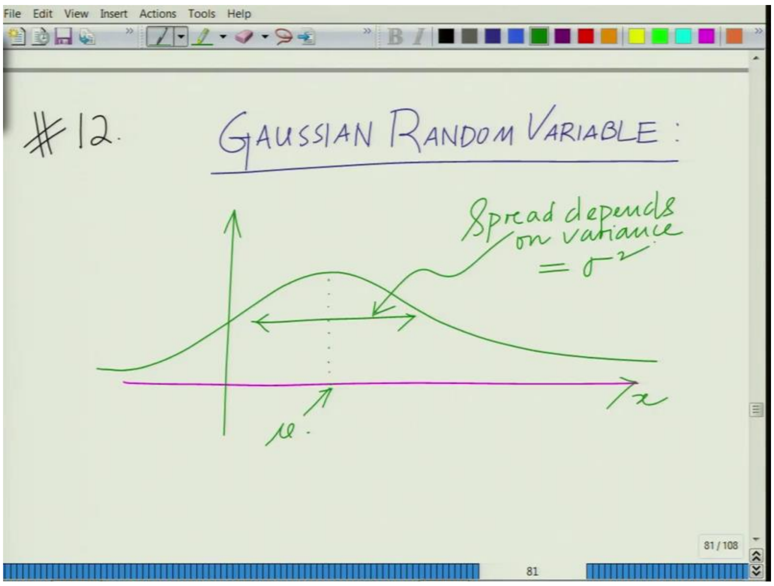
Applied Linear Algebra for Signal Processing, Data Analytics and Machine Learning
Professor. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur
Lecture No. 12
Gaussian random variable; definition, mean, variance, multivariate Gaussian, covariance matrix

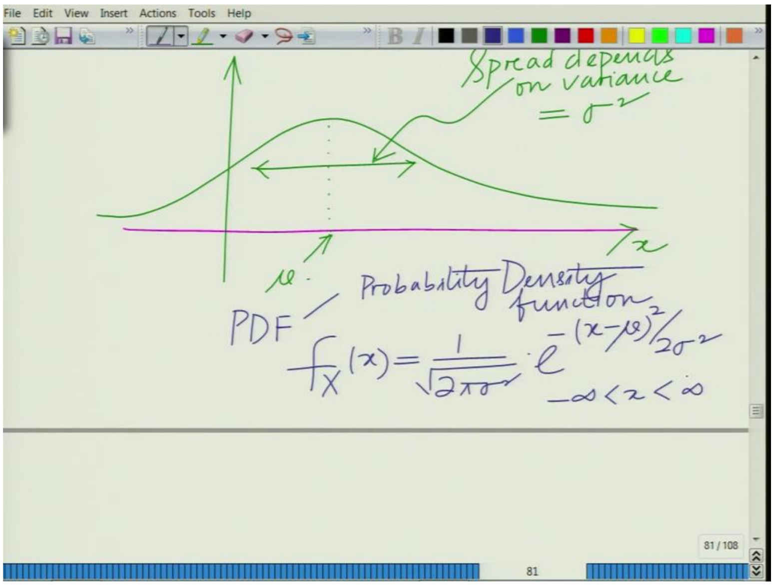
Hello, welcome to another module in this massive open online course. So, in this module, let us start looking at another very important concept in analysis, which arises frequently in linear system analysis and that is of Gaussian random variables and we are going to see that linear algebra linear system analysis is very intricately tied to the properties and analysis of Gaussian random variables and especially Gaussian random vectors.

(Refer Slide Time: 00:45)



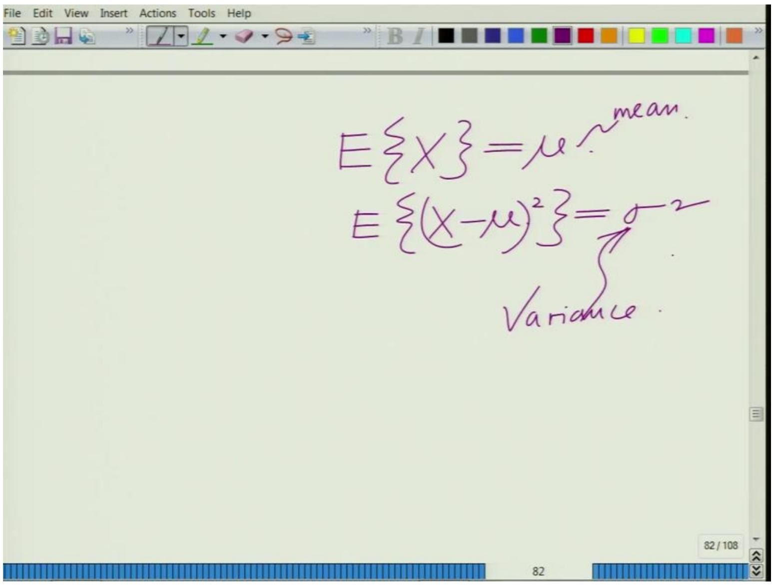
So, let us look at this very important so, there is another very important, what is the Gaussian random variable? The Gaussian random variable is simply if you look at it, many of you might already be familiar it is a random variable, let us say if this is our axis, this is our x-axis and this peak occurs at what is it is called, it is mean that is mu and this spread is controlled by the variance. The spread depends on the variance that is sigma square.

(Refer Slide Time: 02:00)



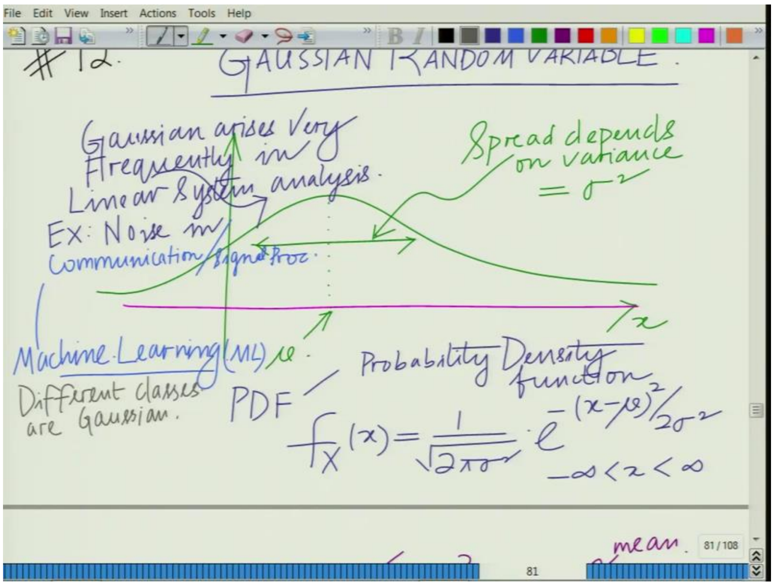
And the probability density function so, the PDF or what we call as the Probability Density Function are the Gaussian random variable is given as f of x of x this is equal to 1 over square root of 2 psi pie sigma square e raised to minus x minus mu whole square divided by 2 sigma square for minus infinity less than x less than infinity..

(Refer Slide Time: 02:53)



And this quantity mean as we said this is where the peak occurs and this is also the expected value of the random variable that is, we look at the expected value of the random variable that is equal to mu and if you look at the expected value of x minus mu whole square that is equal to sigma square this is termed as the variance. So, this is essentially your mean and this sigma squared is termed as the variance of this Gaussian random variable and why we are considering this Gaussian random variable as I have already said.

(Refer Slide Time: 03:31)

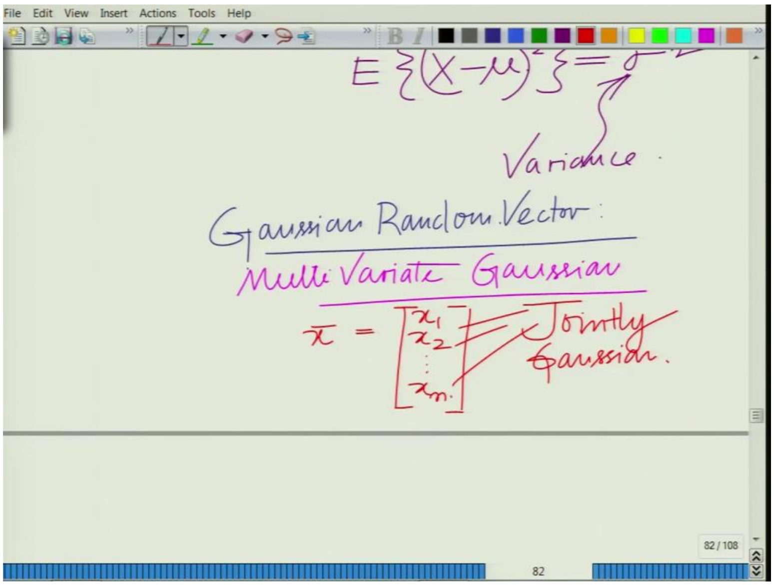


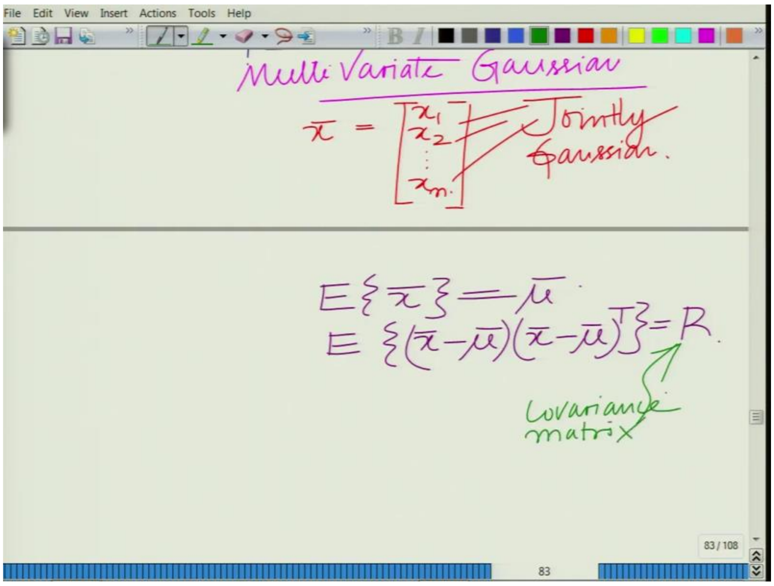
This is almost whenever you look at the applications of linear algebra and linear system analysis, Gaussian random variables, Gaussian random vectors arise very frequently in practical analysis of linear systems. For instance, if you look at any system communication system, signal processing system noise is frequently modelled as a Gaussian random variable or Gaussian random vector. If you look at Machine Learning, the different classes, the samples from the different classes can be modelled as being obtained from Gaussian processes, the samples are as though the different classes can be modelled as essentially your Gaussian random processes.

So, this concept of Gaussian arises very frequently very important arises very frequently in linear system analysis that is applications of linear algebra. For example, noise in communication or signal processing or for instance in Machine Learning the different that is when you talk about ML, different classes can be Gaussian, the objects belonging to the samples belong into different classes can be modelled as basically Gaussian in nature.

So, this frequently arises very frequently arises in analysis of linear systems and whenever, we talk about the practical applications of linear algebra, we have to inevitably talk about Gaussian random variables, Gaussian random vectors and Gaussian random processes.

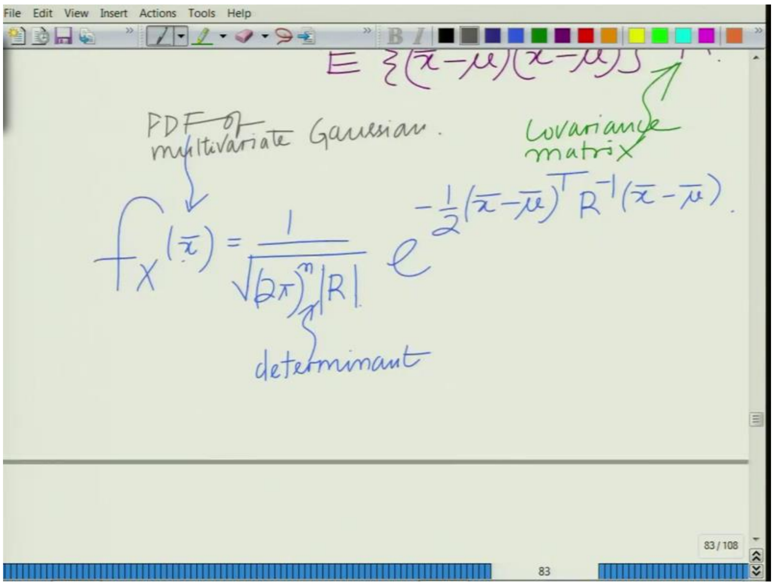
(Refer Slide Time: 06:07)

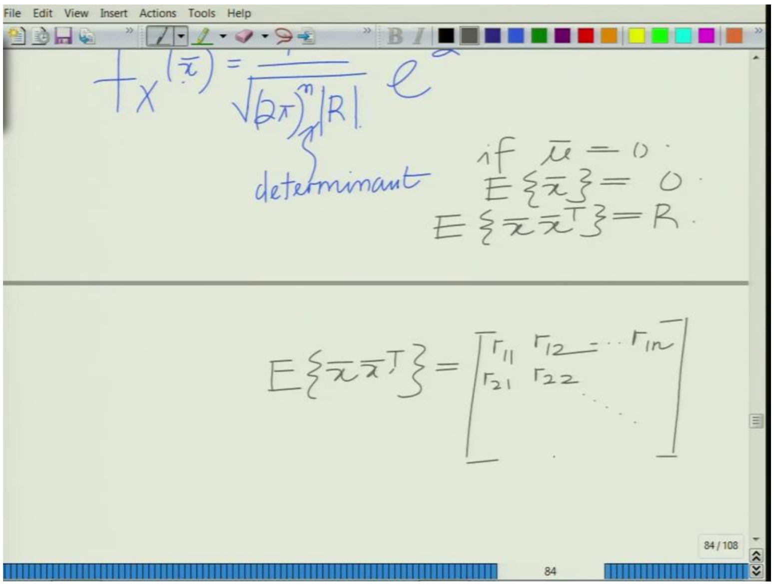




Now, a Gaussian random vector is basically a collection of Gaussian random variables. So, Gaussian random vector is also termed as basically the correct name for this and the technically correct name for this is a multivariate Gaussian where, we have the vector x bar equals x1 x2 xn and these are jointly Gaussian with the mean that is mu that is we have expected value of x bar equal to 0, expected value of x bar equal to mu bar and the covariance matrix is what is termed as a covariance, expected value of x bar minus mu bar expected value of x bar minus mu bar transpose this is equal to R so, this is the covariance matrix this is what is termed as a covariance matrix.

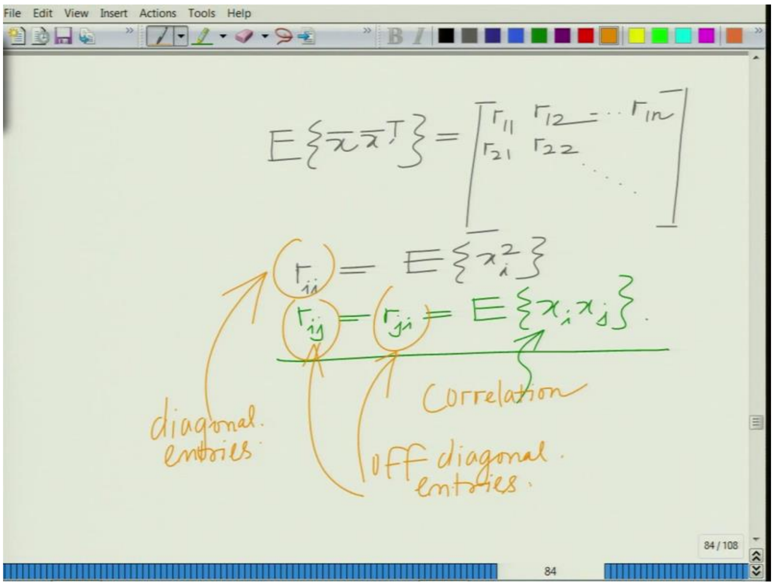
(Refer Slide Time: 07:41)





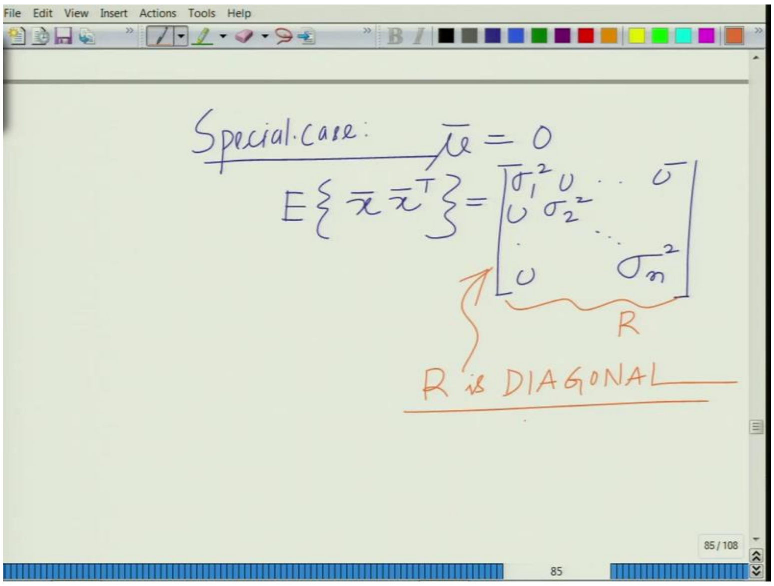
And the probability density function has given us f of x of x bar this will be equal to we are not talking about the multivariate Gaussian f of x of x bar which is equal to 1 over square root of 2 pi. Since, this is an n dimensional vector 2 pi raised to the power of n times the determinant of the covariance remember this is the determinant e raised to minus half x bar minus mu bar transpose inverse x bar minus mu bar so, this is the PDF of the multivariate Gaussian, PDF of multivariate Gaussian, and for mu equal to 0 natural if mu equal to 0 we have expected value of x bar equal to 0 and then, the covariance simply becomes expected value of x bar x bar transpose equal to R.

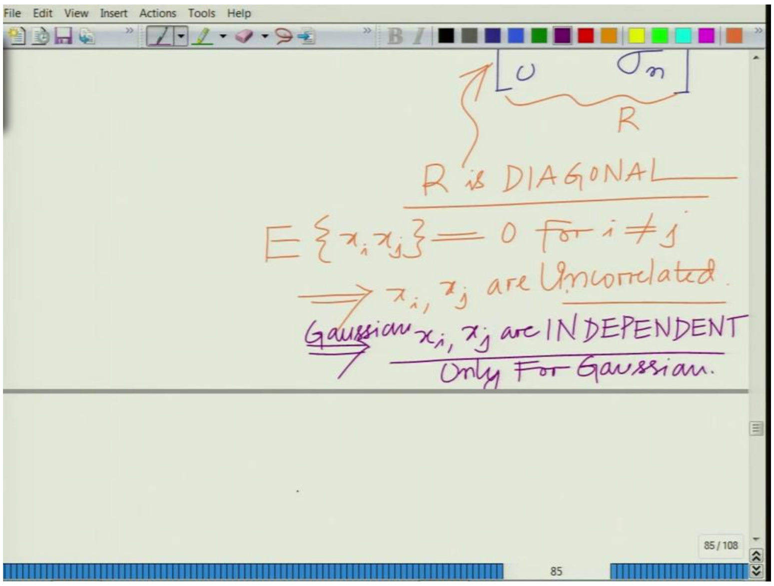
(Refer Slide Time: 09:17)



And now, if you look at the elements of R expected value of x bar x bar transpose equal to r11 r12 r1n r21 r22 so on, you will first see this a symmetric matrix and each rii diagonal element is nothing but the expected value of xi square that is the variance of xi and rij equals rji equals expected value of xi times x. This is basically what, we call as the correlation right or this is basically the correlation between the two random variables xx and x here. So, these are the off diagonal so, these are the diagonal entries and these are the off diagonal entries.

(Refer Slide Time: 10:52)

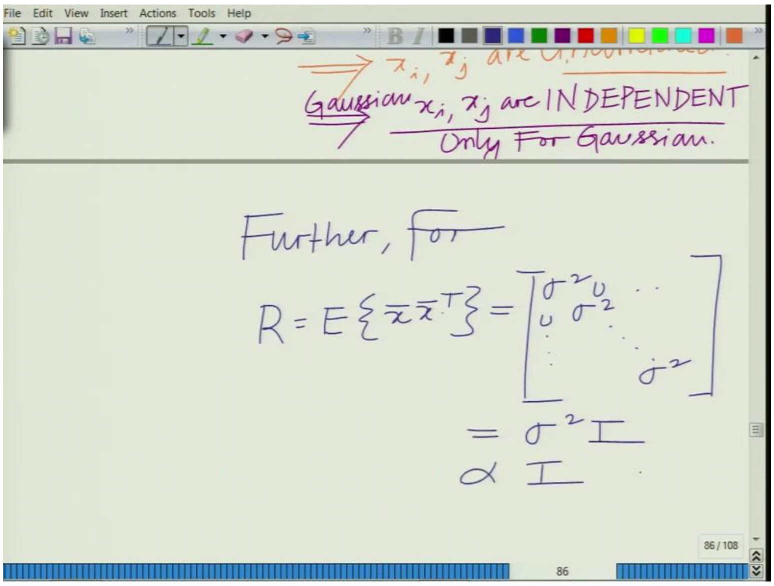


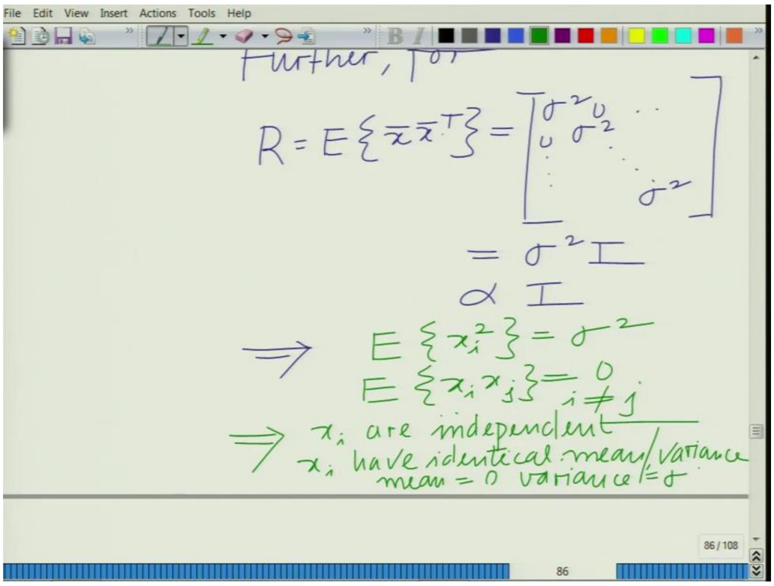


Now, if the off diagonal entries are 0 for a special case now, consider a special case that is we have expected value of special case first consider mu equal to 0 and then, we have expected value of x bar x bar transpose is of the form sigma 1 square sigma 2 square sigma n square that is, this is basically diagonal in nature that is covariance matrix R, R is diagonal in nature which means these different components of the vector the different random variables are uncorrelated because, if you look at expected value of xi to xj for i not equal to j that is equal to 0. So, this implies that expected value of xi into xj equal to 0 for i not equal to j which implies for any random variable xi xj are this implies 0 uncorrelated.

Now, for Gaussian this specifically implies only because it is Gaussian also implies xi comma xj are independent. So, diagonal covariance matrix for Gaussian only for Gaussian remember, not for any general variable because, we are considering a Gaussian random vector if the covariance matrix is diagonal, it implies that, the different components x1 x2 xn these random variables these are uncorrelated and because, they are Gaussian jointly Gaussian, it also follows that, they are independent.

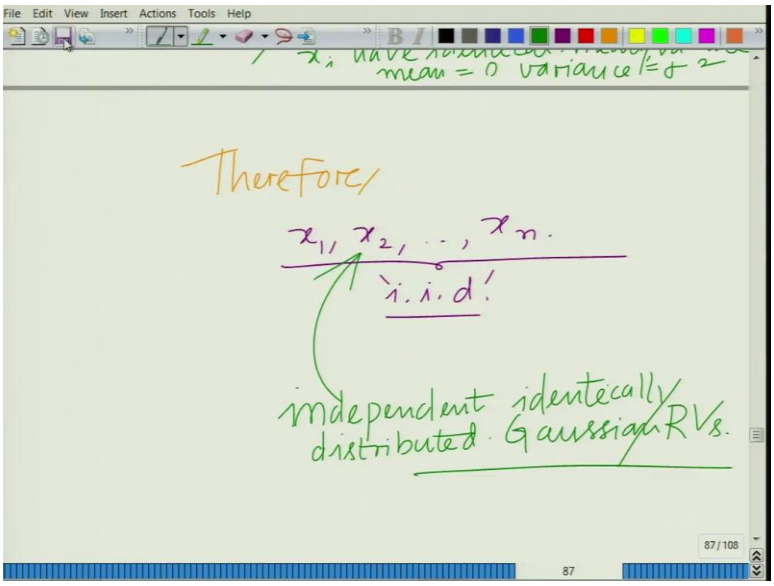
(Refer Slide Time: 13:09)





And further special case is when, we have further when all the variances are equal for R equal to expected value of x bar x bar transpose, this is diagonal and the variances are equal that is this is proportional to identity, covariance matrix is proportional to identity, this implies something very interesting that is each expected value of xi square equals sigma square and the expected value of xi is equal to 0 for i not equal to j implies of course, the xis are independent and all the xi have same mean or have identical mean slash variance identical mean and variance that is mean equal to 0, variance equal to sigma square.

(Refer Slide Time: 14:46)



Therefore, we have these are known as we termed that as x1 x2 xn us termed as was such a situation x1 x2 xn are termed as i i d that is these are independent and identically distributed Gaussian random variables, correct. Identically so, these are termed as i i d that is these are independent identically distributed Gaussian random variables. x1 x2 xn are independent identically distributed random variables and then in that, that is all of them have the mean 0 variance sigma square, they are uncorrelated because, their Gaussian or jointly Gaussian, it also means they are independent and if you look at the covariance that is essentially proportional to identity it is sigma square times the identity matrix.

So, let us stop here and we will continue our discussion in the subsequent modules. Thank you very much.