Economic Operation and Control of Power System

Dr. Gururaj Mirle Vishwanath

Department of Electrical Engineering

IIT Kanpur

Week - 04

Lecture – 16

Hello and good morning everyone, Welcome you all for the NPTEL online course on Economic operation and Control of Power Systems. In today's class we will learn we will learn about economic dispatch problem using dynamic programming approach. In the last class we have learnt about the classical methods to start with the lambda iteration method. Lambda iteration method we initialize the lambda value and that for that specific lambda value we identified what is the generation because lambda is nothing but dF by dP. So, we could able to get the generation of each generators and then check whether the sum of the generation is equal to demand or not. If that is and for that calculated generations if it is matching the demand then that is the optimal lambda. Otherwise if the generation is higher than the load demand then you have to decrease the lambda. On the other hand if demand is higher than the generation for the calculated generated values then you need to increase the lambda. So, the projected you need to project the lambda right first for the first iteration you take you have taken some lambda then you increase the lambda let us say by 5 percentage or 10 percentage then you project the lambda. So, then projecting the lambda is done with respect to the change in error because you have two different values already earlier with the two iterations whatever value you have got the lambda and the error. So, change in the error how much is the change in the lambda. So, based on that you project the lambda then once you project the lambda then you increase the lambda again for the calculated lambda obtain the generation you go on with the same iteration unless and until you meet total generation is equal to total load. So, then we went on to the gradient method because in certain cases where you have complications in terms of the expressions polynomial expressions like there is a fractional degree and other things. So, the lambda iteration is quite difficult. So, we went to gradient search method. The gradient search method the objective is to find the maximum descent right. So, the maximum descent is found out with respect to each variable. Let us say there are n generators then there will be n plus 1 variable right. So, you obtain the gradient function for the gradient function in this case is the gradient of the Lagrangian function right. Such that there is a tolerance limit decided and each variable will be converging closer to its value which is 0 means dF by dP minus lambda should be is equal to 0 as close as possible to 0.

So, then you will converge there will be some tolerance limit that you fix based on that it will converge. And then, but sometimes choosing lambda in the case of lambda iteration method or gradient search method also have some limitations. So, what we go for modified method right, modified search method and then we make a variable as a dependent on other variables and then so that we can have a better conversions. So, and the best approach is we identify it is a Newton's method where we could able to exactly find out that correction factor and with that correction matter factor we could able to converge in a very quicker manner right. Now, in today's class we will discuss about the dynamic programming based approach. Before that let me give you an analogy of investment plans how best you can get the profit by investing in different ventures.

Investment Amount	Profit from Venture			
	I	II	III	IV
0	0	0	0	0
1	0.28	0.25	0.15	0.20
2	0.45	0.41	0.25	0.33
3	0.65	0.55	0.40	0.42
4	0.78	0.65	0.50	0.48
5	0.90	0.75	0.65	0.53
6	1.02	0.80	0.73	0.56
7	1.13	0.85	0.82	0.58
8	1.23	0.88	0.90	0.60
9	1.32	0.90	0.96	0.60
10	1.38	0.90	1.00	0.60

So, that we can use the same aspect same analysis to the power system problem right. So, it is an we called as forward recursion algorithm there are two types of dynamic programming forward recursion and backward recursion algorithm. So, let us take forward recursion algorithm. So, in this you can see here there are four different ventures, four different companies where a person is ready to invest right.

And the investment amount could range from 0 unit to 10 unit this could be no it is just a per unit value you are very familiar with per unit right similar to that. The year it could be 1 dollar or you can consider to be 1 unit means 1000 dollar or 1 lakh dollar it is up to you. So, now find an allocation policy with 10 units to be invested that is the maximum profit right. Now, in this case of the table is given where it is mentioning that if let us say you have 1 unit and you want to invest total this 1 unit into venture 1 then you will get profit of 0.28 and if you invest in venture 4 you So, now find an allocation policy with 10 units to be invested that is the maximum profit right. Now, in this case of the table is given where it is mentioning that if let us say you have 1 unit and you want to invest total this 1 unit into venture 1 then you will get profit of 0.28 and if you invest in venture 4 you 1 unit means 1000 dollar or 1 lakh dollar it is up to you. So, now find an allocation policy with 10 units to be invested that is the maximum profit right. Now, in this case of the table is given where it is mentioning that if let us say you have 1 unit and you want to invest total this 1 unit into venture 1 then you will get profit of 0.28 and if you invest in venture 4 you you are very familiar with per unit right similar to that.

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So, let us take forward recursion algorithm. So, in this you can see here there are four different ventures, four different companies where a person is ready to invest right. And the investment amount could range from 0 unit to 10 unit this could be no it is just a per unit value you are very familiar with per unit right similar to that. The year it could be 1 dollar or you can consider to be 1 unit means 1000 dollar or 1 lakh dollar it is up to you. So, now find an allocation policy with 10 units to be invested that is the maximum profit right. Now, in this case of the table is given where it is mentioning that if let us say you have 1 unit and you want to invest total this 1 unit into venture 1 then you will get profit of 0.28 and if you invest in venture 4 you will get a profit of 0.2. Similarly, at point at venture 2 you will get 0.25 venture 3 you have 0.15 of profit right. Similarly, at different investment level each venture has a different range of profits that is offering to the investor. And it is not you know linearly increasing or decreasing it is a random value that is coming up. For example, if you see here sometimes the investor is investing at venture 1 if it is investing let us say 2 investing 10 unit for the for the venture 1 you should get 0.28 into 5 right if you do the mathematics, but that is not the case here 0.28 into 5 should be 1.4, 2 unit then he is getting 0.45 and reasonably if you think if he is but he is getting 1.38. So, you cannot you know use the simple mathematics to have this analysis. So, this profit varies over the investment and across the different ventures also.

Now, this is the table. Now, given a limited amount of money to allocate the problem is to find the optimal investment allocation. That means if I have 10 units to invest whether is it worthy to invest just all the 10 units in venture 1 or can I invest let us say 4 unit in venture 1, 3 units in venture 2, 2 units in venture 3, 0 units in venture 4 whatever. So, what is that combination of investment in different ventures for a specific amount of investment that a investor is interested to do what is the best combination. So, that is what we are interested to find out now. So now, let x1, x2, x3, x4 be investments in 1 through stage 4. So, that means the profit of investment in x1 is Vx1 the profit once I obtains in investing the venture x2 is Vx2 similarly Vx3 and Vx4 these are the profits. So, x1 plus x2 plus x3 plus x4 is equal to 10 that is a constraint maximum investment that can be done is 10 unit. Now, to transform this into multi stage problem let the stages be x1, u1, u2 and a. So, if you remember dynamic programming we have we divide a problem into different stages and in each stage you have different nodes states in each stage you have different states or nodes and there is a policy. So, we are finding sub optimal policies to reach out to the ultimate policy.

So now, the stage 1 we call it as u1. So, where u 1 is nothing, but x1 plus x2 investment across venture x1 and investment at venture x2 and the condition is u1 should be less than or equal to a, a is the ultimate destination that means the total investment. Obviously, investment of 1 and 2 should will be less than investment of 1 plus 2 plus 3 plus 4 and then u2, u2 is that the next level next stage that includes the investment at the previous stage which is x1 plus x2 plus investment at the third stage which is x3. So, u1 plus x3 will give you u2 and ultimately a is nothing, but u2 plus x4. Now, the total profit is the summation of the cost or the profit obtained with respect to the individual investment. Now, which can also be written as v1 x1 plus v2 of u1 minus x1, u1 is x1 plus x2 that means x2 is u1 minus x1.

Similarly, you express with respect to the different stages and at the second stage we can compute f2 of u1 is equal to maximum. Let us say you are calculating at stage 2 where there is only 2 investment which is considering right. So, that is x1 and x2. So, we are finding the maximum profit at the stage 2 is the profit the maximum profit that one can get with the investment across both the ventures v1 x1 and x2 right. Now, let us try to understand with this problem. So, x1 or x2 or u1 it is varying from 0 to 10, the profit one gets is v1 of x1 by investing at x1 and v2 of x2 which is investing at x2 and f2 of u1 is the maximum of v1 of x1or v2 of x2. Then we will see if the investment is 0 then you will get 0 comma 0 investment. If let us say the investment is 1 unit then v1 of x1 suggest This suggest if you have 1 is 0.28 and v2 of x2 suggest it is 0.25 right 0.28 and 2.25. So, if let us say you have just 1 unit to invest. So, it is common sense that you invest totally in v1 x1 that is x1 and that is coming out to be 1 comma 0 combination unit you invest completely on x1 and 0 at x2. Let us say you have 2, 2 units to invest then the possible combination is you completely invest 2 units in venture x1 or total 2 units in venture x2 or 1 each in x1 comma x2 right. Now, if you invest total 2 units in x2 only then what you will get 0.41 that is what the table is indicating and if you invest total 2 in x1 then you will get 0.45. But let us say you invest in x1 and x2 then what it is coming out to be 0.53. So, which is the best among them investing on x1 and x2 because that is coming out to be 0.53. So, that is what we are considering here you invest on each 1 unit can be invested on x1 and another unit can be invested on x2. Let us take another example. Let us say you have 3 units to invest you have got 3 units to invest.

Now, possible combination are let us say I will invest total 3 in x1 and total 3 units in x2 right or let us say you know 1 comma 2 that is x1 comma x2 or 2 comma 1 which is x1 comma x2 correct. Let us try to see these combinations first. If you invest totally on x1 you will get how much 0.65 total 3 units are invested on x2 you will just get 0.55 profit. Let us say 1 comma 2 combination that means you are investing 1 on unit on an inventory x1 that is 0.28 plus 2 units on x2 which is 0.41. How much it is coming 0.69. Let us say you invest 2 unit on x1 and 2 and 1 remaining 1 unit on x2 then how much you get 0.45 plus 0.25. How much it is coming 0.7. So, among them which is the best combination maximum profit you want right. You do not want to have minimum profit. So, you want to have a maximum profit as from the investor point of view.

So, you will get 0.7 as the best value. So, the combination is 2 comma 1 like that you exercise for all different units of investment across x1 and x2. Then you will get optimal sub policies at this specific stage for the investment in x1 and x2 and that is and this combination is given here. Now moving forward let us move on to the next stage. Let us move on to the next stage. Now, again you have this investment 0 to 10 and from the previous investment you have got the optimal cost maximum profit you have already got right that is 0.28, 0.53, 0.7 up to 1.68. again you have this investment 0 to 10 and from the previous investment you have got the optimal cost maximum profit you have already got right that is 0.28, 0.53, 0.7 up to 1.68. You do not want to have minimum profit. So, you want to have a maximum profit as from the investor point of view. So, you will get 0.7 as the best value. So, the combination is 2 comma 1 like that you exercise for all different units of investment across x1 and x2. Then you will get optimal sub policies at this specific stage for the investment in x1 and x2 and that is and this combination is given here. Now moving forward let us move on to the next stage. Let us move on to the next stage. Now again you have this investment 0 to 10 and from the previous investment you have got the optimal cost maximum profit you have already got right that is 0.28, 0.53, 0.7 up to 1.68. maximum profit you want right. You do not want to have minimum profit. So, you want to have a maximum profit as from the investor point of view. So, you will get 0.7 as the best value. So, the combination is 2 comma 1 like that you exercise for all different units of investment across x1 and x2.

Then you will get optimal sub policies at this specific stage for the investment in x1 and x2 and that is and this combination is given here. Now moving forward let us move on to the next stage. Let us move on to the next stage. Now again you have this investment 0 to 10 and from the previous investment you have got the optimal cost maximum profit you have already got right that is 0.28, 0.53, 0.7 up to 1.68. x2 then how much you get 0.45 plus 0.25. How much it is coming 0.7. So, among them which is the best combination maximum profit you want right. You do not want to have minimum profit. So, you want to have a maximum profit as from the investor point of view.

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So, you will get 0.7 as the best value. So, the combination is 2 comma 1 like that you exercise for all different units of investment across x1 and x2. Then you will get optimal sub policies at this specific stage for the investment in x1 and x2 and that is and this combination is given here. Now moving forward let us move on to the next stage. Let us move on to the next stage. Now again you have this investment 0 to 10 and from the previous investment you have got the optimal cost maximum profit you have already got right that is 0.28, 0.53, 0.7 up to 1.68. This is the maximum profit that you could get by just investing on x1 and x2. You just carry this table as it is from the previous stage. Now there is one more way to do this venture which is available for you and their profit is already been given it is been already defined. Now let us say you start now again from the beginning. You want to invest you have just 1 dollar or 1 unit with you. What is the best combination to invest across x1, x2 and now x3 also. So, with x1 and x2 you already have this value 0.28 with x3 you have 0.15 but you have just 1 unit to invest let us just totally depending upon investment on x the best among x1 and x2 which is 0.28. And what is the combination from 1 and 2 you got 1, 0, 3 is no more relevant now. So it is just 1, 0, 0. And then you let us take another one or two examples to have a better understanding. Let us say you have 2 units to invest. Now again the same philosophy you have to apply either you go for total two investments on F2 of u1 or total two investment on x3 or one each on F2 of u1 and x3. Let us say if you are totally investing 2 units on the best combination of x1 and x2 which we call it as F2 of u1 then what is the profit that you are going to get 0.53. Now if you are totally investing on x3 venture how much you are getting 0.25. So, which is the best again totally you invest on or one each you take you take one each. Let us say one each means 0.28 plus 0.15 you say this is 0.28 for 1 and 1 if you invest 1 unit in x3 which is 0.15 Totally how much you are getting 0.4 if you invest 1 1. But what is the best already you have 0.53 available which is the maximum profit that you can get. So, you better invest totally on the best combination of x1 and x2 again you will get 1 comma 1 comma 0 because in the previous case for unit 2 investment the best combination was not 0 sorry 2 comma 0 or 0 comma 2 it was 1 comma 1 that is what is given here 1 comma 1 and you just retain as it is.

Similarly, you do it for all the investment and you will get the optimal sub policies at stage 3 as per this entries. Now, let us move on to the last stage. So, in this stage because you have 4 ventures now you move to the next stage. In the next stage you have again same 0 to 10 right 0 to 10 and you have the best combination available from the previous one x or considering x1 x2 and x3 put together. Now, we have a new venture to add on which is x4. Now, we will just take randomly any number let us say I will take 3 here right. I have 3 units to invest now 3 units to invest now. Now, if I have 3 units to invest then the possible combinations could be let us say I will just try to figure out let us say I will take total 3 on the best combination of the previous one which is F3 of U2 which is coming out to be 0.7. If I totally invest 3 on x 4 which is coming out to be 0.42. Let us take another combination of 2 plus 1 right that means 2 units from F3 U2 and 1 from x 4 how much it comes out to be 0.73 right. This combination gives you 0.73 right or let us say you take 1 from F3 of U2 and 2 from the x 4 how much it is coming out to be 0.28 plus 0.33 how much it will come 0.43, 0.53, 0.63, 0.61 right. This comes out to be 0.61 earlier you have if you totally invest on F3 of U2 it will get 0.7 or just x 4 it is 0.42 1, 2 combination 1, 2 combination will give you 0.61 but 2, 1 combination will give you 0.73

so that is just take randomly any number let us say I will take 3 here right. I have 3 units to invest now 3 units to invest now. Now, if I have 3 units to invest then the possible combinations could be let us say I will just try to figure out let us say I will take total 3 on the best combination of the previous one which is F3 of U2 which is coming out to be 0.

7. If I totally invest 3 on x4 which is coming out to be 0.42. Let us take another combination of 2 plus 1 right that means 2 units from F3 U2 and 1 from x4 how much it comes out to be 0.73 right. This combination gives you 0.73 right or let us say you take 1 from F3 of U2 and 2 from the x4 how much it is coming out to be 0.28 plus 0.33 how much it will come 0.43, 0.53, 0.63, 0.61 right. This comes out to be 0.61 earlier you have if you totally invest on F3 of U2 it will get 0.7 or just x4 it is 0.42. 1, 2 combination will give you 0.61 but 2, 1 combination will give you 0.73 so that is the best. So that means you invest 2 units on f3 of U 2 right and one unit on x4. If you consider 2 unit investment on f3 of U 2 what is the best combination of for 2 unit investment on f3 of U 2 that is 1, 1, 1, 1, 0 right. Then ultimately you will get x1, x2, x3, x4 as 1, 1, 0 and then a investment of 1 unit on x4 which is 1. So, you will get a combination of 1, 1, 0, 1 you understand this point. Similarly you have to do it for all the investment. Let us summarize whatever we have understood because this is very important to move forward to understand our power system problem right. So, it was not necessary to enumerate all possible solutions instead an orderly stage wise search is used the form of which is the same at each stage. The solution is obtained not only for A is equal to 10 the interestingly but for all the complete set of values from 0 to 10. See your interest was only 10 ultimately you wanted to have this combination 4, 3, 1, 2 but in a way you get at every stage what is the best combination for every investment for every possible investment across different ventures x 1, x 2, x 3 and x4. Why this is important? Let us say you have invested in 4 ventures x 1, x 2, x 3, x4 and due to some reason x4 is totally collapsed that venture is collapsed right. But still you have investment on x 1, x 2, x 3 now you can see what is the best combination on x 2, x 3 itself then for that specific investment for that you can easily look into the table and get the possible best possible combinations. For example, I will say here the total optimal policy contains only optimal sub policies for instance if A is equal to 10 then the best combination 4, 3, 1, 2 is the optimal policy and for stages 1, 2 and 3 for stages 1, 2 and 3 if the x4 is gone then considering 1, 2 and 3 what is the best investment you get 3 units of best unit I mean 4, 3, 1 that is coming out to be 8 units of investment. Let us say x 3 is also gone venture is gone then having 1 and 2 in hand then you have what is the best you can go for 4, 3 investment if you have 7 units of investment to make right if you have just 1 unit left out 1 venture left out which is x 1 and you have to make 4 units of investment then best is 4 correct. So, notice also that by storing the intermediate results we could work a number of different variations of the same problem with the data already computed. Now, we already computed say 1 venture is gone you will have a other best sub policies available with you. Now, we will move on

to economic dispatch using dynamic program that means with the understanding that we have got now we will apply the same thing in power system problem. If non-convex input output curves are to be used we cannot use an equal incremental cost methodology since there are multiple values of megawatt output for any given value of incremental cost. Let us say you have in a specific even in a specific generator there could be multiple valves. So, you can see here the cost the heat characteristics or cost characteristics that you get it is not linear you know it is not smooth basically it is not smooth it is highly non-convex problem. For non-convex problem classical methods that we have used like gradient, lambda iteration and Newton's method have some limitations. So, that is that holds good for gradient approach because 1th first derivative is what required that you can get, but here it comes out to be like double derivative or something like that. So, here obtaining solution with the help of classical methods is little bit cumbersome. So, then we go for either heuristic methods, heuristic methods include particles swarm optimization, artificial intelligence, ANN based approaches. So, there are so many heuristic methods available, heuristics approaches they go for they search the solution actually in a given space. There could be you know some specific search you may not get a exact optimal result, you may get a better result, but it need not be a best result. So, dynamic programming such kind of scenarios can give help you get a best results. So, the dynamic programming solution to economic dispatch is done as an allocation problem. Using this approach we do not calculate a single optimum set of generator megawatt outputs for a specific total load supply rather we generate a set of outputs at discrete points for an entire set of load values. For all the possible load combinations let us say from minimum to maximum we find out the best possible generation values. Now, let us take a problem to understand. Assume there are 3 units in the system all are online, you have 2 generate 3 generators P 1, P 2, P 3 are the generations from each of the generator right and the fuel cost associated with each of the generator is F1, F2, F3 right Their input output characteristics are not smooth nor convex, data are as follows. You can easily check also from here from 50 megawatt let us say for each generation of P 1, P 2, P 3 where generation is varying from we have the data from 0 to 225 megawatt of generation right. If that means 0 generation is equal to 0 the fuel cost are not applicable that is why we have mentioned as infinity error it is not applicable. That means there is no scope for operating these generations at 0 value 0 megawatt of supply. It starts with 50. So, at 50 you see for the fuel cost associated with the first generator is 810 dollars per hour right. For the second generator you have 750, for the third generator you have 806 that means they have different cost for each generation. And with respect if we increase from 50 to 75 you see their variation is also non-linear 810 to 1355 and 750 to 11155 and their proportionate variation is also different and correspondingly if you check from 75 to 100 also you see they are not linear basically it is not linear rise with respect to individual unit also. So, ultimately you have the total fuel cost for each generation right with respect to each generator. Now, let us say a problem is given where you need to identify the best possible generations for the load demand of 310 megawatt. This does not fit the data directly right because there is no generation value exactly equal to 310 megawatt. Either you have I mean any possible combination you take the generation combination would give you either 300 megawatt or 325 megawatt. But your demand lies between 300 and 350. So, we need to interpolate ultimately right. So, let us start with the help of dynamic programming.

So, scheduling units 1 and 2 we find the minimum cost for the function. Let us start with unit 1 and unit 2 and then take up unit 3. Now, the fuel cost or the total cost of F2 is that means at stage 1 is 1 or stage 2 whatever you can consider. So, that is the total cost involved in generation with respect to the first unit F1 of P1. F1 of P1 is something but F1 of demand minus P2 right because P1 plus P2 will be equal to demand plus total fuel cost associated with P2. So, F1 of P1 plus F2 of P2 will give you total cost right considering just unit 1 and unit 2. Over the allowable range of P2 for 100 to 350 megawatt of demand. That means let us consider just P1 and P2 what is the minimum generation that can happen combining P1 and P2? It will be 50 plus 50 right because 50 each this comes out to be Pd is equal to 100 megawatt. What is the maximum that you can get? From P1 you get 200 right. This is the maximum above which it is not possible to generate and for P2 you get the maximum as 150. So, 200 plus 150 will give you Pd maximum as 350 megawatt right. So, for the allowable range of P2 you have the demand variation from 100 to 350 megawatt. Now, we need to save the cost for serving each value of demand that is minimum and the load level on unit 2 for each demand level. Now, yeah this table will give you a better understanding. Now, at first stage we are in first stage. Now, you vary the demand from 0 to 350 megawatt. Now, simply you enter the data here. The actual demand will start from 100 to 350 for the combination of P1 and P2. Then the fuel cost associated for the first generator you just put as it is that you are getting from the previous table from this table right. So, for the generation 1 it varies from 50 megawatt to 200 megawatt minimum to maximum and you have a individual cost which is varying from 810 rupees to 810 dollar per hour to 2760 dollar per hour right.

Now, for this combination now this is with respect to the generation 1. In this entry this row we are varying generation of generator 2 from 50 megawatt which is the minimum generation of that particular generator to 150 megawatt which is the maximum generation of the generator 2 right. Now, for each combination of the generator I mean let us say let us freeze 50 megawatt. For 50 megawatt of fixed operation of the generator 2 you check what is the cost that is that you are getting while parallelly operating with generator 1. The cost associated with the generator 2 for generating 50 megawatt of power is 750 dollar per hour that you get from the table. Now, to start with from 100 to 350 megawatt we are varying from 100 to 350 megawatt because that is the combination that you get of the demand variation from 100 to 350 megawatt.

Let us say you are supposed to operate generator 2 at 50 megawatt right but the load is 100 megawatt. Then what is the remaining generation? 50 megawatt. Now, you calculate the total cost of generation of P1 which is at 50 megawatt and P2 which is 50 megawatt. That comes out to be 810 rupees 810 dollar per hour plus 750 dollar per hour.

It will come out to be 1560 right. Let us say you will go to the next demand level. The next demand level is 125 megawatt but you are just calculating for the fixed 50 megawatt generation. Now, 125 minus 50 is 75. So, now 75 you have to generate by using generator 1. So, what is the cost associated with the 75 megawatt of generation of generator 1 which is 1355. So, 1355 plus 750 will give you the value of 2105. Now, as the total demand varies from 50 to the further values right 50 to 250 you have the data entries here because beyond which you do not get any generation from generator 1, because 50 have already fixed right. 50 have already fixed for generator 1. T he maximum you get from generator 1 is 200. So, 200 plus 50, 250 is the maximum value that you get with the combination of generator 1 and generator 2. Beyond which even if you operate P2 at 50 there is no generation available from generator 1 to support let us say 275 megawatt of generation or 300 megawatt of generation.

You understand this point. Now, you get these values entered here. Similarly, you take the next generation capacity of generator 2 which is 75 megawatt and the cost associated with the 75 megawatt of generation from the generator 2 is 1155 dollar per hour. You just carry out the same iteration like we did for 50 megawatt with respect to this entry. You understand? For example, I will just take one data to help you understand. Let us say you want to operate at 250 megawatt of demand where fixed generation of generator 2 is 75 megawatt. Now, how much is the left out? Which value that I had considered? 250 right. So, let us take 250 megawatt of load demand.

Now, we are dealing with 75 megawatt of generator 2. So, remaining is 175. For 175 you check what is the value? 2427.5. So, 2427.5 plus 11155 that will give you 2582.5 and what is the maximum capacity here again? If you are fixing at 75 the maximum generation available from generator 1 is 200. Now, you can see you can go to the next level here. Earlier we stopped at 250 megawatt. When we fixed generation of generator 2 as 50 megawatt, we could only reach up to 250 megawatt of demand. Now, because you have enhanced the generation capacity of generator 2 to 50 megawatt from 50 megawatt to 75 megawatt, now you can provide an extra supply of 25 megawatt to that demand. So, you can calculate up to 275 megawatt. Similarly, you keep on increasing the generation capacity from its minimum of 50 megawatt to maximum of 150 megawatt capacity and for each combination you get a set of values, right? The set of values that is been tabulated here. Now, having got these values, now you have to check what is the best possible combination of generation for each demand.

Let us say at generation 50 for let us say you are starting from 100 megawatt, right? You are starting from 100 megawatt. This is 100 megawatt. Now, you have only one option available which is generator to operating at 50 megawatt and the best cost is that itself 1560 and this generation associated with this with that is 50 megawatt. Now, you take the next step which is 125 megawatt of load demand. Now, with for 125 megawatt of load demand, now you have two possible generator combination of generator 2 which is either 50 megawatt of operation or 75 megawatt of operation.

Now, you check which is the best. That means whether 2105 is the lowest cost or 1965 is the lowest cost, the obvious is 1965. That means if you have the load demand fixed which is 125 megawatt, what is the best operate of possible operation of generator 2 is 75 megawatt. Like that let us say you have you know this is where you get let us say 250 megawatt of load demand. Now, 250 megawatt of load demand can be catered at four different combinations of generations from generator 2 which is either 50 megawatt, you can generate by using 50 megawatt of generation or 75 or 100 or 125 or even 150 also.

There are five possible combinations are there. So among them, you can check which is the lowest. So, if you want to operate at 250 megawatt, if you want to supply a demand of 250 megawatt, so it is advised to operate the generator 2 at 150 megawatt and remaining 100 megawatt is catered by generator 1. That will give the cheapest cost. So, ultimately you got the best possible generation for each variation in the load demand if you have only two combination of generators which is generator 1 and generator 2.

Now, that is what we have summarized here in this table. Now, let us go to the next stage. Earlier you had just 1, 2. Now, you add upon third generator. Now, total cost is the cost associated with the generation 3, F3 of P3 and total demand minus P3. That means is equal to P1 plus P2 ultimately. So, you have the from previous table, you have the best combination of 1 and 2. Like how we did solve for the ventures investment. You have the best combination of 1 and 2.

Now, you are just bringing in generator 3 also. The same approach you need to follow. There is no difference. The same approach you follow. Let us take one example to understand. Now, let us say you have once, let us take any number. Let us say you have 275 megawatt of load demand to cater. Now, by taking 50 megawatt of generation from generator 3, what you get? 806 dollar per hour and if you take 50 megawatt and 275 megawatt of load demand, the remaining 225 need to be catered by the best combination of generator 1 and generator 2. Now, 275 megawatt minus how much? 50 is 225. Now, what is the best combination that it is clearly indicating from this table? This is 311.5 dollar per hour. So, 311.5 plus 806 will give you 3921. Similarly, you do it for all the generations.

While generation is increased from 50 megawatt to 175 megawatt. You have to kindly note this point because each generator has its own limits. How that limit is, limit you are obtaining is because of the demand which is fixed from within the range of 300 to 325. Now, it is just as it is. Now, you get the best combination of generators, best combination of generation from generator 3 at this stage. Now, ultimately you are interested in last, you are interested in these two entries only because ultimately your concentration is 300 and 325. Between this you need to find out 310. So, I am just asking you to summarize that in a table. So, for 300 megawatt you have the best combination from the previous table 4035, 4377.5 and you have the entries at different levels of generations.

So, you get F3 as 4168. You see here F3 as 4168 and then P3 as 150 megawatt. This is the best combination for 300 megawatt of generation. For 325 you have F2 of double D. F2 of double D is the best combination of generator 1 and generator 2. That is 4377.5 and the best combination of generator 3 is again 150 megawatt but that is coming out to be 4463. Now, you see here finally you get the list for 300 megawatt of generation, 300 megawatt of load demand the cost associated is 4168 and the combination is P3 is 150 megawatt because now you get for 300 megawatt of generation from previous table what is the best generation 150 megawatt and you should check if you subtract 150 from 300 what is the remaining you get 150 megawatt. So, you should check for the best combination of generator 1 and 2 with respect to 150 megawatt of generation. So, that for 150 megawatt of generation you freeze P2 as 100. So, the remaining will be scattered by So, generator 1 that means ultimately you get P3 as 150, P2 as 100 and remaining as P1.

Similarly, you get for 325 megawatt. So, this is the entry you get. Now, ultimately your generation is the P1 is already fixed which is 50 megawatt. And P3 is also already fixed which is 150 megawatt which is not varying. The only variable which is varying is generator 2 that is from 100 to 125 that means you can consider this generator to operate as it is supposed to vary from 100 to 125 and that is what it is indicating by varying from 100 to 125 it is giving the best possible solution. Now, you rather than operating at 125 you operate from 100 to 110 that is it because your load demand is not 325 it is just 310 megawatt. So, for the 310 megawatt, for generator 2 you have to interpolate that is it.

For 100 megawatt of generation you have a specific cost for generator 2, for 150 you have another sorry for 125 you have another cost. So, you just have to interpolate what is the cost? Tentative cost that is 1478. So, you have total cost of generator 1 for 50 megawatt which is 810. For generator 3 which is operating at 150 megawatt you have this cost. So, you just have to add these 3 to get the total cost of operation which is 4286 dollar per hour. For this specific load the combination of generator 1, 2 and 3 will give you the best cost which is 4286 dollar per hour. I hope this is clear to you. So, that is it for today. Thank you very much. .