# Economic Operation and Control of Power System

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**Week - 03**

**Lecture – 13**

A very good morning. Welcome you all to the NPTEL course on Economic Operation and Control of Power System. Today we will be focusing on power flow analysis and during our last lecture we talked about Newton-Raphson load flow analysis which is purely for AC circuits. And today we will talk a little bit more about modified Newton-Raphson and how we can simplify the iterative technique to save time and which is renamed as fast decoupled. And we will also look into a very simple load flow analysis popularly known as Gauss-Siegel. I am very confident that you must have studied Gauss-Seidel and fast decoupled as well as Newton-Raphson during your undergraduate studies.

However, considering the economic operation as I told you from my introductory class, you need to know the power flow in each and every line and probably the next couple of lectures will be evident how to determine the transmission losses and who will be contributing those losses as a generator and all these things are possible only when you know the power flow analysis. So once you know the voltages, angles, impedances or admittances of each and every transmission line that will help you to calculate the current and hence power, both real and reactive power that flow in each and every corridor of transmission or distribution lines. So first of all, let us understand a little bit about what is decoupled power flow. You might have realized during the previous lecture, the Jacobian which is a very complicated matrix and the size of the Jacobian is keep on increasing when the size of the system is keep on increasing.

So and then you have to take the inverse of each Jacobian which is a tedious task and along with that calculating each and term, each and every term within your Jacobian become also an important issue. So calculation of Jacobian elements and hence inversing those Jacobian again and again become a very very tough task. So some of the cases where we are interested to have the load flow solutions but not necessarily to be 100% accurate, we can compromise little bit, so if there is a scope of compromising your load flow solution for some extent, then probably and to save time and computing facilities, we can move into decoupled power flow where couple of assumptions can help us to reduce your Jacobian elements and hence its inversing. Now when you see the Jacobian elements both P and Q, they are being, we have to carry out the partial differences in P and Q with respect to voltage and delta that is angle. I want to also highlight some of the textbooks do focus on delta as an angle and some cases they write theta, so do not get confused about delta and theta, they are all same delta I, delta J or theta I or theta J until and unless they are the voltage angles, okay or the phase angles.

Now the moment you say that P is a function of both V and delta and Q is a function of both V and delta, then you need to differentiate both P and Q with respect to V and delta to create your Jacobian. But we all know that the voltage magnitudes, they are truly dependent on your Q, means the Q is responsible for your voltage magnitude and similarly the P is responsible for your phase angles. So if you say or if you assume that the P is independent of voltage magnitude or there is no interactive or dependency between P and voltage and similarly there is no dependency between Q and phase angle, then probably the differentiation of those components P with respect to delta and Q with respect to, sorry P with respect to voltage and Q with respect to phase angle can be neglected. So that is my bus number one and two. So you can assume that the real power is independent of voltage magnitude and the reactive power is independent of phase angles.

Now one more thing that the angle difference delta I, delta J or theta I, theta J is usually very very small. So one can say that cos of theta I minus theta J, that is cos of zero, which is close to one, that is one more assumption we can say. Or one can also say that sine of theta I minus theta J become zero. All right. So one can also assume that this is going to be zero.

We can also say that the g I k sine of theta I minus theta J, which is also extremely less compared to b I k because the R and X ratio you know. We can also assume that the Q I, which is quite small compared to b I I, b I squared. So once you do that, then probably your resulting the power flow equations will reduce drastically, which has been reflected in equation number 13.3.

$$\frac{\partial P_i}{\partial \theta_k}=-|V_i||V_K|B_{ik} \quad , \quad \frac{\partial Q_i}{\frac{\partial |E_K|}{|E_K|}}=-|V_i||V_K|B_{ik} \qquad \ldots(13.3)$$

Now once you do that, once you do that, once you do that, then probably the Jacobian matrix as you have learned, which has many elements.

So you had an element here, you had element here, you had element here, you had element here. But now if I say this is my P, P with respect to V for example, or P with respect to theta. And then I can say P with respect to V and I can say Q with respect to theta and Q with respect to V. So what will happen actually? So these terms know because we have assumed they are independent and the relation is not very intact. So they will actually die down.

And your Jacobian matrix become slowly, you will have actually a small component of matrix in diagonal shape and they become actually zero elements. So indirectly the whole Jacobian matrix, now you have let us say theta and b as a variable. All right. So now what will happen actually the whole Jacobian equation of your Newton-Raphson will be decoupled, will be decoupled by two different matrices and one could be your b dash and the second could be your beta. So imagine if the problem is of 10 by 10 size, then it could have been, you know, you can break them into two 5 by 5 problems or you get much lesser than that.

So similarly once you have these equations, you can again obtain theta. You can take inverse of this element now. And then similarly you can obtain the voltage by taking inverse of this element and this process will continue. So there is no change in the process, but it is extremely simplified to be solving load flow compared to Newton-Raphson. So the simplified version of Newton-Raphson load flow is very popularly known as fast decoupled and most of the practical applications they use fast decoupled load flow as a industrial tool for obtaining voltages and angles at different systems.

Now quickly look into the power flow solution algorithm. So what you need to do, begin power flow solutions, okay. And then you build b dash and b double dash matrix that we learned earlier and then you can calculate what is, take the first expression and then determine what is change in theta and then you keep on incrementing theta till you get a solution and then you can go for, solve the equations for delta E and then you keep on changing your new E with respect to the change in delta E calculated. And then you repeat the cycle till the problem is completely converged. So in the previous case of Newton-Raphson, we do perhaps actually see to that both theta and voltages are within its tolerance level simultaneously, but here we do it separately.

Now moving to Gauss-Seidel method, this is a very, very basic and very, very popular, very popular. So the Gauss-Seidel what we do, the equation of, you know, real and reactive power flow, you know, the injected power, injected power at a given bass of a bass system, pk plus jqj can be calculated as vk ik conjugate, so it is vk times summation of j equal to 1 to n plus ykj vj conjugate. And then if you simplify properly, you can calculate the voltage at each bus can be solved for by using Gauss-Seidel method. The equation for voltages is given by vk alpha, where alpha is nothing but my iteration number, okay, maybe 0th iteration, 1st iteration, 2nd iteration, so on. Now what we do, similar to your previous case, in Newton-Raphson you have to assume certain data.

So here actually with certain assumption of your initial values, okay, of voltages and real and reactive power null values, you can calculate the vk. Once you calculate, you keep on substituting in the original equation and move iterative wise step number 1, 2, 3 till the difference in my voltage from the previous iteration to the next iteration become negligibly small, okay. So once we do, we stop it and we say these are the voltage solutions of my equation. But one interesting thing, you see it is like a compromise. If you go for Newton-Raphson as you could see, you can get the solution with very, very few iterations.

You can get the solution with very few iterations, no doubt about it. Now if you go for decoupled, the number of iterations will increase, okay, and if you go for Gauss-Seidel, it will further increase. So maybe let's say it is 3 iterations, this is 10 iterations and this is 20 iterations, okay. The order will increase significantly. But please remember the computational time within each iteration of Newton-Raphson will be significantly higher compared to decoupled and compared to Gauss-Seidel, okay.

So this is how you compromise. Either you spend time in each iteration, reduce the number of iterations or you go for less time in iterations and go for maximum number of iterations. Now here I am skipping this slide because we have already understood what is linear or DC power flow. And one more important part, sometime you need to model the voltage dependent loads. Means if your load is dependent because sometimes we say P plus jQ which is my load, P plus jQ, let's say which is 2 plus j1.

That's fine, fixed, okay. So we say irrespective of the voltage because this load has been connected at a particular bus and this is my voltage of this bus R and this is my Pi and this is Qi. Now the question is if this Pi and Qi become 2 plus j1 fixed irrespective of the voltage at this pass, then we can say that the load is independent of the bus voltages. However, in most of the cases, this load become dependent on P. That means the Pi become a function of Vi, alright, plus j times Qi become a function of my Vi.

If that is the case, then it is a slightly complicated problem and practically most of the loads are voltage dependent, especially electronic loads in general. For analysis, we have assumed that the load is independent of the bus voltages, that is constant power. However, the power flow can be easily extended to include voltage dependency with both real and objective load. This is done by making Pdi and Qvi as a function of Vi, okay. So this is how we have modeled the problem.

Now this one small example, you can check it out and then you can go for a specific example to help you understand how the loads are dependent on your voltages. Now there is one interesting term known as dishonest Newton-Raphson, okay. So it is something like cheating Newton-Raphson, okay. Newton-Raphson probably force you to take Jacobian inverses, okay. But one can proceed with the same Jacobian without taking the inverses and that is known as dishonest.

So means you are not following the procedure, you are doing something and but hopefully you can do the inverses rarely, maybe three iterations, once you can inverse, maybe six iteration you can do twice inverse, something like this, okay. You can skip your inversion to reduce the complication sometimes. And researchers have felt that even through this process you get the solution with reasonably long duration time but it is possible. Since most of the time in the Newton-Raphson iteration is you have to spend a lot of time for inversion of the Jacobian. One way to speed up the iteration is to only calculate the Jacobian or inverse occasionally, okay.

Calculate the Jacobian and inverse occasionally means don't do it each and every iteration, do it in an interval, okay. Maybe you can say every alternate time I will inverse or every three times I will inverse, okay. So this is known as dishonest Newton-Raphson. I do not want to complicate your life by teaching so many load flow solutions which are not too much necessary to understand economics, to power system economics. So probably but as a student it is better to understand these are the scope of solving a lot of the solutions for a graduate or an undergraduate student.

So this is one of the example that you can go through to understand better. Decoupled load flow we discussed in the beginning but just as a review, so this is what the clear picture because in the beginning I haven't given you those details of the matrix representations but now it is clear. So there I have written actually in pen but now it is clearly visible to you. You can see the elements are dou P upon dou theta. Then you have dou P upon dou V, Q upon theta, Q upon V.

So assuming that you know these terms are existing and these terms are not existing, all right. So then it become completely decoupled problem and you can solve it similar to Newton-Raphson. So this is fine. Of diagonal Jacobean elements. So the same thing again theta ij is considered to be very, very small.

R is less than x. So gij is less than bij. So you could now see these equations, P upon V and Q upon theta are indirectly these elements become 0, okay. These elements become 0. So what will happen now your Jacobean inversion become easier, your decoupling become simpler and then you can solve this problem quite easily. So to summarize the fast decoupled power flow, by continuing with our Jacobean approximation we can actually obtain a reasonable approximation that is independent of voltage magnitudes and angles.

So P and Q we can separate it out based on their dependency and make the Jacobean quite simpler, all right. This means the Jacobean needs only be built or inverted once. This approach is known as fast decoupled power flow, popularly known as FDPF. FDPF uses the same mismatch equation as standard power flow.

So it should have same solutions. The FDPF is widely used particularly when we only need an approximate solution very quickly, all right. So this is fine. And then one numerical example, you know, what you can do, dear students, you can take this example which has been given to you. It's a very simple problem, three bus system and you could see there is a slack bus and two loads and there are three lines with some resistance and reactances given to you. And slack bus parameters, load parameters are also given to you, the value of P and Q at bus number two and three.

Now the question is try to solve this problem using, you know, Gauss-Seidel and using Newton-Raphson and as well as fast decoupled, all right. So you can try to solve this problem using all the three different algorithms and check your voltage and angle at each and every bus, okay. And if they are, they must be very, very close to each other. So that will boost your confidence that you have really learned properly the lower solutions and practiced on a practical problem which is quite small. So you can solve this problem and this is the MATLAB program.

We try to, you know, write a MATLAB program to solve this problem. So the data which has been given in the form of MATLAB, you can even write a simple program to solve this and there are open source available. You can also try solving Gauss-Seidel, fast decoupled, Newton-Raphson which are in public domain and you can practice a small problem, enter the data, get the voltage solution, solve manually, solve through your algorithm and check whether they are actually matching. So with this note, we are coming to an end with some of the load flow solutions we discussed today and Thank you very much for your kind attention. Thank you.