Economic Operation and Control of Power System

Dr. Gururaj Mirle Vishwanath

Department of Electrical Engineering

IIT Kanpur

Week - 04

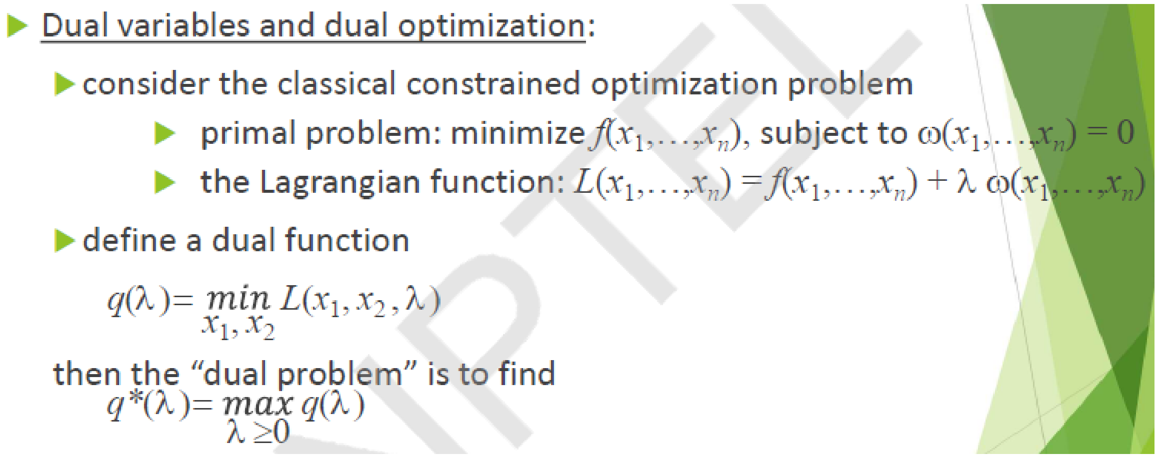
Lecture – 20

Hello and good morning everyone. Welcome you all for the NPTEL online course on Economic Operation and Control of Power System. So, in today's class we will discuss about unit commitment problem solving using Lagrange relaxation approach. So, the dynamic programming method of solution of the unit commitment problem has many disadvantages over large power system with many generating units. In the previous class we have took up a unit commitment problem to solve using a dynamic programming approach. So, there were 2 to the power of N minus 1 combinations right.

So, among this based on strict priority method we just picked up N number of combinations itself and then we could able to solve the problem and then we compared with the 2 to the power of N minus 1 combination again and we saw there was some minor difference with the cost. So, but if let us say if there are 100s of generators and they have their own combinations of turning on or turning off, the problem will be very tedious to pick up which combinations would be visible. So, N number of combinations you may take up, but still that may not give you the solution which is close to the optimal value. So, the difference of solution may be very large.

So, in that case Lagrange relaxation makes more realistic approach, it makes more realistic solution such that you can have a solution which is close to optimal solution right. So, in the Lagrange relaxation technique these disadvantages can disappear, this method is based on dual optimization problem. Now, we will discuss about dual optimization approach. So, another way to solve an optimization problem is to use a technique that solves the Lagrange variables directly and then solves all the problem variables themselves. This formulation is known as a dual function or dual solution.

The Lagrange multipliers used in it are called dual variables. In the case of convex functions this procedure is guaranteed to solve the problem. Now, consider the classical constraint optimization problem, there is a primal problem which is minimization of the cost function which has a variables x1 to xn subject to a constraint which is omega of x1 comma up to xn is equal to 0, this is a constraint. Now, what is the Lagrangian function? There is classical technique, Lagrangian function is nothing but the cost function which is objective function that need to be minimized plus the Lagrangian multiplier with multiplied with the constraint right. Now, we are defining a dual function.



Now, let us say P star is equal to or J star, you can take any denotation, J star is equal to, this is a primal form. This is minimum of x maximum where lambda is greater than or equal to 0. Now, this is your Lagrange function, F0 of x plus lambda into Fi of x. This is your Lagrange multiplayer, this is your cost function, objective function, this is your objective function. Given that the constraint Fi of x, Fi of x should be less than or equal to 0.

You observe these two things, lambda should be greater than or equal to 0 and the constraint Fi of x should be less than or equal to 0. In that case, P star or J star which is primal form which is expressed as minimum of maximum of the Lagrange function. This is your, ultimately this is your Lagrange function. You see, this is your Lagrange function total put together. This is L, right? That means minimum of the total variables x, maximum of L is nothing but minimum of function itself, provided that Fi of x, the constraint is less than or equal to 0.

That means, this is just a mathematical representation of the original objective function F0 of x which can be expressed in terms of the Lagrange function which is maximum of Lagrange function and then you obtain minimum of maximum of Lagrange function that is same as the original function, minimum of F, F0 of x. This is the original problem, right? You need to minimize the, minimize the cost. Now, if you see in the primal form, you can skip the constraint also, right? The Lagrange function you already have that can be expressed in terms of this primal form ultimately, right? Now, how this is valid? Now, if you see here, you just take up, now this is minimum, you keep as it is. Now, let's take up this maximum of F0 of x plus lambda into Fi of x, right? How this is just F0 of x? It is very simple. Let's say your constraint is Fi of x is less than or equal to 0, right? Now, if Fi of x is less than or equal to 0 and this is true.

First case is Fi of x is less than or equal to 0 and this is valid, this is true. Then what will happen? Fi of x is negative let us say. So, lambda is always positive that is we have already defined. So, if lambda is positive, Fi of x is negative, then this term will be negative, right? So, maximum of the cost function which is positive plus this, how do you get this to be maximum? How do you get this to be maximum? Once you have lambda is equal to 0, then you have this overall function will be maximum. If you take any lambda which is greater than 0, then Fi of x is already negative, then this will not take maximum of this function will be always less compared to the Fi of x.

So, then you get when Fi of x is less than or equal to and lambda is greater than or equal to 0, then this maximum of this overall function will boils down to Fi of x itself, right? And then let's say, if this is not true, Fi of x is not less than or equal to 0, right? That means, if it is positive, if it is not negative means it should be positive, if this is positive, then lambda is anyway positive. So maximum of F0 of x plus this term can go to any value, it can go up to infinity, right? So, ultimately you are finding maximum. So, that means when this is not valid and lambda is greater than or equal to 0, the function will be, will take up a value of infinity. So, if this is true, then the function would take a value of F0 of x. Now, this is the whole essence of maximum of the Lagrangian function.

Now, again what you are doing is P star of, P star is equal to J star is equal to minimum of this maximum of Lagrangian function. Now, minimum of either the essence of this is either F0 of x or infinity among these two which should be the most minimum one. No, you have two option. On one hand you have a function which is a finite function and another side you have a infinite number. Among this you have to pick up a minimum number, minimum value.

Which one would you pick up? It is F0 of x because infinity is always a higher number, right? So, minimum of this will be F0 of x only. That is your original function which is a cost function. So, the primal form which looks very complicated which is nothing but minimum of maximum of the Lagrangian function boils down to the function original function itself which is F0 of x provided the constraint is within the limit. Fi of x is less than or equal to 0, right? And then there is also a, there is a dual form of it. This is a primal form and there is a dual form.

Now, the dual form is expressed as D star of, D star is equal to maximum of where lambda is greater than or equal to 0, minimum of F0 of x plus lambda i Fi of x. This is what is dual form because I have already defined in the previous slide, if you just go through here, previous slide, there is a dual form. Dual function is nothing but minimum of Lagrangian function, right? So, and then you obtain maximum of Q of lambda that is your dual form. So, that means maximum of, minimum of Lagrangian function is a dual form, right? And that is what I am just putting up here. Now, the dual form is maximum of, minimum of, this is your Lagrangian function.

So, in a nutshell, I would like to summarize, dual form is maximum of minimum of Lagrangian function whereas primal form J star is nothing but minimum of Q of lambda

Maximum of, maximum of the Lagrangian function, you got it? That is what we have understood. Minimum of, maximum Lagrangian function is a primal form, maximum of, minimum of Lagrangian function is the dual form. Now, we are using this mathematical, you know, understanding of dual and primal form to solve the unit commitment problem which is more complex actually, right? Using, this is called as Lagrangian relaxation approach. Now, let us try to understand the procedure.

The solution involves two separate optimization problems, right? The first requires us to take an initial set of values for x1 and x2 and then find the value of lambda that maximizes Q of lambda, right? So, Q of lambda is the dual function and then take this value of lambda and holding it constant, we find values of x1 and x2 that minimizes lambda of Lagrangian function x1, x2, lambda. That means, first you initialize the variables then obtain lambda for the values that you have taken such that the, for that specific lambda you get maximum of Q of lambda and then take this value of lambda and holding it constant then you obtain again x1 and x2 such that you minimize now the overall Lagrangian function. Now, this process is repeated or iterated until the solution is found, right? Let us take up a simple problem but here it is a continuous function especially why we are discussing this Lagrangian relaxation approach is very much important when there is a problem because unit commitment problem is a discontinuous problem, generator is on or off, it is a discontinuous thing. Now, so for discontinuous kind of problem you cannot have differentiation directly, so you should deal them individually that is why we split the problem and then you use the dual optimization technique to solve the problem. Now, this is a continuous problem, continuous you know function, so here there is no concept of integer number where generator is on or off that is not there, so we are just taking a simple problem.

Now, this is a function that need to be minimized subject to a constraint, right? Now, this is your Lagrangian function, this is objective function plus lambda into constraint. Now, the dual function is minimum of Lagrangian function, right? And this minimum of Lagrangian function you can easily obtain because you can now differentiate, it is a continuous function. Now, you can take up, you differentiate this Lagrangian function with respect to each variable x1 and x2, you get that means dou L by dou x1 is you get as 2 lambda basically with where x2 is kept constant, you just do it 0.5x, I will just explain here 0.5x1, right? Plus or minus lambda is equal to 0 or x1 is equal to 2 lambda, right? Similarly, you differentiate this Lagrangian function dou L by dou x2 where x1 is kept constant, x1 is equal to constant then you get x2 is equal to lambda by 2, right? That is what we have got here.

Now, you apply this x1 and x2, apply this x1 and x2 and obtain Q of lambda, that Q of lambda is nothing but minimum of Lagrangian function of this, then you apply this x1 is equal to 2 lambda and x2 is equal to lambda by 2 to this expression again, right? Then you get Q of lambda is equal to minus 5 by 4 lambda square plus 5 lambda. Then the dual problem is Q star of lambda which is maximum of Q of lambda, maximum of a function means you have to again differentiate, right? So, how do you get the maximum? Then you partially differentiate Q of lambda with respect to lambda. Then because now as I already told, now you keep x1 and x2 constant, differentiate with respect to lambda which is a variable and then you obtain 5 by 2 lambda minus 5 is equal to 0 where lambda is equal to 2. Now, you put lambda is equal to 2 to this expression then Q of Q star of lambda will come out to be 5. Let us take another problem which is closer to our unit commitment problem statement.

Now, let us say there are, there is a function which has 2 variables x1 and x2 and there are 2 variables which are integer type, u1, u2 either 1 or 0. This indicates that generator could be on or off, right? So, this is the function given by:

- Minimize: $f(x_1,x_2) = 0.25x_1^2 + x_2^2$ subject to $\omega(x_1,x_2) = 5 - x_1 - x_2$
- the Lagrangian function: $L(x_1, x_2, \lambda) = 0.25x_1^2 + x_2^2 + \lambda(5 - x_1 - x_2)$
- the dual function: $q(\lambda) = \min_{x_1,x_2} L(x_1, x_2, \lambda) \rightarrow x_1 = 2\lambda$ and $x_2 = \frac{\lambda}{2}$

$$q(\lambda) = -\frac{5}{4}\lambda^2 + 5\lambda$$

- the dual problem: $q^*(\lambda) = \max_{\lambda \geq 0} q(\lambda) \rightarrow \frac{\partial}{\partial \lambda} q(\lambda) = \frac{5}{2}\lambda - 5 = 0$

$$\lambda = 2$$

$$q^*(\lambda) = 5$$

x1 and x2 are continuous real number and u1 is a binary number either 1 or 0.

Now, how do you solve this problem? It is a discontinuous you can see here because you have, you are adding a binary number 1 or 0. You cannot straight away differentiate and solve the thing. You have to separate it out. Now, what you will do is just because there are 2 variables u1 and u2, what is the possible combination? Either you have 00, 01, 10, 11, 4 combination. Now, you try for each of the combination.

- Given $J(x_1, x_2, u_1, u_2) = (0.25x_1^2 + 15)u_1 + (0.255x_2^2 + 15)u_2$

  subject to $\omega = 5 - x_1u_1 - x_2u_2$

  and $0 \le x_1 \le 10$

  $0 \le x_2 \le 10$

  Where, x_1 and x_2 are continuous real numbers, and

  $u_1 = 1$ or 0

  $u_2 = 1$ or 0

Now there are 4 possible solutions:

1. If u_1 and u_2 are both 0, the problem cannot have a solution since the equality constraint cannot be satisfied.
2. If $u_1 = 1$ and $u_2 = 0$, we have the trivial solution that $x_1 = 5$ and x_2 does not enter into the problem anymore. The objective function is 21.25.
3. If $u_1 = 0$ and $u_2 = 1$, then we have the trivial result that $x_2 = 5$ and x_1 does not enter into the problem. The objective function is 21.375.
4. If $u_1 = 1$ and $u_2 = 1$, we have a simple Lagrange function of

$$L(x_1, x_2, \lambda) = (0.25x_1^2 + 15) + (0.255x_2^2 + 15) + \lambda(5 - x_1 - x_2)$$

Then the problem cannot have a solution since the equality constraint cannot be satisfied. You go to the previous this thing. If u1 and u2 is 0, then omega is equal to 5. So, that is not a feasible approach. If u1 is equal to 1 and u2 is equal to 0, we have the trivial solution that x1 is equal to 5 and x2 does not enter into the problem anymore.

The objective function is 21.25. That means, now let us put u1 is equal to 1 and u2 is equal to 0. u1 is equal to 1 and u2 is equal to 0. What do you get? At x1 is equal to 5, you get omega is equal to 0, right? I mean x2 is 0, right? u2 is 0. This is gone. Then 5 minus x1 is equal to 0, right? That means our x1 is equal to 5.

So, that for that means there is a trivial solution possible for x1 is equal to 5. Now, you put this value of x1 is equal to 5 and u1 is equal to 1, u2 is equal to 0, you put these numbers to this original function. You get a value for j, which is nothing but 0.25 into 25 plus 15, right? Into 1 plus 0.

Anyway, this is gone. This is nothing but for this combination of u1 is equal to 1, u2 is equal to 0, there is a solution possible where x1 is equal to 5, the equality constraint is met and the objective function is coming out to be a number which is 21.25, right? Now, take another example u1 is equal to 0, another case and u2 is equal to 1 here and here also you are getting some number x2 is equal to 5, x1 does not enter into the problem and the objective function value is 21.375. Now, last possibility is u1 is equal to 1, u2 is equal to 1.

We have a simple Lagrange function. Now, both x1 and x2 will play a role and this comes out to be like this and if you solve, the resulting optimum is x1 is equal to 2.52488 and x2 is equal to this value and lambda also you solve because now it is a simple problem. You have equations that to be solved, you have x1 and x2, you just get it is a linear problem to be solved and then you get the lambda is equal to 1.2624. With this, an objective function value will be 33.1559. Now, for all these combinations, for 00011011 which is that combination which is giving the most minimal cost or most minimal number that is 10 because at 10 you are getting 21.25, 01 you are getting 21.375, at 1, 00 is not feasible at all. Then for 11, you are getting 33.1559 which is very costly. So, then you can remark the optimal value is 1, 0 and x1 is equal to 5. So, this is what we are freezing. So, what is the remark? In the unit commitment problem, there are variables that must be restricted to two values, either 1 or 0. When there are more than a few 10 variables, there is a systematic way to solve this problem using the dual form because it is just two variables x1 and x2 that means two generators basically and you either switch off or here the most economical solution is saying you have to switch off the generator 2, x2 is 0. I mean x2 is not entering the problem because u2 is 0.

Why do you consider only u1? That is what it is saying. So, but if there are hundreds and thousands of generators and they have their own start up time and the constraints are so many, then this is not so simple as we do it manually. You cannot check the combination 0001 like this. How to solve this? Now, we go for iterative approach. Now, iterative form of Lagrangian relaxation method, the optimization may contain non-linear or non-convex functions.

The iterative process based on incremental improvements of lambda is required to solve the problem. Select a arbitrary starting lambda. Then solve the dual problem such that:

$$q(\lambda) \text{ becomes larger}$$

We are following the same approach. Then update lambda using a gradient adjustment. Lambda t is equal to:

$$\lambda^t = \lambda^t + \left[\frac{d}{d\lambda} q(\lambda)\right] \alpha$$

If you remember the gradient function, we need to find out the deepest descent. We need to find out that slope which will give you the minimum value. We need to find out.

But here, it is a non-convex approach. If you see, the convex means this one where I have explained also in the previous class. Take any straight line between these two points. Whatever the curve is there, all the points lies below this straight line. This is a convex problem. So, what is concave or a non-convex? Most concave, this the concave problem or non-convex problem is this.

For any point, the points lies above the straight line. Now, in the case of convex problem, you find out the descent in such a way that you would find out the minimum value. The objective was to find out the minimum. But in the case of non-convex problem, the objective is to find out the maximum descent. It is exactly the opposite.

That is why the dual problem is just a mirror image. Either you are finding out the minimum. In the dual problem, you need to find out the maximum. Got it? So, that means this, through this, that is the reason why we were telling choose a particular value of lambda, we will get the maximum of Q of lambda. Find closest to the solution by comparing the gap between the primal function and the dual function. The primal function as we have discussed, that is nothing but maximum of Lagrangian function.

This is primal or we call it as J star. The dual problem is Q star. This is nothing but maximum of minimum of L. You understand? They are just like exactly opposite. Now, now find the closeness to the solution by comparing the gap between the primal function and the dual function, the gap between J star and Q star. And the relative duality gap that means J star minus Q star by Q star in practice, the gap never reaches 0.

Primal form: $J^* = min(\max_{\lambda \geq 0, \lambda = c} L)$; Dual form: $q^* = max(\min_{\lambda = con} L)$

- Relative duality gap: $\frac{(J^* - q^*)}{q^*}$, in practice the gap never reaches 0.

That means you need to find out the difference so that the difference should be as close as possible to 0. So, that means what we are eventually doing, we are trying to find out how closely or how tightly the dual and a primal function is coupled to each other. That means let us say for a specific value of lambda, if this is a primal function J star and you are plotting with respect to lambda, let us say. Because lambda we are keeping constant in both dual as well as primal form. Now, let us say this is a dual form, sorry, primal form J star and for a specific lambda, you get the most minimal value.

For the same lambda, let us say this is your Q star, dual form.

$$q^*(\lambda) = \max_{\lambda^t} q(\lambda) \quad \text{where} \quad q(\lambda) = \min_{P_i^t, U_i^t} L(P, U, \lambda)$$

For the same lambda, you are getting the maximum, then they are tightly coupled. But on the other hand, this is your J star, this is with respect to lambda. Let us say you are getting minimum value at this specific lambda, but in the case of dual form, you are not getting the maximum at the same lambda rather you are getting at some other point, then they are not tightly coupled. So ultimately, we are trying to find out that gap between primal and dual form that means we are finding that optimal lambda which gives the minimum value for the primal form and the maximum value for the dual form by iteration.

We are trying to solve this by using iteration. Now this is the loading constraint. Now time period starts from t is equal to 1 to t because load is changing from t is equal to first interval to t is equal to the last interval, t is equal to t. So what is the loading constraint? Summation of load minus generation put together should be is equal to 0. Now this is for all time period. Now if you see here, earlier if we were not considering this unit commitment considerations were not there in the constraint.

Now you need to add this commitment of the load, commitment of the generator which is either on or 0. That means U i of t varies from either 0 or 1. So that multiplied with the generator available for that time and then the summation of load minus algebraic summation of the load minus total generation for that specific time interval should be is equal to 0. And the limits also should be applicable that means U i of t, the ith generator for the t time whether it is on or 0 and if it is based on this, this particular generation should be within the limit of minimum and maximum for all generators for all time to come. And unit minimum, uptime and downtime constraints need to be taken into consideration.

- Loading constraint

$$P_{load}^{t} - \sum_{i=1}^{N} P_i^t\, U_i^t = 0, \ \forall_i = 1,\ 2,\ \dots,\ T$$

- unit limits

$$U_i^t P_i^{min} \le P_i^t \le U_i^t P_i^{max}\ , \forall_i = 1, 2, \dots, N \text{ and } t = 1, 2, \dots, T$$

- the objective function

$$\sum_{t=1}^{T}\sum_{i=1}^{N}\left[F_i(P_i^t) + S_{start-up_{i,t}}\right] U_i^t = F(P_i^t, U_i^t)$$

Now the objective function would be for all time period where the load is changing from t is equal to 1 to t and for all the generation, summation of the generation put together, the fuel cost of all the generator plus the start up cost multiplied by the commitment either 1 or 0. That is your objective function that should be minimum. That means the start up cost and the fuel cost of all the generator for any specific interval should be minimum. Now in a similar way, the economic dispatch problem will be in terms of 3 set of variables now.

Earlier it was just only P and lambda, X, lambda. Now along with P you also have U that need to be finalized, that need to be optimized. Now you can either keep a generator on or off, but which one to consider, whether to be on or off, that is optimization thing that need to be done. And if you need to keep the generator on, let us say you decide it to be on, then how much should be the value of the generator? That decides this P. And ultimately you get the lambda which is the most minimal one.

Now there are 3 variables. So that means there is a cost function plus the Lagrangian multiplier multiplied by the constraint. This is the constraint. For all time interval, this is need to be obtained. The Lagrangian function is nothing but the objective function plus Lagrangian multiplier with the multiplier with the total constraint.

Summation of generation is equal to total load. Now unit commitment requires that the minimization of the Lagrangian function subject to all the constraints. The cost function and the unit constraints are each separated over the set of units. Now there is a cost function and the unit constraint, they are separated now. That means what is done with one unit does not affect the cost of running another unit as far as a cost function, unit limits and the uptime and downtime constraints are concerned.

That means now the individual generator can be dealt in a separate way. And the loading constraint is a coupling constraint across the all the units. The Lagrangian relaxation procedure solves the unit commitment by temporally ignoring the coupling constraint. You can ignore the coupling constraint and deal the individual generator to minimize its cost. Now the dual procedure attempts to reach the constraint optimum by maximizing

the Lagrangian with respect to the Lagrangian multiplier. That means this is a dual function which is maximum of Q of lambda where Q of lambda is equal to minimum of the Lagrangian function.

We are following two step process. The step number 1, find a value for each lambda which moves Q of lambda towards a larger value. That is what we need to find out. Q of lambda, if you plot against lambda, I told obtain the lambda which will give you the maximum value. Now assuming that lambda found in step 1 is fixed, find the minimum of L by adjusting the values of P and U. Because if you see here in the original Lagrangian function, there is a objective function f of Pi of Ui of t and there is a Lagrangian multiplier multiplied by this.

If you fix, at one time you consider lambda is a variable, another if you consider lambda is variable, you fix P and U. If then you fix P and U and consider lambda is variable. First consider P and U fix and lambda is variable and then lambda is constant and P and U is variable.

Now this is your Lagrangian function. This is the objective function. The generation cost of each generator plus the start-up cost for all time, for all the generator put together multiply with their commitment plus this is the Lagrangian multiplier multiplied by the constraint. I am just splitting it out:

- In a similar way to the economic dispatch problem
  - $L(P, U, \lambda) = F(P_i^t, U_i^t) + \sum_{t=1}^{T} \lambda^t \left[P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t\right]$

This is the constraint. Summation of load is equal to generation. That need to be 0. Summation of load minus summation of generation for all time should be 0. This is the constraint. Now what we are doing is just a mathematical jugglery. Now you keep this as it is. Summation of t is equal to 1 to n, 1 to t, summation of i is equal to 1 to n and this is the objective function minus you take this, you take this to because these variables are same, the summation term is same.

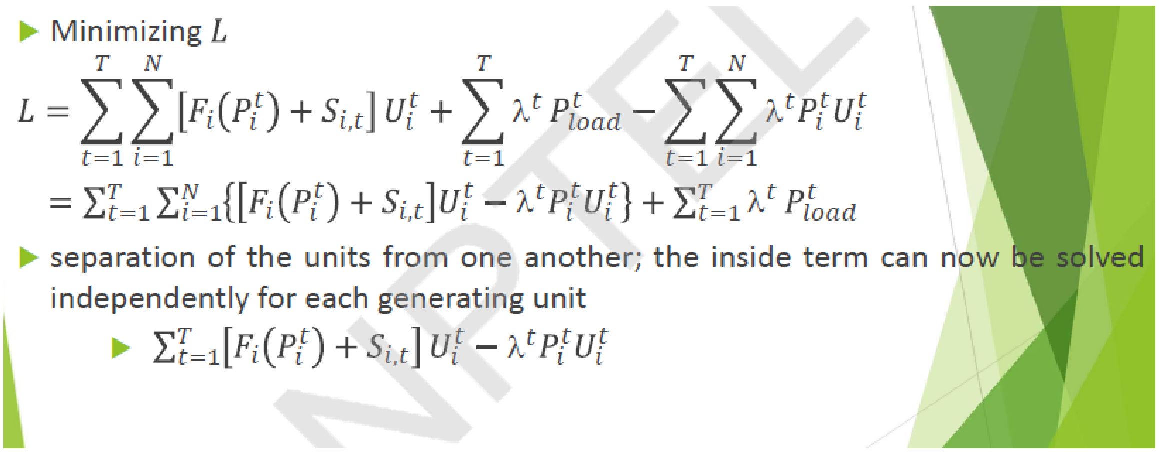
You just take this and put in one side and then summation of t is equal to 1 to 10, 1 to t, lambda t, P load of t. Separate it out. That is it. Now separation of the units from one another, the inside term can now be solved independently for each generating unit.

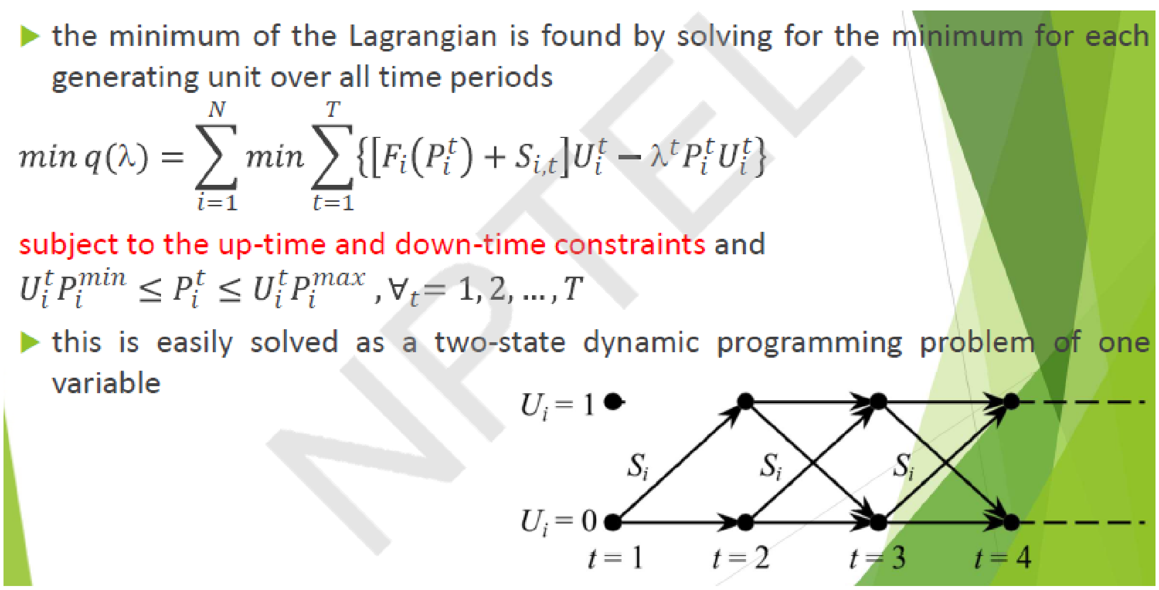
Now you take up this summation of i is equal to 1 to n. That means what you get? Summation of t is equal to 1 to t and this cost function Fi of Pi of t plus start up cost into Ua of t minus lambda t Pi of t Ua of t. This is for one specific generator and that you can deal for separate generators. Now the minimum of the Lagrangian is found by solving for the minimum of for each generating unit for all time periods. If you find out the minimum of each generator for all time periods and put together, then you are eventually getting a minimum of all generators. So that means minimum of the dual function Q of lambda is nothing but summation of i is equal to 1 to n minimum of this function that we have got in the last slide.

Now subject to the uptime and downtime constraints also. This is easily solved as a two state dynamic programming problem for one variable. Now you have got just, you have to solve only one specific generator and you should do it for all the generators. Now for each generator, now the problem will be a dynamic programming problem. In the sense for all time you need to obtain the most possible combination.

That means whether the generator should be either 0 or 1. Let us say we are just dealing with one generator, generator number 1. Now from time is equal to 1 interval to time is equal to let us say 10th interval. You need to find the best possible combination of this generator whether it should be in the 0 state or the on state. And that can be obtained by using dynamic programming where if Ua is equal to 0, you find out what is the function value and for Ua is equal to 1, you find out the function value. Based on that, whichever is the minimum negative because at Ua is equal to 0 that means unit is off and the cost function is 0 only.

But for Ua is equal to 1, it should be less than 0. Then only you can pick up Ua is equal to 1 as the most minimal only. If it is greater than 1, then you can choose Ua is equal to 0. That means you can consider that particular generator to be off for that particular time interval. Minimizing the function with respect to Pi of t at the Ua of t is equal to 0 state.





That means it is off state. The minimization is trivial and equals to 0. That is what I was telling. If generator is off, then the cost will be 0 only. Now at the Ua of t is equal to 1 state, the minimization with respect to Pi of t is minimum of Fi of Pi plus lambda t Pi t.

This is the Lagrangian function, minimum of the Lagrangian function.

Now you differentiate that means you get d by d Pi of the Fi of Pi of t is equal to lambda t. Now there are three cases to be considered for Pi of the optimal value of the generator and the limits. Let us say if the optimal value of the generation is less than the minimum value, then minimum of this function can be obtained by considering not the optimal value of the generator but the minimum of the generator because it has violated the constraint. If this is within the limits, whatever optimal value that you are getting, if it is within the minimum and maximum value, then you consider this as Pi of itself. If Pi maximum, that means if the generation, the optimal generation that you get for the i th generator is greater than the maximum, then you consider it to be a maximum value, not the optimal value.

The constraint need to be considered. Now the two state dynamic program is solved to minimize the cost of each unit. For Ua t is equal to 0, the minimum is 0. Therefore, the only way to have a lower cost is to have the cost function or the Lagrangian function, Fi of Pi plus lambda into Pi of t should be less than 0. Now, using lambda, lambda must be carefully adjusted to maximize Q of lambda and various techniques use a mixture of heuristic strategies and gradient search methods to achieve a rapid solution.

- gradient component $\lambda^t = \lambda^t + \left[\frac{d}{d\lambda} q(\lambda)\right] \alpha$

  where $\frac{d}{d\lambda} q(\lambda^t) = P_{load}^t - \sum_{i=1}^{N} P_i^t U_i^t$
- heuristic component
  - let λ be adjusted upwards at one rate and downward at a much slower rate
  - $\alpha = 0.01$ when $\frac{d}{d\lambda} q(\lambda)$ is positive
  - $\alpha = 0.002$ when $\frac{d}{d\lambda} q(\lambda)$ is negative

For the unit commitment problem, lambda is a vector of lambda t. At each hour, you need to obtain the lambda. Now we follow the simple gradient approach. What we do is gradient component is lambda of t is equal to lambda t plus, this is a rate of change of Q lambda, you need to find out the maximum of this slope, right? This is multiplied by alpha. This is the acceleration factor where d lambda, d by d lambda of Q lambda is this one.

Summation of load minus generation is equal to what you get as d by d lambda of Q lambda. This is the slope. Then heuristic component is let lambda be adjusted upwards at one rate and downward at a much slower rate. Ultimately, you need to find out this Q lambda versus lambda, right? This is the maximum value. Let us say earlier this is here at this point lambda. Now, you find out the rate of the slope of Q lambda, d by d lambda, this is lambda, d by d lambda, rate of change of this slope of Q lambda.

If you are increasing, if this value is positive, that means you are increasing, you are going towards the maximum value. Then you add, that means you consider alpha is equal to 0.01. It is a big number. You are going in a right direction. That means if this slope is negative, that means you are going in opposite direction, then slope is negative. If slope is negative, then you consider alpha is very small number, 0.002. That means you decrease the lambda to come to the maximum value.

Earlier you were here, you are going to reach the maximum lambda. So, you are increasing the lambda because this was positive, slope was positive, then alpha was positive, then the next time, next lambda will be higher than the previous lambda. That means you are reaching the maximum value. If the slope is negative, then alpha should be again positive number only. You cannot take alpha as a negative value because now you are going in the opposite direction.

You need to decrease the lambda to come because you need to decrease the lambda to come to the maximum value. That means what you do, you take a smaller number. Ultimately you need to find out that optimal lambda. So, the relative size of the gap between the primal function and the dual function is used as a measure of the closeness to the solution. So, we will take up the problems in the flow chart in order to understand this in a better way in the next class. Thank you very much. .