Economic Operation and Control of Power System

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Week - 05

Lecture – 21

Hello, good morning everyone. Welcome you all for the NPTEL online course on Economic Cooperation Control of Power System. Today we will continue our discussion with us Lagrange Relaxation and using Lagrange Relaxation solving a unit commitment problem. So just to recap, we have discussed about dual variables and dual optimization. So, in the considering the classical constraint optimization problem, we have defined primal problem which is minimization of a function subject to a constraint, this equality constraint which is equal to 0. And the Lagrangian function as we are very much familiar about Lagrangian function is nothing but the objective function plus Lagrangian multiplier into the constraint.

And then we have defined a dual function. Dual function is Q of lambda which is minimal of the Lagrangian function and then maximizing it. So, then the dual problem will become maximum of minimum of the Lagrangian function which is depending upon three variables. Here in this case let us say x 1 and x 2 are the two variables and lambda is another one.

So, x 1, x 2 and lambda. So, closeness to the solution is determined by the gap between the primal function and the dual function. Primal function is nothing but minimum of maximum of Lagrangian function which will come out to be the original function itself that we have derived in the last class. This is nothing but your objective function right. So, the primal form is the objective function and the dual form is maximum of minimum of the Lagrangian function right.

And between them you are you are finding out the gap. So, that gap in a way is nothing but your constraint that we will get to know in the later part of the slides. So, in a way we are minimizing the cost while meeting out the constraint. Minimizing the cost while meeting out the constraint while we are subjecting into a very complex system where multiple generators are present and we need to optimize whether these generators need to be turn on or turn off during a specific time interval. It is not just you know Lagrangian typical Lagrangian function where you have in a classical optimization problem, you have just to meet out you know summation of generation is equal to load and minimizing the cost.

It is not just that. It is even more complex where there are multiple variables pitching in and especially the discontinuous function because of the presence of the state whether to turn on a generator or to turn off a system. It is a discontinuous state. So, we need to optimize in a very systematic way using Lagrangian function. So, I will explain the procedure in a while.

So, this is the loading constraint. This is the load, load of at the specific time interval P load t and the summation of generation P i of t along with their state whether they are turned on and turn off. All of them put together it should be 0, right? Algebraic summation should be 0. And there is another limits where individual generation of a generator should be within the limits of minimum and maximum and this should be applicable for all the generators for all time interval, right? And now the objective function will be to minimize the overall cost of all the generators for all time interval. See, this is for all the generators.

This indicates for all time interval for the ith generator at a time interval t plus the start up cost of the ith generator at the time interval t while optimizing its turning on or turning off. So, that is the overall objective function that need to be minimized. So, now you can see here you have three variables. First variable is set of the generators P, then their states operating states whether turn on or turn off and then the Lagrangian multiplier. In the previous constraint optimization we just had P and lambda, U was not there.

So, now this is the objective function cost minimization and subject to the constraint. Unit commitment requires that the optimization minimization of the Lagrangian function subject to all the constraints that is objective anyway. Any optimization function the objective is to minimize the cost while meeting out the constraint. The units are decoupled here that is operational cost of one unit does not affect the cost of running another unit. Proof I will show in the next slides.

So, the loading constraint is a coupling constraint across all the units. So, only constraint which couples between the individual generator even though they are dealt individually, right? You are maximizing the cost of individual generator. So, but the coupling constraint is the load, right? So, that means summation of generation should be is equal to load in a way ultimately. So, the Lagrangian relaxation process also unit commitment problem by temporarily ignoring the coupling constraints. So, the dual procedure attempts to reach the constraint optimum by maximizing the dual function with respect to the Lagrangian multiplier.

So, this is the dual function, right? We follow the two steps procedure. The step one is find a value for each lambda which moves q of lambda towards a larger value. So, if you remember I have discussed about this. This is the primal function let us say is j star and this is lambda, right? So, this is a minimal function and there is a q star which is a maximum function. So, ultimately you need to obtain a lambda, obtain a lambda where q star will be maximum and that indicates in a way j star as a function is also minimum.

So, the objective is to obtain the maximum point and then you obtain that specific lambda. So, if they are tightly coupled that means then j star minus q star by q star if it is close to 0 that means we say that j star and q star are converging at a same point. So, we need to obtain this optimal lambda that means that optimal lambda in a way indicates the cost is minimized while meeting out the constraint, right? So, find out a value for each lambda which moves q lambda towards a larger value. That is a dual function, right? Maximizing the maximum of minimum of Lagrangian. step two is assuming that lambda is constant found in step number one then you keep it as So, a fixed value find the minimum of Lagrangian function.

You first actually there are three variables we are keeping one as constant and then optimizing the other variable like that. So, now we are fixing lambda you take some random value of lambda fix that lambda and for that fix lambda now you minimize Lagrangian function. Lagrangian function is in a way dependent upon two variables now. One is u and p. Now you then considering once you fix the lambda you get a value of p because d l d of f by d p is equal to lambda d f by d p is equal to lambda.

If you fix the lambda you get the value of some generation, right? So, now you got the generation you got a lambda and then you obtain you optimize the u because both are fixed now. You optimize the u whether to turn it on or turn it off for the entire time interval. Then based on that you again get the values another values of generations then you again optimize the generations with the economic load dispatch and then with the updated value of economic generation you again obtain the lambda it is a iterative process actually. I will take you to various examples so to help you understand. So, see you see here find the minimum of l by obtaining suitable u i and then adjusting the value of p i to meet the load demand.

Now here is a proof why they are decoupled. So, minimizing Lagrangian assuming lambda to be fixed, right? So, this is your Lagrangian function there is total generation number of generators are considered for the entire time interval, right? Now this is the fuel cost and start up cost and this is your state operating state. So, now this need to be minimized, right? So, subject to the constraint. Constraint is for all time interval the total load demand should be equal to the total generation for all the time interval considering all the generators into picture, right? You see here you just have to rearrange the equation so that you will come out with this expression. Just take this summation term outside of the load then you put this inside so you will get this expression, right? So, now summation of t is equal to 1 to t from first time interval to the last time interval.

This lambda into p load this term is a constant term because lambda is fixed. You assume a lambda then you work it out. Then load is a fixed term, fixed quantity for a specific time interval. So, this overall term is constant, right? So, this term is constant. All the other term inside the second bracket are related to one generator only.

If you see here, this specific if you take out this summation i is equal to 1 to n. That means this is for individual generator. See, for the first generator you get the same expression, for the second generator also you get the same expression. So, this expression is common for all the generators. So, the inside term can now be solved independently for each generating unit.

Hence decoupling is achieved that means summation of time is equal to 1 to t for each generator you can have a individual optimization. Now, the minimum of the Lagrangian is found by solving for the minimum of for each generating unit over all the time periods. So, minimum of Q of lambda need to be obtained. So, this is your function now, right? Because anyway the load is constant that we have taken it out. Minimizing a constant term, constant let us say x is constant and y is a variable.

You need to minimize this function. Let us say for example, this is nothing but minimizing a variable one, right? Just y. So, we can just simply ignore the constant. So, we have just removed the load part from this. That is why we told the load gain constraint is temporarily been not considered, it is ignored.

So, subject to the unit limit constraints where each generator at any given point of time should be within their limit. That need to be taken into consideration. Now, this is easily solved as a two state dynamic program problem for one variable. Now, lambda is fixed, you got the generator generation of that specific generator of that specific time interval. Now, you need to optimize whether the generator need to be on or off.

That is being done by using a dynamic programming two state problem. So, what we do is you consider u is equal to 0 and then u is equal to 1. Then you find out the value. And this value if it is whichever is the minimum, you consider that. So, I will explain you in detail for this expression basically.

You need to minimize this, right? You need to minimize this. That means f of p for one specific generator for that time interval, right? Plus its startup cost. Anyway, startup cost is also constant. It is not a startup cost is a variable term. So, we can ignore this startup cost also.

Forget this startup cost for time being, right? Then what do you get? f into u minus lambda into p into u, right? This is what you got. Let's say u is equal to 0. u is equal to 0 means you consider that to be off because both the terms are multiplied by u. This is overall 0, right? That means for u is equal to 0, this expression for all the generator is coming out to be 0. So, if you need to consider the generator to be on, the only option for it to be the overall value should be negative, right? For considering u is equal to 0, you get 0 by putting into this expression, right? Now, u is equal to 1, either you may get 0, 1 or negative, 0 positive or negative value, right? If you consider, let's say, if you get a value positive, then that positive value will be lesser than the 0 value because minimum of 0 and a positive value is 0 itself, then better to consider the generator to be off.

Let's say you get 0, then you may consider generator to be in the on state because you need to meet out the load demand eventually. So, keep the generator in on. But if you get the value of negative, it is 100% guarantee that you need to consider this value, you need to consider that specific time interval, the generator should be in the on state, not in the off state. Now, this is just a mathematical interpretation and now I will give you a physical or actual practical understanding of why this we need to consider this to be a negative, right? So, just for this explanation itself, you see, I am saying F of P into U, U is anyway 1 now, I am considering it is on, F of P minus lambda into P. This I am saying it should be less than 0, right? That means F of P should be less than lambda into P.

That means lambda should be greater than F of P by P, got it? That means this lambda you have already chosen fixed. You see here, you already chosen some random lambda to start with. Now, F of P by P for this specific generator I, what is this? F of P by P is nothing but if you can recall, Lagrangian multiplier for that generator is F of P by P only, in a way it is d by d, dP of F of P. That means this F of P by P that you are getting for the generator is nothing but that lambda of that specific generator at this point of time we can assume like that. That means, if that assumed lambda is the initial lambda that you have assumed, so that and the lambda that you are getting for this generator being considering to be an on state, that lambda original lambda is higher than this lambda.

That means you need to minimize the lambda. Lambda indicates what? It is operating cost overall. So, that means now this generator is giving you a hint that you better turn on me, me as a generator, turn on a generator such that the overall cost would be lesser than the previous lambda. You understand this point? So, that means we are in a way considering the generator to be on so that we can reduce the operating lambda. That is what it means, considering it to be negative is more economical than considering to be at a off state that is giving a value of 0.

Now, this is what the explanation is. At minimizing the function with respect to P i of t, U i of t is equal to 0 state, the minimization is stable and equals to 0. And for one state considering it to be on, the minimization with respect to P i of t is minimum of this function. Then as I explained, you would get if you make it as if you differentiate and get a value of negative, then that indicates that that specific lambda of the generator will be cheaper than the overall lambda. So, but in a way we should also taken into consideration that at any given point of time the generation should be within the limits. Let us say the optimal generation that you get, if it is less than or equal to minimum, then what do you get? If it is less than the minimum, you consider it to be minimum because you cannot operate less than the minimum.

If you get the generation within the limits, you consider that to be as the value. If it is greater than P of, then you consider P maximum. So, once you optimize the turning on or turning off state of a generator through dual optimization, then through economic dispatch obtain the generation and that generation should be within the limits. Got it? Now, the two state dynamic problem is solved to minimize the cost of each unit.

U i of t is equal to 0, the minimum is 0. Therefore, the only way is to have the negative value and maximizing the dual function by adjusting lambda. Lambda must be carefully adjusted to maximize Q of lambda. So, various techniques use a mixture of heuristic strategies and gradient search methods to achieve a rapid solution. For the unit commitment problem, lambda is a vector of lambda t and each time interval you have a lambda. So, that means unit to maximize the lambda, that means maximize the lambda means it will give you a hint unit to maintain generation is equal to load ultimate.

So, you get here. Now, using gradient search method, you are updating lambda at any given point of time. After every iteration, you are updating the lambda. So, you have taken the previous lambda, then you obtain the slope d by d lambda of Q lambda. That means if this slope is rising edge, if this is positive means, the slope is positive means you are rising.

I will just show you, take this point. This is rising. That means this is indicating you that you need to increase the generation basically. You need to increase the generation to come out to a point where summation of generation is equal to load. So, that means you aggressively move forward.

That is why considering alpha is equal to 0.01. If you are going in a negative slope, that means it indicates that you need to come back. So, that negative slope means already d by d lambda Q lambda is negative. So, that means with respect to the previous lambda, you are decreasing the lambda now. That means you are going in a opposite direction. So, every lambda you are operating, lambda t plus, this is the slope.

I will just consider the same slope as m into alpha. So, if this slope is positive, alpha is also positive number. That means the next iteration lambda will be higher than the previous iteration lambda because you are adding it. So, that means you are increasing the lambda. This is with respect to lambda.

Now, you see here this is Q of lambda. Now, you are going in a positive direction so that you are increasing the lambda so that you are reaching the peak because you need to maximize the dual function. Now, if it is the slope is negative, so next iteration lambda will be lesser than the previous iteration lambda because this is negative and alpha is positive. That means you are coming back to reach the peak. So, that is the essence. Now, once you get this updated lambda, now the relative size of the gap between the primal function and the dual function is used as a measure of the closeness to the solution.

That means you need to, you need, this is the correction factor or this is that parameter which indicates when to halt the iteration. So, J star minus Q star. This should be as close as possible to 0. Now, I am just giving a flow chart representation. You see here start, peak starting value of lambda for all time interval t is equal to 1 to t.

Then for each generator, you obtain the loop with the two state dynamic program with t stage, with all that stages, solve for each generator whether it should be on or off. Now, solve for the dual function Q star because you need to find out the dQ by d lambda. And then solve the economic dispatch for each one using the committed units. Let us say the 100, out of 100 units, you say 50 units gives you an indication that they can be turned on because other 50 units, the 0 is the most minimal value. So, then solve the economic dispatch for those 50 units and then calculate the primal function lambda of t and then obtain J star minus considering lambda is constant, J star minus Q star by Q star, you obtain this.

If this threshold is, if this gap is greater than the threshold, you keep some threshold value. Then you again update the lambda and go for the next iteration. Otherwise, you have converged for all time interval and just obtain the results, that is it. You see here, start with all lambda t values set to 0, calculate P i of t. Now, if you have lambda d by d P by F of P is equal to lambda, then you get the generation, correct? And also check the generation limit within whether they are in within the limits or not, simple procedure.

And then, next is minimization step. Now, you need to minimize, then maximize, right? So, first minimization, minimization of Lagrangian function. Then now, you got the lambda, you got some generation and then you minimize U because out of three variables, two you have fixed, lambda and U. Now, you have only lambda and P you have got. Now, you need to optimize U, that you are doing using a dynamic programming, using the inequality constraint. I told you, this function F of P plus S minus lambda P should be equal less than 0, then you consider it to be on.

If this is greater than 0, then you consider it to be off state, that I explained you. So, then having, now you got all lambda U and P, but still for those cases where the summation of generation is greater than the load, maximum generation is greater than the load, that is a feasible option. Then you carry out economic dispatch for further minimize, to further minimize the lambda or Lagrangian function. Because the generation that you have considered is based on the lambda which is assumed, that is not the optimal lambda.

Now, we are moving towards optimal lambda. Now, you say you got set of generation, you say now this, find out the economic dispatch among this generation combination. And once you get this, if all these generators cannot meet the hourly demand, if they meet the hourly demand, then you do the economic dispatch. If they do not demand, if they do not meet out the hourly demand, load demand, then what you have to consider is, consider some random very high number. Now, J star you need to minimize, right? Now, you consider some random number as 10,000. Because if you see, J star minus Q star by Q star, this is what the threshold value that you will find out.

Because J star considering some very large number, that means it is indicating that you are not converged, then you go for a next step. It is just a mathematical understanding. You can consider J star to be 1 lakh, 1 crore also, but it takes more time, little bit. So, then using gradient method, you find out the slope, then you obtain lambda. The next lambda need to be updated, because you have to go for the next iteration.

Considering now P and U as constant. So, at any given point of time, you consider two variables as constant and then you are just third one. So, then you calculate d by d lambda of Q lambda and then you check here, summation of load minus generation, right? So, if it is positive, then you increase lambda, you consider alpha to be 0.

01. If it is negative, you consider alpha, it is 0.002. Then maximize the dual function by updating this lambda, right? Then you obtain this error, the gap, duality gap, J star minus Q star by Q star. Then follow the same step. We will take an example to understand. Now, there are three generators which are given to you. Three generators that are given to you, they have their fuel cost and their limits.

And there are four hours, T is equal to 1 to 4, right? So, each hour has their own load demand, right? Now, let me start by considering lambda as 0. First, you have to assume lambda, consider lambda as 0 now, right? So, if you consider lambda as 0, then you obtain d by dP of F of P. That means, let us say for the first generator, it is 10 plus 0.004 P 1 is equal to 0. Right? That means, you get P 1 as negative. P 1 is equal to negative because P 1 is equal to minus 10 by 0.000004. You get some negative value that is this minus 2500. Similarly, you will get P 2 and P 3 also as negative, right? But whether practically can you have a negative generation? Not possible, right? So, but generation cannot be negative, so the generators will be kept offline. So, you need to turn it off, right? So, perform dynamic program by calculating F of P plus S minus lambda of P.

So, even if you put these values, you get this as positive and then that does not have any sense. So, you need to consider this to be an on-off state, right? Now, for the iteration 1, since if there is not enough generation, arbitrarily set the cost for each hour to a high value. Here is J star is 10,000 because we are not able to meet the generation is equal to load with this being the generator in the off state. Correct? So, you consider for each hour, for each hour, lambda is 0 and U 1, U 2, U 3, these are on the states which are 0 now.

You are not turning on and then that means generation is off. Then you obtain dQ lambda by d lambda. Obtain dQ lambda by d lambda. That is nothing but your load.

I will just show you what is dQ here. You see here, this is your Q. You differentiate with respect to lambda. Now, this is off state. So, this is 0. This entire term is 0. U is off. For the first iteration I am saying, this is 0 and this is also multiplied by U. This is 0. Correct? Now, you differentiate only this term with respect to lambda, you get only load, right? So, dQ by d lambda is nothing but the load demand at each interval.

That is what I have put here. You get these values. These are nothing but the load table. And this is P 1 ADC, P 2 ADC, P 3 DC. This indicates, this is the generation you have got with respect to the assumed lambda. This table indicates the more optimized generation after economic dispatch. Now, because we cannot carry out economic dispatch because generation is not equal to load, I mean because the generation are negative and we cannot turn it on, we turned it off.

So, then P 1 ADC, P 2 ADC, P 3 DC is also 0. We need not have to optimize now. There is no question of optimization when being that regenerator is in off state, right? Then you consider fuel, the F of P as some random number 10,000, right? Now, what is Q star? Q star is, because you need to find out J star minus Q star by Q star. Now, J star is summation of all the fuel cost. Now, 10,000 plus 10,000 plus 10,000 plus 10,000 is 40,000. Obtain Q star. Obtain Q star means, for P 1, obtain the value, put the value of P 1, P 2, P 3 which is 0 and there is startup cost which is also considered 0 and this is also 0.

So, this term is 0 and here lambda is 0, right? Because it is the first iteration. Lambda is 0, multiply by 0, whatever may be is there, it is 0. So, 0 plus 0 is 0. So, Q star is 0, J star is 40,000.

So, but in the denominator you have 0, you get infinity. You get infinity. That means error is too high. It indicates you need to go to the next iteration. It is not converged because threshold you might have kept some 0.005 or whatever. So, infinity is greater than 0, this threshold you go for an next iteration. Now, in the next iteration, you obtain updated lambda. First, you start with the lambda only, right? Go for updated lambda. Updated lambda is as I have already told, lambda plus, sorry, lambda is equal to previous lambda. I will just mention like this. Lambda 1 is equal to lambda 2, second iteration, lambda 1 plus dQ by d lambda multiply by of alpha, right? So, previous lambda was 0.

Obtain dQ by d lambda, you have already done that. What is the dQ by d lambda? That is nothing but this load values. You multiply it by of because dQ by d lambda is positive here for all. That means it is indicating you should go in a positive direction. You should increase the lambda. So, dQ by d lambda by d lambda, now what is the alpha? I have told if it is positive, you consider it 0.01. Move aggressively. So, that means 17 into 0.01 because previous value was 0.

So, next lambda will be 1.7, 5.2, 11 and 33.3. You got it, that is what the value I have put here. You got the new lambda. Now, for this new lambda, you obtain P because if you have lambda, you can easily obtain P. dF by dP is equal to lambda. For each interval, you have a different generation.

Now, I am just giving an example for the third generator. Like that you need to do for all the generators. There are three generators, 1, 2, 3. So, for the third generator, you obtain F of P minus lambda P. That is what you need to do because what we are doing? Dynamic programming, now you have to go for dynamic programming.

Before dynamic programming, you get the value of generation. First, you get lambda, you assume lambda. Then you get the value of generation for this lambda. Then you optimize by using dynamic programming. If it is negative, you are considering that to be on. If it is zero, you are considering it to be zero because you are considering the most minimal one.

Now, you got the lambda. Using dF by dP is equal to lambda, you get the value of P. It is very simple. I hope you got this actually. You see here. This is the fuel cost for each generator.

Now, for the first generator as we did, 10 plus 0.004 P 1 is equal to not zero now. For the first interval, it is 1.7. So, now you obtain the value of P 1. Correct? Similarly, you get for the second generator and the third generator.

For the third generator, for example, if you see here, 6 plus 0.01 is equal to what is the third? Value of this thing. It is 3.33 is equal to lambda for the fourth hour. It is 3.3. For the fourth hour, it is 3.3. But you can see here easily you can obtain for the fourth hour, you will get the again generation as negative. So, for the third hour, you see here lambda is, you know, this value is 11. 11, right? Because dQ by d lambda into lambda you are getting 11, right? So, for the third hour, for the third generator, for example, I am saying, for the third hour for the third generator, what you are getting? 6 plus 0.01 is equal to into P 3 is equal to 11. That means P 3 is positive here because 11 minus 6 is 5, 5 by 0.0, you get some positive number. Similarly, for the second generator, you take 8 plus 0.05, P 2 is equal to 11 for the second generator at the third interval.

Correct? Again, you get positive. That means there is some generation happening. You can consider that to be like that, you know. So, there is some generation, it is a positive value you are getting. So, anyway, I am doing for the third generator. That is the reason you get for the only third hour, you are getting negative. This summation is coming out to be negative, F of P minus lambda P. For other values, other interval, you are getting F of P minus lambda P is positive. F of P minus lambda P is positive. That means you need not have to consider that specific time interval as a turn on state for that generator. Correct? So, you get this table for the second interval, sorry, for the second iteration, you get the, you have the lambda, lambda as these values. For the first hour, second hour, all the three generators, you turn it off, especially for the third hour, the generator 2 and generator 3 are showing off.

They are indicating they can be turned off, turned on because their value is negative. F of P minus lambda P is negative. That is the most minimal one. So, that means their maximum generation is 400 and 200. Correct? That is, we have got from the table and then you obtain dQ lambda by d lambda again, dQ lambda by d lambda. Now, here also you cannot optimize because at the third hour, what is the total load? The third hour, the load demand is 1100. Third hour load demand is 1100 because even turning on second generator and third generator, the maximum load that you can cover is 600.

So, you cannot carry out economic load dispatch. So, there is no question of optimizing this generation for the second iteration for all the generators for all time interval. So, now you get the Q of lambda. Similarly, what you got in the previous one, here you use this expression, obtain Q of lambda earlier you got 0, but since now there is some generation, you get a different value.

It is not 0. You get the value of Q of lambda. But because we are not considering all the generator to be in the, I mean the fuel cost is high because now we are not turning on, you get the value of J star as 40000. Right? Now, J star minus Q star is some value 1.67. Now, you go here. Now, again the next iteration, you change the lambda and then you slowly get, you know, get the value of generation like how you got for third interval.

Now, it is second interval also it is appearing. Now, you get this that the summation of generation in the third interval, you can see here in this specific case 600, 400 plus 200, 1200. It is greater than 1100. That means now you can carry out economic load dispatch. That means very simple D f1 by D p1 is equal to D f2 by D p2 is equal to D f3 by D p3 is equal to D l by D lambda, now.

which is equal to lambda. Right? You carry out this economic dispatch. Like the previous very classical optimization technique. You get the value, optimal generation of p1, p2, p3. Right? So, again you update Q, you update J. J is not just 40000. It has decreased now you can see here because you got an economic value.

So, the first and second, third, there may be remaining 10000 only. You need to find out that values. So, ultimately you get J as 36000, Q as 18000.

You get the difference which is 0.965. And you subsequently carry out for all the iterations. You carry out the iterations. The procedure remains the same. Obtain lambda, updated lambda, then get the p. Then with that p you optimize whether it to be turned on or turned off by you going for that expression f of p minus lambda.

If it is positive, don't turn it on. If it is negative, turn it on. Then you get, you carry out the slope dQ by d lambda. Obtain J star, Q star. Again the same procedure you follow. At the last, you know, you see here over a period of time it decreases. The error decreases. The error decreases and you get the values turning up for all the, the low demand is being met over a period of time because you are increasing the lambda.

You are increasing the lambda means you are increasing the generation because you are rising, rising, right? You are increasing the generation. So, at time, at every interval you get the generation which will meet out the low demand. So, gradually you would obtain the set of generation. This is the most optimized generation for all the time intervals.

So, the remarks, the commitment schedule does not change significantly with further iterations. However, the solution is not stable. Oscillation of unit 2, right? There could be some small oscillations. The duality gap does reduce after 10 iterations.

The gap reduces to 0.027 if you carry out for 10 iterations. So, good stopping criteria would be then when the gap reaches 0.05. So, you can consider as minimum as you like, but it takes more iteration. So, this is what is the procedure of Lagrange relaxation and solving a unit commitment problem where you have three set of variables, one is generation lambda as well as the turning on or turning off the states u. So, with that we will conclude unit commitment using Lagrange relaxation. Thank you very much.