Economic Operation and Control of Power System

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Week - 03

Lecture – 14

Very good morning dear students and today I welcome you all to the course on Economic Operation and Control of Power System which is NPTEL driven. And we are getting into a new topic today which is slightly different than what we talked about during last couple of lectures. And the topic is calculation or determination of transmission losses. Now before I do get into this you must be surprised to know that why we are keen to calculate transmission losses of a power network and for a course of economic operation and control of power system. In the beginning itself I told you what is the main objective of economic operation. We do have loads that loads need to be catered over consumers and we have generators which are situated thousands miles away and those power need to be transmitted through transmission lines and then catered to the city point.

As an example if the consumers are in Delhi and if the generator is situated in Kota, so from Kota to Delhi the power need to be transmitted. So we are supposed to know how much energy we lost due to the transmission. So as you know that I square R loss which is a part of your system loss. So if the consumer demands 500 megawatt then I have to generate more than 500 megawatt or less than 500 megawatt.

To accommodate the losses I have to generate more than 500 megawatt so that even after the losses the loads can be met. So it is very important for me to know to calculate what is the transmission loss for delivering a particular amount of power in a given corridor. Now one more challenge I test to this. In the case of economic operation we optimize the generation output so the cost is minimum. And if those optimal generation setting points that you have determined through your algorithm and if they are even after accommodating the transmission losses able to meet my load then the system is optimized.

But if I obtain a solution which is optimal but not able to cater the consumers as designed then there is no fun in solving the optimal solutions. So the next phase what I think in couple of classes earlier we must have learned a determination of generation output in optimal manner without considering losses but slowly today we will focus on determination of optimal generation points of different generators to meet the loads including transmission losses. So that is very very interesting topic. Please focus next 30 minutes for a better understanding. So what I wanted to tell you here the first question to understand the concept let us consider a simple numerical example to understand and there are two generators the incremental fuel cost for two generating units G1 and G2 are given by the following equation.

So as I told you the characteristic the cost function will be given in the form of a function of generators outputs PG1 and PG2. Don't get confused about the suffix and super suffix so PG1 means the generation output of the unit 1 PG2 means generator output of the unit 2. Now the equations are given to you IC1 which is 25 plus 0.2 PG1 and IC2 which is 32 plus Now having those equations my responsibility is to determine PG1 and PG2 where PG1 and PG2 are the real powers generated by different units. The economic allocation for a total load of 250 megawatt.

So what is my responsibility to meet 250 megawatt neglecting the transmission losses. So what we do is quite simple the incremental cost the equations are same so PG1 IC1 equal to IC2 and if you solve these equations you will get actually and if you simplify further you will get PG1 minus PG2 equal to 35 and the summation of PG1 and PG2 must be 250. So if you solve these two equations and you will get PG1 which is 142.5 megawatt and the PG2 which is 107.5 megawatt.

IC1= IC2

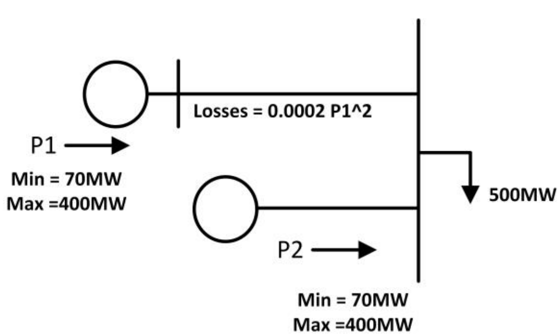
PG1- PG2 = 35 …(14.1a)

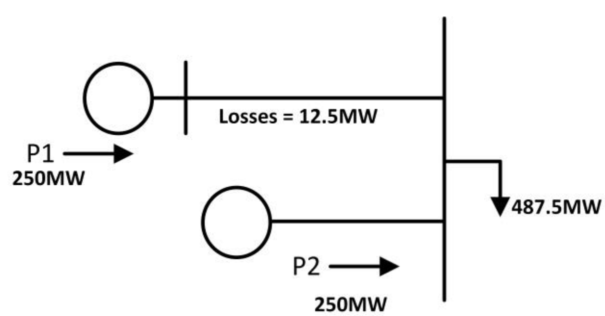
and PG1+ PG2 =250 …(14.1b)

From above equations 14.1a and 14.1b we get:

PG1= 142.5MW and PG2=107.5MW

So if you add these two what I am getting actually I am getting PG1 plus PG2 which is 250 megawatt am I right.





As well as if I substitute the value of PG1 and PG2 that I have calculated in those two cost equations and that will be the minimum cost all right. So that is through Lagrangian multiplier so the moment I say to the IC1 equal to IC2 this you have obtained through the basic Lagrangian multiplier equations where you have to differentiate both the equations equal to 0 and so through which we can determine the values for PG1 and PG2 where there is no losses we have considered okay. But this is not enough if I do consider the losses then probably whole calculation need to change. So let us consider a two generator system example where the two generators are connected to a particular bus and one generator which has been connected through a transmission corridor to a particular bus and the second generator connected to the same bus without any transmission line.

So if you look at this example of actually figure 14.1 so the first generator which has been connected through a transmission line where we possess a loss which is close to 0.0002 P1 square and one can also consider a similar example where we have a generator P1 and generator P2 this transmission loss instead of a function of P1 directly it has been given to you the loss is going to be 12.5 megawatt.

$L = F_1(P_1) + F_2(P_2) + \lambda(500+P_{loss} - P_1 - P_2)$ …(14.1c)

$F_1(P_1)$=400 + $2P_1 + 0.002{P_1}^2$ …(14.2)

$F_2(P_2)$=400 + $2P_2 + 0.002{P_2}^2$ …(14.3)

$P_{loss} = 0.0002{P_1}^2$ …(14.4)

Now my argument here is that if the total let us get into a very practical example the losses on the transmission line are proportional to the square of the power flow that is what is been mentioned the losses are function of your generation output that is P1 square and if you have to cater 500 megawatt then each generator will produce let us say 250 each for an example but because of the loss of 12.5 megawatt so you generated 500 megawatt but you may not be able to get those as an output at bus number connect bus at which they have been connected because of the losses 12.5 if the energy available to you will be approximately 500 minus 12.5 that is 487.5 megawatt will be available to you. So the losses in the transmission line are proportional to the square of the power flow and the generating units are quite identical that is what we have assumed hence the production cost is modeled using a quadratic equation we can say production cost is AP square plus BP plus C where ABC are the coefficients and P is my generation outputs.

$\frac{\partial \mathrm{L}}{\partial \mathrm{P}_1} = \frac{\mathrm{dF}_1(\mathrm{P}_1)}{\mathrm{dP}_1} - \lambda(1 - \frac{\partial \mathrm{Ploss}}{\partial \mathrm{P}_1}) = 0$ …(14.5)

$\frac{\partial \mathrm{L}}{\partial \mathrm{P}_2} = \frac{\mathrm{dF}_2(\mathrm{P}_2)}{\mathrm{dP}_2} - \lambda(1 - \frac{\partial \mathrm{Ploss}}{\partial \mathrm{P}_2})=0$ …(14.6)

P1 + P2 – 500 – Ploss = 0 …(14.7)

7.0 + 0.004P1 + λ (1-0.0004P1) = 0…(14.8)

7.0 + 0.004P2 - λ = 0 …(14.9)

P1 + P2 – 500 – 0.0002$P_1{}^2$ …(14.10)

Solution:

P1 = 178.882

P2 = 327.496

Production Cost:

F1(P1)+ F2(P2)=4623.15₹ or$/h

Losses: 6.378 MW

If both the units were loaded at 250 megawatt we would fall sort of the 500 megawatt requirement my load point because some percentage of the power which has been consumed by the transmission line in the form of losses. Now we try to make this problem little more interesting now where should the extra 12.5 megawatt be generated who will be able to generate the 12.5 to me the 500 megawatt is the load point loss is going to be 12.5 so we must generate 512.5 megawatt of power out of these two generators so that even after the loss of 12.5 megawatt the load of 500 megawatt can be met. Okay so to do that we can again go for the Lagrangian equation L which is f1 P1 plus f2 P2 plus lambda times now the load is no more 500 the load in the past just 500 but now it is 500 plus losses and minus P1 P2 is going to be my equality equation which has been placed at lambda times that equality constraint.

So what we have to do basically my P1 plus P2 equal to 500 megawatt and then we say 500 minus P1 minus P2 times lambda this is what we do in our earlier studies we must have learned but because of losses now what I'm writing is that P1 plus P2 will be equal to 500 plus P loss okay so that means 500 plus P loss minus P1 minus P2 times lambda this will be the equation I have to take care okay that's why you could see 14.1 C look like this and two cost equations which is given to me P1 f1 P1 is given to me in the form of quadratic equations f2 P2 is given in the form of quadratic equation and P loss in general a function a square function of the generation output which has been given as 0.0002 P1 square why there is no P2 because the second generation which has been connected directly to the bus without any transmission line and hence there is no loss incurred out of the generation two to cater the load at the same bus means I generate use it the same point so I will not experience any transmission losses if it is very far away then I do experience transmission loss now once you have this then you can carry out economic dispatch with losses derive the similar equations dou L upon the P1 the Lagrangian equation need to be differentiated with respect to P1 with respect to P2 so we'll have actually d of f1 P1 upon dP1 minus lambda times 1 minus dou P loss upon dou P1 okay because when you differentiate P2 with respect to P1 they are 0 when you differentiate P1 with respect to P1 this become 1 so to simplify you will have an equation which is 14.5 and similarly you will have a question 14.6 and 14.7 which is very clear P1 plus P2 must be 500 plus P losses and those three questions 14.7 14.8 and 14.9 we need to solve okay and perhaps actually when you solve this equations right inside you can get the P1 value which is 178.882 and the P2 value which is going to be 327.496 now if you add this two I mean you can easily see to that actually P1 plus P2 is going to be slightly more than 500 and the losses because we are not mentioning the loss will be close to 12.5 megawatt here we are saying the loss will be a function of your P1 square so if you solve this equation now you are experiencing a loss in the transmission line which is 6.378 and P1 plus P2 must be 500 plus 6.378 and if you calculate the cost the cost of generation will be 4623.15 rupees or dollar per hour is for your convenience now continuation in continuation to generator systems with generator 1 supplying all losses because that is what you have discussed here everything is being provided by the generation 1 generation 2 is not providing anything now in a simple way what we can do let's ignore the economic influence of losses and run unit 1 up until it supplied all the losses means let's say there are two generators and you want that the first generator has to provide everything okay means 250 250 each balanced and the first generator will provide all the losses of 6.378 in the previous case but here you are not optimizing but you are saying distribute equally in 250 250 and then try to add all the losses to the generation number 1 so what will happen in that case your P1 will be 263.932 and P2 will be 250 and the cost will be 4661.84 here I want your attention I wish to have your attention here what we are doing basically so the previous cost if you see which is 4623 now it is 4661 so what has happened now the cost of generation to cater 500 megawatt has been increased significantly because we have not optimized the generation output of unit 1 and unit 2 if I would have optimized means my first generation must give me 178 the second unit must give me 327 then the cost will be minimal but if if I do not apply my optimization tool and I say no actually let me do it randomly or one of the generators do have a limit for example if you see that only 250 will be supplied by the second generation due to a constraint let's say the maximum generation in this example the P2 is supposed to cater 327 that is what the numerical optimal value is but practically my generator can only provide 250 understand okay the second generation the moment I say it can provide 250 then all the generation will be transferred to the first unit and hence once this become 250 the second has to cater 263 and will slowly deviate from our optimal point to suboptimal point and the cost will be increased from 46, 4623 rupees per hour to 4661 rupees per hour so this is a wonderful example for all of you to understand that in case of economic dispatch without loss we can easily solve the problem but when you accommodate losses then the generation sum of all the generators will be load plus the losses now how to get my losses the losses could be a numeric value or it could be a function of my P1 or P2 so when it is a function of P1 or P2 then using the previous equations you can solve those equations and you can obtain P1 and P2 and its cost as well as the losses but if you impose a constraint on a particular generation the upper limit then you are now moving to a point which is suboptimal so because of a constraint the cost will increase right so you are not allowing the cheaper generators to produce more power okay if you allow the cheaper generation to generate more power then the cost will be minimum but when you restrict due to practical implications then the cost will increase so I think this is a very good example to understand how my cost will vary if I move from optimal points and how also I can accommodate my losses in my generation output to meet a particular load using economic dispatch with losses now one more problem you see there are two things you may say I'm interested to minimize cost one may also say I'm not worried about cost but I want to minimize my losses because loss is also a interesting thing that you have to minimize okay so if you have to minimize my losses to minimize the loss solution for this case you simply run unit number one down and unit number two up as far as possible so please focus on if I have to minimize my losses then

what I will do because there is no loss connected to my unit number two this generator do not possess any losses so I will try to extract the maximum power out of my second generator and once its maximum limit is fixed then only I will divert rest of the power to my first generator so if the highest limit of the second generator is 400 megawatt theoretical so then I will take everything from P2 because I am irrespective of the cost equation even it is expensive I'm least bothered my interest is to take all the power from both the generator in such a manner the transmission losses will be minimum okay to do that I will certainly take all the powers which is not adding any losses to my network that is take everything from P2 and rest will be taken from P1 so you can now see the cost is going to be four six five five we have seen that the cost is significantly increased but the losses reduced drastically to two point zero eight four so we have seen three examples in one example the loss is twelve and then another example using the losses six then another example using this is two megawatt but the cost increase from four six three two four six six six two four six five five so one interesting conclusion that we can derive here the losses and the cost they are dependent so if you compromise your cost you can reduce the losses and if you do not compromise your cost then you will increase your loss but you can minimize the cost so the characteristics are quite interesting so it will look like this okay and this is my multi objective optimization problem ideally what we do we try to understand how to minimize both that is the best thing one can achieve but it is impossible to minimize both but for certain extent we can compromise so that a reasonable compromise in cost will lead to significant reduction in losses if you can achieve that there is nothing like hope you all have understood this interesting concept on generation output on a given power plants connected to a system with minimum losses and minimum cost now we slowly get into an interesting fact known as coordination equations now the objective is to minimize:

Minimize: $L = F_T + \lambda\phi$ ...(14.11)

Where: $F_T = \sum_{i=1}^{N} F_i(P_i)$ ...(14.12)

$\Phi = P_{load} + P_{loss}(P_1, P_2 \ldots P_N) - \sum_{i=1}^{N} P_i$ ...(14.13)

Solution: $\frac{\partial L}{\partial P_i} = 0$ for all $P_{imin} \leq P_i \leq P_{imax}$

$\frac{\partial L}{\partial P_1} = \frac{\partial F_i}{\partial P_i} - \lambda(1 - \frac{\partial P_{loss}}{\partial P_i}) = 0$, $\frac{\partial P_{loss}}{\partial P_i}$ => is called the Incremental Loss

$$\left(\frac{1}{1-\frac{\partial P_{loss}}{\partial P_i}}\right)\frac{dF_i}{dP_i} = \lambda$$

$Pf_i = \left(\frac{1}{1-\frac{\partial P_{loss}}{\partial P_i}}\right)$ is called the penalty factor for bus i

Note: If Losses increase for an increase in power from bus i, the incremental loss is positive and penalty factor is greater than unity.

l equal to the main function f of t plus lambda times psi and f of t is nothing but summation of all the cost functions of all the generators that is summation i equal to 1 to n f i p i and psi is nothing but p losses the function of p 1 p 2 p i now what I wanted to highlight previously the loss is a function of p 1 only but practically it may be a function of all the generator outputs in a complicated network okay so the p loss equation could be the function of p 1 squared could be a function of p 2 could be a function of pn or mixture of all these generators all right so the solution when you take differentiation with respect to p i and p i is less than or equal to p i max and p i mean so you'll be having this now you're getting this equation which is very important dou f of upon dou p minus lambda times 1 minus dou p loss upon dou p i equal to 0 and if you further simplify you can get 1 upon 1 minus dou p loss upon dou p i multiplied by d f of upon d p equal to lambda okay so previously in my without loss expressions I had only this portion this equation is without losses now when you incorporate losses into the system then the equation will look like 1 upon 1 minus dou p loss upon dou p i times d f of d f upon d p equal to lambda so what has happened now the equation got modified because of losses and hence the 1 upon 1 minus dou p loss upon dou p i is known as p f i which is penalty factor and that penalty factor is due to losses if losses increase for an increase in power from bus i the incremental loss is positive and hence the penalty factor is greater than unity when the denominator is less than 1 so then the whole penalty factor become more than 1 so that is what actually you know losses increase for an increase in power flow that means this element is positive when this element is positive 1 minus the positive element will give you some element which is less than 1 and 1 upon less than 1 will be more than 1 so my penalty factor will be more than 1 now let us bit of theoretical understanding when we did not consider transmission losses the economic dispersed problem was solved making incremental cost at each unit the same the concept of penalty factor can still be used and will have the following effect for p f i greater than 1 which is positive increase in p i results in increasing losses and if p f i is less than 1 that is positive increase in p i results decrease in losses and this is one of the typical in olden days actually researchers used to plot this characteristic of incremental cost and using the roller they used to roll on a paper they used to roll on a paper and then get the optimal point for my p1 p2 because there was no computer at the time okay when there was no computer this method which has been used to get solutions optimal solutions generation output of p1 p2 p3 using this graphical method you'll be very happy to know in early days still that time also people used to optimize they used to instruct generation one this much then this and two is this much then should see this much and these two solve manually by using a roller okay but just for a better understanding what I wanted to highlight there are two two characteristics in each okay one is having you know positive impact okay and p f i which is positive in the first case second case p f 2 is also positive and the third case the p f 3 is negative so the original case you know with the penalty factor that you could see the dotted line the dotted line which is the penalty factor if the penalty factor is negative then the characteristic drops and the financial factor positive the characteristic goes up okay the original characteristic moved original characteristics moved original characteristic drop because of positive positive and positive means more than one and less than one okay so you know p i dash which is dispatched ignoring the losses if you ignore the losses you can obtain p i dash and p i double dash is based on my inclusion of losses if you solve a numerical problem and plot this characteristic you can experience these characteristics better now further the loss formula how do you because the moment you say the loss equal to or the loss is in depending on the square of piece p1 square p2 square p3 square the question is how do you obtain or how do we know that loss is a function of p1 square p2 square p3 square we all know it the loss is dependent on p1 p2 p3 because you can change your p1 p2 p3 we have seen from previous examples that the losses have keep on changing when I have changed my p1 p2 values but the question is what about the coefficient of those squares if I say p loss p loss which is 0.002 p1 square I know p loss or we all know p loss is a dependent on p1 but how do I get those coefficients how I have derived it so for a given system these coefficients can be determined okay very very old history so let us understand how they have been obtained and this is known as B matrix loss formula because these coefficients are known as B coefficients so practical methods for loss and incremental loss calculations earlier automatic dispatching was performed by analog computers and the loss formula was stored in the analog computers by setting precision potentiometers the equation for the B matrix loss formula is as follows we say p loss which is p transpose B times p plus B naught transpose p plus B 0 0 means we have three coefficients B coefficients one is a square term or the second is actually a non-square term and third is a constant term the previous equations now can be rewritten as p loss which is summation of i to j p i b i j p j plus summation i b i o p i plus v o now if you substitute this equation when my phi equation will be minus summation i equal to 1 to n p i plus p load plus the loss equation which has been derived from 14.15 and now if you differentiate these equations so you will get actually d f of p upon d p minus lambda times 1 minus 2 because square loss so become 2 summation j b i j p j minus v i o and then this equation 14.17 become a standard equation for me to solve it further and then economic dispersed with updated penalty factor so what we do when we solve economic dispersed problem with losses then we try to update the penalty factor as one of my variables okay so the process as you could see we can start and then you know we can assume all the variables then we can calculate what is the demand and then we can calculate the penalty factor and then we determine.

What is lambda and then we solve for each p using the penalty factor because penalty factor in p they are interrelated okay and once we get this value you get a new lambda and then increment this lambda and repeat this cycle till you get your final solution so this is the step by step problem we have already solved it but as an algorithm we can also understand how penalty factor can be used to obtain economic dispersed solutions for a given system now let us move to reference bus penalty factor the B matrix assumes that all load currents confirms to an equivalent load current and that the equivalent load current is the negative of the sum of all generator currents and that means the B matrix when you calculate the total loss which is p t b p plus v o t p plus v o and this v v naught v o o or v naught naught they are the coefficients okay then the incremental loss at generation number i which is dou p loss upon dou p i so those equations need to be differentiated with respect to p i okay so the third element becomes zero the second element become constant in the first element become constant as a power variable now the incremental loss is the change in losses when an incremental is made in generation output as for the derivation the incremental loss for the bus i assume that all the other generators remain fixed by original assumptions the load currents all confirm to each other and always balanced with the generation then the implication in using B matrix is that an incremental increase in generator output is matched by an equivalent increment in low now what we need to do basically we have to calculate those elements for a given system how those coefficients can be calculated if they can be calculated then my loss element is known to me and hence i can calculate the optimal output of each and every generator the alternative approach to economic dispense is to use a reference bus that always moves when an increment in generation is made on a different generator bus suppose we change the generation and pass by delta p i means we had 500 megawatt now.

I'm changing to 505 okay so what is my new output now the p old plus this is delta p i to compensate for increasing delta p i the reference bus just drop by delta p reference means i do have n number of generators one generator increase its output the second generator must drop the reference bus must drop because the reference bus is always in the process of accommodating all the dynamics that happens in rest of the generators okay so my p new reference will be p reference old plus delta p reference which is p reference with minus delta p i plus delta p losses so if one can calculate the beta i one can calculate the beta i based on equation and beta i similar to inverse of my penalty factor which is 1 minus 2 p loss upon 2 p i so beta is the reference bus penalty factor beta is the reference bus now all generators are in economic dispense when a shift of delta p megawatt from any generator to the reference bus result in no change in net production cost where delta p is arbitrary small when it is very very small probably there is not much change you experience but if there is a significant change that will result in into cost increase significant now if you calculate the delta p production cost which is nothing but my d f of p upon d p i times that the p i plus d p reference upon p reference delta p reference so this is an expansion of two different generators one is p i the other is p reference or p 1 and p reference you can calculate delta p reference which is minus beta times delta p i which is very simple you can also calculate the production cost using beta as a variable and then you can cut equating the production cost equal to 0 you will land with this equation and from this equation the penalty factor may be derived using Newton Raphson power flow now it is important the objective of this course is to determine the optimal generations to be the load with and without losses so with losses means there is a penalty factor added to my equations and those penalty factors now can be calculated using Newton Raphson power flow so if you understood very well what is Newton Raphson power flow you can call the Newton Raphson power flow an integrated network connected to many generators and then you can capture the penalty factor and then use those penalty factor for your optimal solution and get the generation outputs so finally you can conclude these are the generation output of a given system to meet my load economically accommodating all the losses of the network okay.

That is the basic objective of this course the ratio of change in power on the reference bus when a change delta p is made so you can now get this equation 14.2 and 14.29 for my delta p references and to carry out matrix manipulation a bit of adjustment you can have the equation 14.31 and 14.31 the term dou p ref upon dou qi and dou p ref upon dou pa dou p ref upon vi dou vi are derived by differentiating the standard power flow equations for the reference bus so similar to a Jacobian matrix the term dou qi and dou pi as well as dou vi upon dou pi are from the inverse Jacobian matrix so this is a slightly interesting we are using the Newton Raphson power flow concept to determine the penalty factors so finally by transporting we will come to an equation which is of left hand side though p reference one though p1 with respect to q with respect to p2 with respect to q2 with respect to pn and with respect to qn which is J inverse with respect to theta v everything okay so 14.33 become a very very interesting equation for me to be solved using Newton Raphson row concept Newton Raphson optimizations and that will help to determine the optimal solutions so with this we stop here bus penalty factor direct from an AC power flow and we'll discuss in the next class how to get all these solutions in a better so we stop here.

Thank you very much you