL online course on Economic Operation Control of Power Systems. In today's lecture, we discuss about hydrothermal scheduling, one of the important topic of discussion which is hydrothermal scheduling. So just to have an introduction, coordination of hydro units to optimal dispatch is tedious as hydro power plants are not same. So, multiple hydro power plant, each hydro power plant has its own geographical constraints and you know having coordination among these hydro units is not a very easy job. Hydro power plants are different from each other due to their geographical locations.

And hydro power plant power generation is interrelated to the control of the flood waters and regular water release for agriculture purpose. See, hydro is always relevant to the livelihood, right. So, just it is not power generation. Power generation is one of the aspect of hydro power plant, I mean the reservoir.

So, reservoir has multiple other purpose mainly agriculture and you know irrigation and so many other things. So, the constraints need to be met while we also optimally utilize the hydro resource for the power generation. So, the water release by the hydro power plant is an essential constraints to avoid damage in downstream areas also. Let us say you want to have maximum generation using hydro power plant. So, you release lot of water and that should not lead to any flood in the downstream areas and they may get you know affected.

So, that should not also be done. So, there are two types of hydro scheduling, one is long term schedule and a short term schedule. So, here long term forecasting of water resources and water release is considered where minimum of typical scheduling range considered is around 12 months or 1 year, right. So, long term water available water resources considering all the seasons. So, different seasons what are the possible water accumulation and their possible utilization for multiple other purpose to sustain humanity and other livelihood.

So, that also we need to be in consideration while we have long term scheduling. Whereas, a short term includes hourly scheduling of all generation sources so as to achieve minimum production cost. So, that is objective. Now, here the water inflows availability of the unit and the load demand is considered to be as a known parameter while we carry out this short term scheduling. So, what is available water inflow and what is the load demand and what is the status whether generation hydro power plant is available for generation or not these things are considered to be a known parameter before which we go for short term scheduling.

So, some of the challenges. So, in long term this includes storage of water in multiple seasons as I told where the long term challenge include meteorological and statistical analysis. You should know the weather forecasting right. So, when there will be heavy rainfall, what would be the possible water accumulation in the reservoir dam. So, this depends upon meteorological and statistical previous statistics analysis.

So, if there is any error in those statistical analysis that will also have a effect on long term scheduling. Whereas, short term in the case of short term water inflow forecast would depend upon snow melt expectations and near term weather forecast. So, let us say you plan for next 1 hour then there would be sudden snow melt and there could be huge in rush in the water inflow. So, that may hamper your short term scheduling this is something unexpected and also it depends upon near term weather forecast. So, there is a long term weather forecast and a near term weather forecast.

So, this has an impact on long term and a short term. So, there are certain policy that need to be considered for long term scheduling. For the long term drawdown schedule a fundamental policy selection must be considered. So, the policy would be based upon whether the water should be released assuming the worst possible prediction of the situation. The worst possible situation would be let us say there is a drought and you do not have water for next 3 months there is no rainfall coming up.

So, still can you able to sustain the agriculture and drinking purpose, fishery so many other constraints are there. So, would you able to manage that you cannot you know straight away utilize all the available water to you know there is a demand in electricity I will just use it and use the available water and meet out the load requirement. It is not just electricity that matters. So, and also if there is a heavy inflow so that also we need to be considered. These extreme conditions of large water accumulation and minimum water accumulation in the reservoir need to be considered while we go for any policy or long term scheduling right.

So, some of the key features of hydro thermal scheduling. Now, the cost can be reduced by replacing coal with hydro right. So, that is the objective we need to minimize the cost. It is not only cost, it is also the you know reduction of carbon footprint. So, less than 5 megawatt we consider let us say there is a hydro power plant of capacity less than 5 megawatt.

This is considered to be in the in the category of renewable energy sources right. So, above 5 megawatt we though hydro is renewable thing, but it has lot of impact it has lot of other impacts that could be estimated in terms of its impact on the geographical locations you know there are so many environmental effects that may be anticipated if there is a huge water body present. So, anyway we can consider for time being that hydro can be considered to reduce the overall cost because hydro is free of cost basically right. There is water that is coming. So, there is you need not have to purchase water to fill your dam.

So, you can reduce the cost by of thermal power generation while optimally utilize the available water resources right. And the constraints considered are running out of water in the reservoir and inadequate water for navigation. So, that means let us say there are two extreme cases, you utilize all the water or you do not utilize any water right. If you utilize all the water then there could be a problem that you do not have any water available in the reservoir in the reservoir that is what I am saying running out of water in the reservoir. And inadequate water for river navigation and let us say utilize maximum water and there is no sufficient water for river navigation.

Ultimately there is a cycle you should have minimum water flow, there should be an optimum water flow that need to be taken into consideration as well. That means there should be a water flow that completes the path from the water it is started from wherever it is started. Let us say let us take an example of Ganges. So, Gangotri where water the Ganges starts right and then there is a water inflow and you hold water at certain place, but Ganges has to complete her path finding her destination. So, then there is a water that is seen its destination at the sea, then there is a rainfall and then it gains field.

So, this cycle need not to be hampered river navigation should be proper. And then over usage of water is avoided to ensure ample water is preserved to meet the necessary constraints. If you excessive utilize the water then you may not have water for summer season. So, over utilize and under utilize both are problematic right. So, some of the techniques scheduling is an optimization problem in the context of unknown factors as a water inflows, load and steam or hydro units availabilities.

So, these are the unknown factors what is the exact water inflow, what is the exact load demand and what is the availability of steam or hydro power plant. So, unknown factors should be addressed statistically and long range scheduling involves optimization of statistical variables. You optimize the statistical variables you know if you do not have any specific data available with you. Using dynamic program through which the total long range operation time is simulated. For example, one year for a given set of conditions, it is that is what long range scheduling is considered to be.

And composite hydraulic simulation models are used to represent several reservoirs. You carry out certain hydraulic simulation models, then composite I mean multiple hydraulic simulation models to represent multiple reservoirs. So, that you can have a estimation of what could be the dispatch and production cost based statistical models right. Now, what are the short term scheduling techniques? Given a set of starting conditions for example, I will give you reservoir level. This is the reservoir level in a particular dam.

The optimal hourly schedule that fulfills the desired objectives such as cost minimization is sought while meeting hydraulic steam and electric system constraints. Thermal power plant have their own constraints right. So, start up time you know all these things are there, maximum and minimum generation. So, down time, up time and all these things we have discussed right. So, and simultaneously hydro power plant have their own constraints as I have already told how much could be the water in flow and based on that how much you know the river navigation need to be also be maintained.

So, there are so many constraints, geographical constraints that need to be met. Considering all these constraints, the objective would be to reduce the overall optimal, overall cost of thermal power generation. So, that is the objective of the short term scheduling. Part of the hydraulic constraints may include meeting the end point conditions at the end of the scheduling interval in order to conform to a previously established long term water release schedule. These end point conditions have to be taken into picture.

So, let us say what are these constraints? To understand the requirement for the operation of hydroelectric plants, one must appreciate the limitations imposed on operation of hydro resources by, so these are the typical constraints. One is recreation also, these also need to be considered like people go for Rishikesh for rafting right. So, it is very famous spot. So, recreation if you hold all the water with you, then you do not have any water for recreation.

So, and flood control right. So, you need to manage your flood, what is the such that you know there will be minimum damage in the downstream areas and the fisheries because most of the families they depend on their livelihood because of the fisheries. So, that also we need to be taken into consideration and river navigation as I told there should be a closed cycle hydro cycle that need not to be hampered right and cemetery you should have power generation also and there should be daily water supply need to be fulfilled. So, there are so many constraints that need to be taken into consideration. So, let us try to understand some figures. The amount of energy available in a unit of stored water, the potential energy say cubic foot is equal to the product of the weight of the stored water times the height that the water would fall.

It is a basic formula potential energy is nothing but mass into gravity into height right. This is nothing but your weight, weight is nothing but mass into gravity right. So, that is what here it is mentioned is equal to the amount of potential energy is nothing but the product of the weight of the water stored that is let us say 62.4 lb times the height and that the water would fall. So, amount of the volume of water and the weight of the water and as well as the height.

So, let us say 1000 cubic feet of water, the volume of water accumulated in the reservoir falling a distance of 42.5 feet, this is a distance has the energy equivalent of 1 kilowatt hour. If you have to generate 1 kilowatt hour of energy, 1 unit, 1 kilowatt hour means it is 1 unit right. If you want to have 1 unit of energy, electrical energy, the estimation is 42.5 feet of height and around 1000 cubic feet of volume of water.

In other way, it could be 42.5 cubic feet of water, the volume of water, but the distance is large, the height is large, 1000 feet. Both have the same figure, it would be 1 unit of power generation, electric power generation right. Now, this is how a typical hydro power plant looks tentative. So, it is there is a four bay right, four bay means before the dam right and after means after the dam. So, there is a four bay and you this is your water body, this is your reservoir right and there is a trash rags and intake where the water can flow through this dam from the reservoir to the downstream.

So, there is a in gate and there is a water pipe, this is nothing but penstock. So, there is a penstock and after which we connect a turbine multiple varieties of turbines are there depending upon the head, Kaplan and Francis and propeller turbines, reaction turbines so many things are there. So, there is a turbine. So, you simple there is a potential energy, you convert it into kinetic energy using a penstock and you throw this water or you focus the water towards a particular turbine. Turbine rotates and generates the electricity and there is a you know what kind of generator we use, we have already discussed.

There are two types of synchronous generator. So, among them there is cylindrical or non salient pole and there is a salient pole synchronous generator. There are two types of synchronous machines we have discussed. So, cylindrical is basically used for thermal power plants and salient pole synchronous machine is for hydro power plants right. So, where the speed is relatively lower compared to the cylindrical machine.

Now, you have generator, you generate the electricity and pump it into the grid and whatever the water is already used, the energy is already used. So, now, the water is left out to the after way. There is a draft tube and it will complete its part, now the river navigation, it will join the river basically right, it will join the river. So, you see here there is a gross head that means, there is a downstream water bed and there is a reservoir. Actually, the kinetic energy depends upon it depends upon the potential energy height difference right.

So, it depends upon that fore bay and the after bay height difference that is what we call it as gross head right. This is a gross head of course, there will be also friction losses that I will discuss, then you will have a net head right. So, this is what a typical hydro power plant looks like. And let us consider some overall aspects of the falling water as it travels from the reservoir through the penstock to the inlet gates through the hydraulic turbine down the draft tube and out to the tail race at the plant exit, this is what we have discussed. Now, the power that the water can produce is equal to the rate of water flow, what is the speed of water flow in cubic feet per second times a conversion coefficient that takes into account the net head, the distance through which the net head means the distance through the water falls less the losses in head caused by the flow.

So, there is a there are two types of efficiencies that is considered here and there is also conversion efficiency of the turbine generator that means, so there is a feet cube, this is speed of the water per second right, water inflow multiply by net head right and the efficiency of generator right. So, there is a gross head that I have discussed already, this is the difference between the height of fore bay and after bay, but during the water inflow there are so many factors that need to be considered, there is a friction due to the friction losses in the penstock and there could be spillage, there could be some water loss, it is not that exact volume of water is rightly available for your electricity generation at the downstream. So, what we do is in order to consider this aspect, we reduce the overall gross head, let us say there is 100 feet, we reduce the overall head itself to 95 feet, anyway amount of electricity generation is depending upon the head. So, in order to accumulate or accommodate these losses, water losses, so we will reduce the overall height. So, that is what is this net head and generator has its own efficiency also.

So, overall electricity generation will be ultimately depending upon all these factors. I will give you an example here. Let us say flow of 1 feet cube per second falling, this is the speed of water, falling 100 feet, this is the height, has the power equivalent of 8.5 kilowatt. If the flow caused loss is, loss in head was 5 percentage, that means 5 feet you reduce.

Then the power equivalent for a flow of 1 feet cube of water per second, the same speed will be the same with the net drop of 100 minus 5 that is 95 feet, that is your net head would have the power equivalent of turbine generators are typically in the range of, let us say I am considering efficiency in the range of 85 to 90 percent of the generator. At the best efficiency operating point for the turbine generator, so 1 feet cube second falling 100 feet would typically develop about 7 kilowatt at most. This simple figure, let us say 1 feet cube per second considering 100 feet, you get around 80.5 kilowatt. What you need to do is, say 1 feet cube per second, there is no difference.

So, you forget about this speed, 8.5 kilowatt considering 95 feet, that means you consider 0.95, 95 by 100, earlier it was 100 into the efficiency you may consider 0.85 or 0.9, that is the efficiency.

You get this around in the range of 6.9 to 7.2 kilowatt, that means approximately 7 kilowatt. If you consider 8.5, sorry 85 percent efficiency, you get 6.9 kilowatt and if you get consider 90 percent, you get 7.2 kilowatt. So, this is what the ultimate power that you get actually, it is not exactly 8.5 kilowatt. So, the hydroelectric projects consider consists of a body of water impounded by a dam, the hydro plant and the exit channel or lower water body. Now, the energy available for conversion to electrical energy of the water impounded by the dam is a function of the gross head, that is the elevation of the surface of the reservoir, less the elevation of the after bay or downstream water level below the hydroelectric power plant, this I have already told. Now, the head available to the turbine itself is slightly less than the gross head due to the friction losses in the intake penstock and the draft tube.

This is what is the reason why we consider net head rather than gross head. This is usually expressed as the net head and it is equal to the gross head less the flow losses which is measured in feet of head. The flow losses can be very significant for low head plants. Let us say 10 to 60 feet, this is the overall head and if you also subtract 1 or 2 feet, then that will have a significant impact in the amount of generation. So, there are two things, one is very low head plants and the plants where the penstock is of very long length, penstock is also very long, then what happens? There is also significant amount of losses because of friction. So, in these two cases this has a have a lot of impact. Now, the water level at the after bay is influenced by the flow out of the water reservoir including plant release and any spilling of water over the top of the dam or through bypass race phase. During flooding conditions, the rise in after bay level can have a significant and adverse effect on the energy and capacity of power capacity of hydropower. The reason being you see here during flooding condition the rise in after bay level. If I already told there is a four bay, this is four bay, let us say this is after bay level. Due to the flood there is a water spillage over the reservoir and there is a significant amount of after bay height, there is a increase in the after bay height.

That means the overall grass head will now decrease. That means if ultimately the power generation depends upon the height difference, the potential difference between the reservoir and the after bay. So, now if there is due to any reasons there is a shoot up in the after bay water outflow, then there is a overall reduction in the power generation. The type of turbine used in hydro plant depends upon the design head for the plant. By far the largest number of hydroelectric projects use reaction type turbines and for medium head, see we have we are categorizing into three levels between less than 60, between 60 to 100 and sorry 1000 and greater than 1000. So, for medium head we use Francis turbine, for low head the propeller turbine is used.

The more modern propeller turbines have adjustable pitch blading, Kaplan is there to improve the operating efficiency over a wide range of plant head. The typical performance results in an efficiency at full gate loading of between 85 to 90 percentage. The Francis turbine and the adjustable propeller turbine may operate at 65 percent to 125 percent of rated net head as compared to 90 to 100 percent for the fixate propeller, some figure. Another factor affecting operating efficiency of hydro units is the megawatt loading.

See it also depends upon how much you are loading. Every plant has its own optimum design. So, even if you say transformer, let us take a example of a transformer. When a transformer it is designed to operate at this maximum efficiency depending upon some loading conditions. So, the power transformer usually it operates at rated load, right.at rated load, right. So, the efficiency, the peak efficiency is designed in such a way that the constant losses and the variable losses takes place at the rated load condition, right. Whereas the distribution transformer rate is not loaded at its rated capacity all the time. That is why we go carry out all the efficiency, right. So, this distribution transformer is rate operate, designed to operate at its maximum efficiency less than its rated load, right. So, similarly hydro power plant has its own optimum design based on the height, you know, the gate sufficiency, there are so many things which are considered here.

And the loading capacity, what is the maximum, what is the rated loading at which the efficiency could be higher, all these things are considered. Now at light unit loading, the efficiency may drop below 70 percentage also, right. These ranges are often restricted by vibration and cavitation limits. And at full gate may rise up to 87 percentage.

See the difference, some 70 to 87 percentage. If the best use of the hydro resources is to be obtained, operation of the hydro unit near its best efficiency gate position, position of the gate and near the designed head is necessary. There is a designed head and what is the best efficiency gate position, that also we need to be considered. Therefore, the unit loading and control of reservoir four way are necessary to make efficient use of hydro resources. Unit loading should be near best efficiency gate position and water release schedules must be coordinated with reservoir inflows to maintain as high a head on the turbines as the limitations on four way operation will permit, right.

See, next there is incremental water rate versus power output. This is power output, let us say in megawatt, right. So, this is incremental water rate. See, you see here every unit, these are different units, you may consider different hydro power plants. So, the different units have different incremental water rates.

You see here, let us say I will take the example of unit one. So, even a small increase, there is a constant and there is a increase rate. There is a minimum and maximum, I told you, above maximum there is shoot up in the water, incremental water rate. Still, the minimum and maximum capacity of the hydro power plant, it is constant. This we have already discussed in the initial slides.

So, above which each hydro unit have its own typical characteristics. You see here, so let us say unit one after certain capacity, there is a small increase in water, small increase in the power demand. So, increased incremental water rate, it is very drastic actually. So, the water inflow expected, the amount of water expected is quite large for a small variation in the load demand. But there are certain units you see here, it is not as large as in the case of unit one.

So, that means the efficiency also matters between different hydro power plants. So, considering all these hydro power plants put together, it is a very challenging thing. So, I will discuss that right here. Typical plant performance for a medium head, 4 unit plant in South America is illustrated. Those are 4 units of South America.

The incremental water rate is expressed in acre feet per megawatt hour. The rise in incremental water rate with increasing unit output results primarily from the increased hydraulic losses with the increased flow. This is because of the losses, right. So, and every unit has their own efficiency. So, a composite curve for multiple unit operation at the point would reflect the mutual effects of hydraulic losses and rise in after wave with plant discharge. Very careful attention must be given to the number of units run for a given required power output.

And the objective is to utilize the best efficiency hydro power plant compared to the others. So, that is what is mentioned here. So, high head plants typically over 1000 feet uses impulse or pelton turbines also. So, in such turbines the water is directed into spoon shaped buckets. There will be buckets like this on the wheel by means of one or more water jets, you throw water actually in the buckets and buckets would rotate and you get the generation.

So, in the operation of a hydroelectric power system, 3 general categories of problem arises. One is this depends on the balance between hydroelectric generation, the thermal generation and the load. There are 3 things hydro, thermal and load. Systems without any thermal generations are fairly rare of course, that does not exist also. The economic scheduling of these systems is really a problem in scheduling water releases to satisfy all the hydraulic constraints and meet the demand for electrical energy.

It is not a small job. You have so many constraints I have already discussed of hydro power plant and thermal unit also have their own constraints. So, they are scheduling the load demand while meeting out these constraints and simultaneously achieving cost minimization. It is a very tedious job. So, techniques developed for scheduling hydro thermal systems may be used in some systems by assigning a pseudo fuel cost to some hydroelectric power plant. So, what they do is they also consider hydro power plant also has some fuel input, some fuel cost though actually there is no fuel cost.

Pseudo means it is false. So, then it is very easy for them. Now, this also have some fuel cost and high thermal also has fuel cost. Now, you have economic dispatch. This is one way of approach. Then the schedule is developed by minimizing the production cost as in a conventional hydro thermal system. In all hydroelectric systems, the scheduling could be done by simulating the water system and developing a schedule that leaves the reservoir levels with a maximum amount of stored energy.

Now, the objective typical hydro thermal scheduling would be to have maximum amount of stored energy in the reservoir still you meet out the hydro thermal scheduling. In geographically extensive hydroelectric systems, these simulations must recognize water travel between water travel time between the plants also. Now, hydro thermal systems where the hydroelectric system is by far the largest component may be scheduled by economically scheduling the system to produce the minimum cost for the thermal system. Let us say there is significant amount of hydro and there is small amount of thermal. This is one of the cases and the objective would be to reduce the minimum cost of thermal power generation.

So, the largest category of hydro thermal system include those where there is a closer balance between the hydroelectric and thermal generation resources and those where the hydroelectric system is a small fraction of the total capacity. The most of the cases it so happens the thermal is large and hydro is less. In this system, the schedules are usually developed to minimize thermal generation production cost recognizing all the diverse hydraulic constraints that may exist. Now, here you need to also consider the hydroelectric, hydraulic constraints, the water constraints.

You see here, there is a load. Now, you have hydro and thermal. This is a water flow, Q is represented as water input, there is a fuel input, thermal and hydro. They put together meeting out the load demand. Now, hydro units are not self-sufficient to meet the total load demand, right, over a period of time but with a large capacity. Therefore, thermal units should also run parallely to meet the excess load demand.

Hydro units are scheduled such that the total production cost from thermal units will be minimum. You see here, hydro units is capable to supply load at any time step but not the total load of the entire time period. That means, let us say at any given point of time, there is a time sample, right. Just take one time sample, one time duration. Whatever is the load demand, the hydro may able to meet out the load demand.

But the hydro may not be able to cover the complete load demand over a complete time range. So, it is not sufficient enough. So, it need assistance of thermal power plant. So that means you can see here, hydro, the maximum hydro capacity is higher than the load demand for a specific given time sample. But if you consider the energy, energy is power into time, right.

So, the total energy available in the hydro power plant may not be sufficient. That means it is lesser than the total energy demand over a time period of the schedule, right. Therefore, required energy from thermal power plant. Now we are trying to come out with a picture, what is the estimation of thermal power plant requirement. So, the total energy demand from the load minus the energy that can be supplied from the hydro.

This is what the energy that need to be catered by the thermal power plant, right. So, let us say there is a load demand, this is the entire time duration, the hydro is present for all the time duration. But thermal is operated for only specific duration, right. Within this entire time range, for specific time only it is operated just to meet out the excess load demand which hydro cannot be able to meet out, right. So and in that case, what should be the maximum or the optimal utilization of thermal power plant, so that the cost of thermal power plant can be reduced. Where T is the total scheduling period, this I have already told and T t is the number of time intervals the thermal power plant is run.

Now entire amount of hydro energy should be utilized such that the cost of running the thermal plant is minimized. This is the objective. So, that is minimum fuel cost from time is equal to T is equal to 1 to T t, it is not total time, it is time of operation of thermal power plant itself. The fuel cost of the specific power generation into the time that means the energy, total energy cost should be minimized. Subject to the constraint, what is this constraint? The energy demand which is E is nothing but the energy demand of the load minus energy that the hydro can supply.

That minus the total generation, total energy supplied from the thermal power plant is equal to 0. Then only you get the ultimate power generation is equal to load, right, thermal plus hydro put together total will be equal to the load demand. Now what is the Lagrangian function? This is a fuel cost that need to be minimized, thermal fuel cost and there is lambda into this constraint, right. And then this is a very simple procedure. So, partial differential of Lagrangian equation with respect to the power generation you get a constant which is lambda.

This means that the thermal plant should be run at constant incremental cost for the entire period it is on. This is one such consideration where if hydro is available for all the time of dispatch then how much effectively that you can thermal, you can utilize the thermal power plant so that you can have minimum operation cost. And that we come out with a picture that it should be a constant incremental cost. As hydro power plant is large and thermal unit is there only to support it, this is for the first consideration we are considering, hydro is large and thermal is less. Therefore, let the thermal unit is run at a constant power generation P s star at all time steps.

Let thermal be running at its constant power generation then we can have a minimum cost. And that we need to find out what is that optimum generation. Therefore, we will obtain that. Therefore, total thermal energy production cost is this F t is equal to the, this is the objective function, right. So, that means F of P s star you take it out and this is a total time, delta t is the time sample.

Let us say from t is equal to 1 to t is equal to t t where thermal is operating. So, you say you have here hourly dispatch or something like that, let us say it is 10 hours, right. So, the N s is 1 10 times now, right. Each sample time is 1 hour, total duration is 10 hour. So, the N s is summation is equal to t is equal to 1 to t t delta t that is N s.

So, F of P s star into N s, this is the energy, right. Now, and then you see here, now thermal plant fuel function is given by, this is a quadratic expression, fuel cost expression. So, you put, you replace F of P s star with this quadratic expression. This multiplied by summation of t is equal to 1 to t t into delta t is nothing but N s, you get this expression. And then, now what is summation of t is equal to 1 to t t P s t delta t? This is nothing but E you see here, E minus, this is a constraint, right, E minus the total energy of the thermal power generation is equal to 0.

That means E is nothing but this, right. That is what we have put up here. E is nothing but summation of t is equal to 1 to t t P s t into delta t and that is nothing but if you know P s t is a constant term, you take it out, then you get the total time sample which is N s. So, P s star into N s is equal to E, whereas N s is nothing but E divided by P s star. Now, you go back to the previous expression. Now, this is a quadratic expression into N s was there, now you replace by E by P s star.

Now, we are ultimately finding out that optimal generation. Now, you differentiate, you need to find out optimum P s star, right. So, you differentiate this expression with respect to P s star. Then you get this expression which is equal to 0. Then you get the value of that power generation from thermal power plant which depends upon this constraints, this constants A and C.

That is it. If you know this, you know from the fuel cost, they have already given you this values constraints, right. A P s square by B P s plus C square root of C by A will give you the value of what is what should be the optimum generation of a thermal power plant. Considering hydro is large and it is available for all the time duration. Now, this last thing, hydro power plants are not large typically and should also be scheduled along with the thermal units for the entire period of time.

All the thermal units are combined and represented as a single thermal unit, right. So, let us say this is the picture. There is a water inflow at time t, R t is expressed as water inflow and because of there is a water inflow, there is a water volume at end of time t, this is at the reservoir forebay, right. And there is a water spillage also. This is considered as S t which is wastage and there is water discharge at time t. The water discharge happening through this penstock and afterwards there is a generation and there is a water outflow.

So, that is Q t. So, let us denote these parameters now. Now, objective function is to minimize total energy production cost where hydro is less and thermal is quite large. Now, this is the objective function minimum fuel cost, right. So, that means you need to minimize the fuel cost for all the time duration.

From t is equal to 1 to t, you observe here it is not just t t. Now, you are operating thermal for all the time duration. Earlier you operated thermal for only specific time duration t t which is less than t. Now, your objective is to minimize the fuel cost for the entire time duration. So, subject to there is a constraint here. What is that constraint? For the entire time duration also you are operating water now t is equal to 1 to t, right.

This is the volume of water at any specific time interval. This is a time sample delta t. So, at any given time the volume of water that is being discharged that you accumulate together for the entire time duration. So, that is the Q total, the total discharge for the entire time duration. This is the total water discharge constraint and this constraint has also some limit.

So, there are two limits. For every time sample there is a limit that is what is given here Q minimum to Q maximum. This is for every time sample and put together for the entire time duration also there is a total limit. How much water that you can ultimately throw it out. So, there are two constraints with respect to the hydro, right. And there is this typical constraint total generation should be equal to the total load plus total loss.

This is hydro thermal put together load plus load demand plus losses need to be met. So, these are the typical constraints. So, now the Lagrange function is summation of is equal to t is equal to 1 to t is a fuel cost of thermal power plant plus lambda into the constraint equality constraint generation should be is equal to load. And there is a additional thing which is coming up which is nothing but the constraint of the hydro. Gamma into the total you know amount of discharge of water should be equal to the maximum discharge that is possible that is allowable. Now, hence there are coordination equations which are written here.

One is with respect to the thermal power plant another with respect to the hydro power plant. So, delta t you just have to partially differentiate with respect to P s of t this Lagrangian function, right. So, there are two constraints coordination equations D f P s of t by D P s of t plus lambda into with respect to the loss.

This is due to the loss function. We have discussed about penalty factor and other things. So, that is equal to Lagrangian multiplier. Similarly, you have another coordination equation with respect to the hydro. Earlier there was only one generation now we have two. Now, this complex is quite complex. Iterative problem is solved by using lambda gamma iteration method.

I will just give you a brief and then we will discuss in detail about the next class in the next class. So, this is a flow chart how they carry out lambda gamma iteration. Select some initial values of lambda, gamma and the initial thermal power generation P s of t. This is the first time sample t is equal to 1 and then solve the coordination equation that I have already discussed earlier. There are two coordination equation one with respect to thermal and one with respect to hydro and then you check whether the summation of generation is equal to load demand plus losses or not.

And then if this is made with this is within the constraint threshold. So, if this is achieved then the lambda value is freezed that is the optimal lambda. Otherwise you project the lambda such that ultimately you come up with a point where total generation of hydro and thermal put together is equal to load plus losses. Now, lambda value is freezed and then for this lambda then you find out the volume of water because hydro generation is freezed at this point of time. Now, for this hydro generation what is the amount of water for that specific time interval t is equal to 1.

And then you now check whether you have already reached the total time interval or not. If not yes, if not then you check if you have not reached the ultimate time interval then you increase the time interval then again solve the coordination equation, freeze the lambda and then you have for ultimate t is equal to 1 to t for all time interval you have total volume of water available with you. And now you check that constraint whether the total amount of water that you can dispatch is within the limits or not. If that is there you have that hydraulic constraint is also met then you get the final dispatch schedule of thermal and hydro. If that is not the case that means the hydro constraint is violated then you have to project the gamma that means the value of gamma is projected change and then you have to restart the entire dispatch.

That means we fix the hydraulic constraint and then solve for the other constraint which is generation is equal to load. Then if that is met then the gamma constraint is satisfied, the hydraulic constraint is satisfied otherwise you need to reschedule it. So, we carry out the iteration again basically. So, this is what is a typical lambda gamma iteration method for short term hydro thermal scheduling problem. It is quite tedious and MATLAB if you write a program it is even more complex compared to the just a thermal scheduling. So, with this we will conclude and we will take up some of the examples in the next class. Thank you very much.