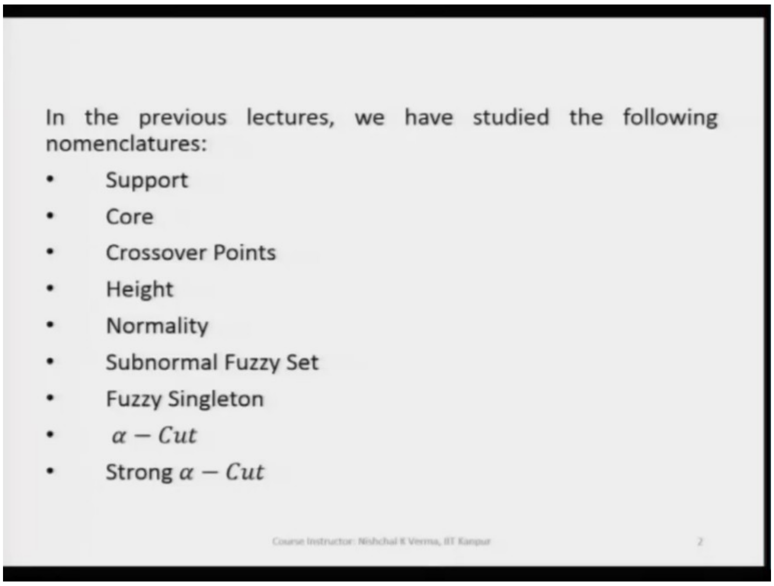
Fuzzy Sets, Logic and Systems and Applications

Prof. Nishchal K. Verma
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 09
Nomenclatures used in Fuzzy Sets Theory

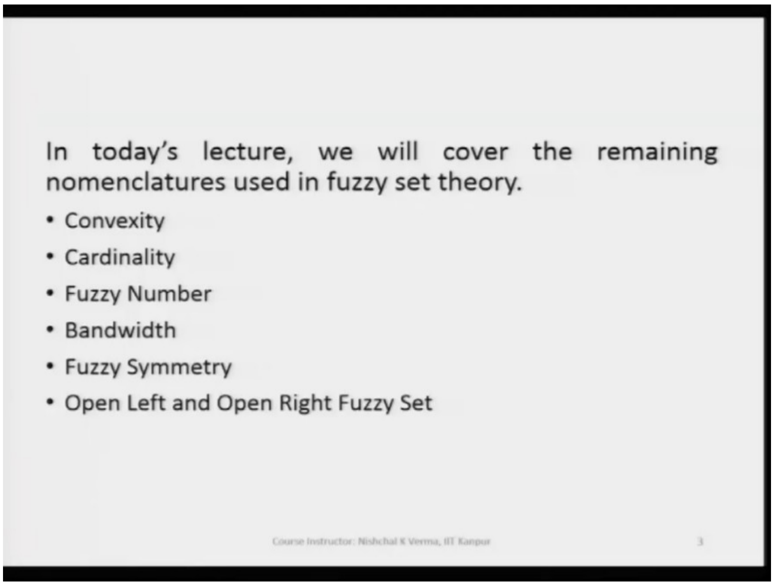
So, welcome to lecture number 9 of Fuzzy Sets, Logic and Systems and Applications. And in the Nomenclatures used in Fuzzy Set Theory we have already covered so many terms in the nomenclatures that are used in fuzzy set theory in previous lectures. So, these terms that were used that were discussed so far in previous lectures are support, core, crossover points, height, normality, subnormal fuzzy sets, fuzzy singleton, alpha cut and a strong alpha cuts.

(Refer Slide Time: 00:48)



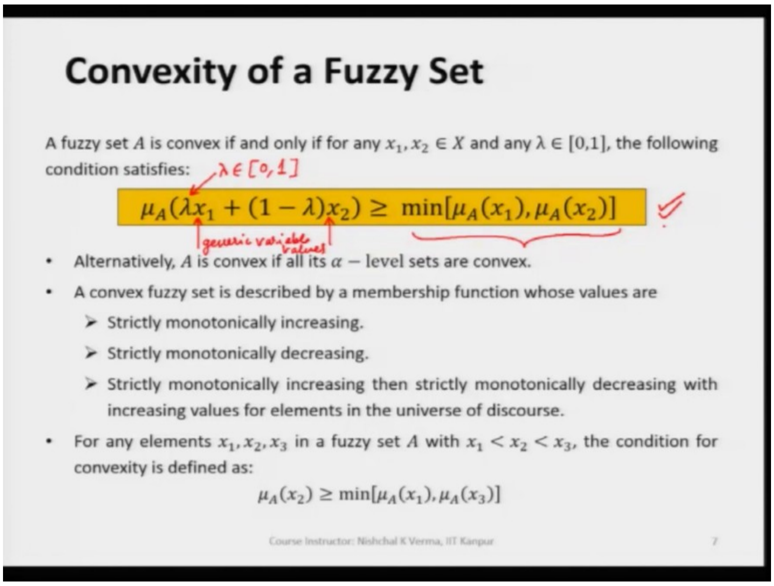
So, in the fuzzy set theory these terms are very very important once we know these terms we you know when we are going through the literatures understanding of these literatures. The literature is related to fuzzy set theory becomes very easier to understand.

(Refer Slide Time: 01:24)



In today's lecture we will cover the remaining nomenclatures that are used in fuzzy set theory. And that are convexity, cardinality, fuzzy number, bandwidth, fuzzy symmetry, open left and open right fuzzy sets.

(Refer Slide Time: 01:49)



Now, we have another term which is convexity of a fuzzy set in the nomenclature. And convexity of a fuzzy set is also very important characteristic of a fuzzy set. And this is used very commonly in fuzzy systems theory. So, convexity of fuzzy set can be checked or can be defined by this relation. So, this relation if this is satisfied if the if this relation is satisfied. We can say a fuzzy set is a convex fuzzy set.

So, what is this condition? Condition here is that if we have a fuzzy set A and we choose any two points as the generic variable values of this fuzzy set say x_1 and x_2. And we also choose a parameter λ and this λ can be any value here, in between this λ is in between or it can be 0 and so, it can be 0 to 1. So, this is the interval of λ that is given.

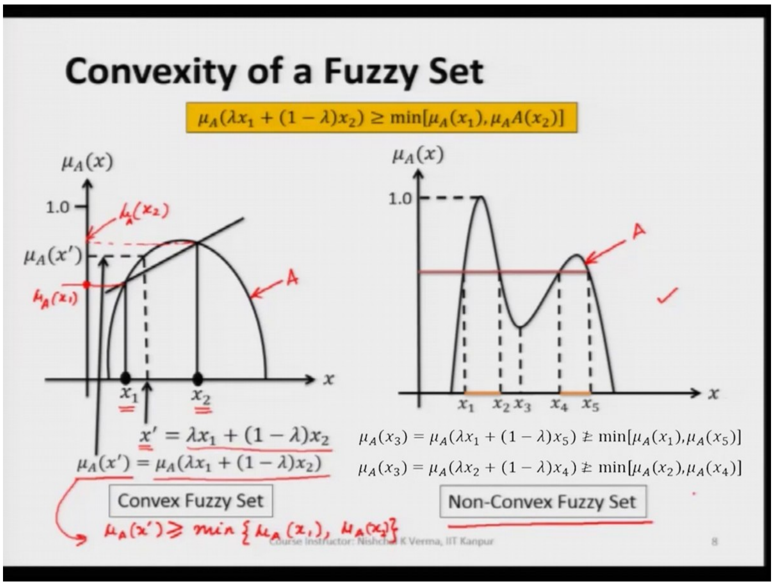
So, this way and x_1 as I mentioned this generic variable values. So, x_1 and x_2 are any two generic variable values. So, now we have λx_1, λx_2 and if we have these three values then if we take μ_A. So, please understand this A signifies the fuzzy set A, that is being undertaken and with respect to this fuzzy set A only we are taking x_1, x_2. So, that is why A is mentioned here. So, now if we take $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq min\,[\mu_A(x_1), \mu_A(x_2)]$.

So, if we find minimum of $\mu_A(x_1)$ and $\mu_A(x_2)$. And then as I already mentioned that $\mu_A(\lambda x_1 + (1-\lambda)x_2)$. So, this $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq min\,[\mu_A(x_1), \mu_A(x_2)]$.

So, if this condition is satisfied we can say a fuzzy set A is a convex fuzzy set. And this is a very important property of a fuzzy set which many times we use while defining a fuzzy set. So, a convex fuzzy set basically is the strictly monotonically increasing and then or in other words I would say a convex fuzzy set is a fuzzy set whose membership function is strictly monotonically increasing or monotonically decreasing.

So, this way we defined a convexity of a fuzzy set. So, we need to remember this criteria this condition here like to repeat this condition, which is $\mu_A(\lambda x_1 + (1-\lambda)x_2)$. And this value this will be basically the value which will be some I mean right from 0 to 1 can be any value and this should be either equal to or greater than min ¿.

(Refer Slide Time: 07:35)



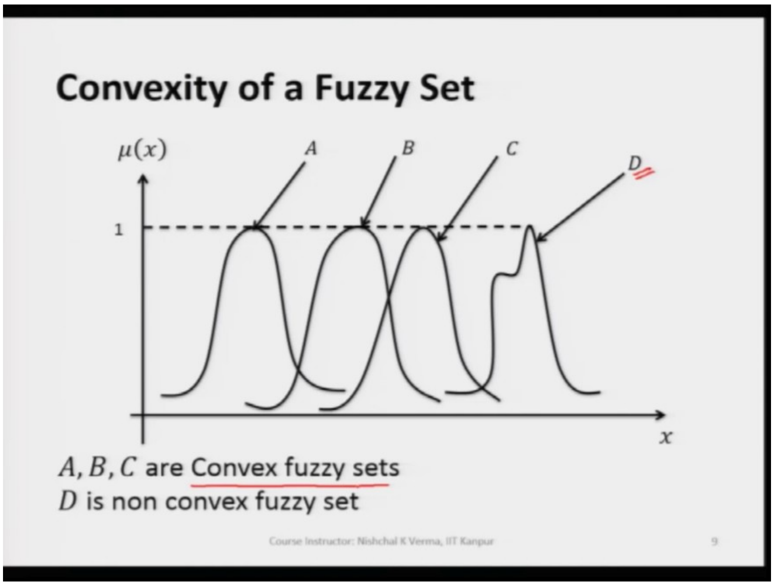
So, this is how convexity of a fuzzy set is defined. And this can be understood even better by taking this example. So, here we have two diagrams and in the first diagram here we have a fuzzy set A, in this fuzzy set A if we are taking any two point any two points x_1 and x_2, and if we choose any value of λ from 0 to 1. So, if we let us say we find after substituting the value of λ and x_1 and x_1.

Let us say we find some value which is x_1 sorry x'. So, we have x' and then $\mu(x')$ is equal to this thing. So, we have computed $\mu_A(x¿¿')¿$. And this must be either equal to this value $\mu_A(x¿¿')¿$ must be I can write it here $\mu_A(x¿¿') \geq min\{\mu_A(x_1), \mu_A(x_2)\}¿$. So, it means that we have this value of this value is here $\mu_A(x_1)$ and this value here is the value which is $\mu_A(x_2)$, I can write here A also because this for fuzzy set A.

So, this way you will take minimum, minimum of these two points should always be less than or equal to $\mu_A(x¿¿')¿$. So, if this condition throughout this fuzzy set for all the points all the any points any generic variable value for this fuzzy set. If this satisfied this condition is satisfied we can say this fuzzy set is a convex fuzzy set.

And if we take another example here where let us say this is a fuzzy set another fuzzy set which is A. So, apply the same logic same condition we will not get the this condition satisfied. So, $\mu_A(x¿¿')¿$ that is coming out for this case the other case for non convex fuzzy set case this will be less than the min of $\mu_A(x_1)$ and $\mu_A(x_2)$. So, for all non convex fuzzy sets, for all non convex fuzzy sets we for all non convex fuzzy sets we will not get this convexity condition of a fuzzy set satisfied.

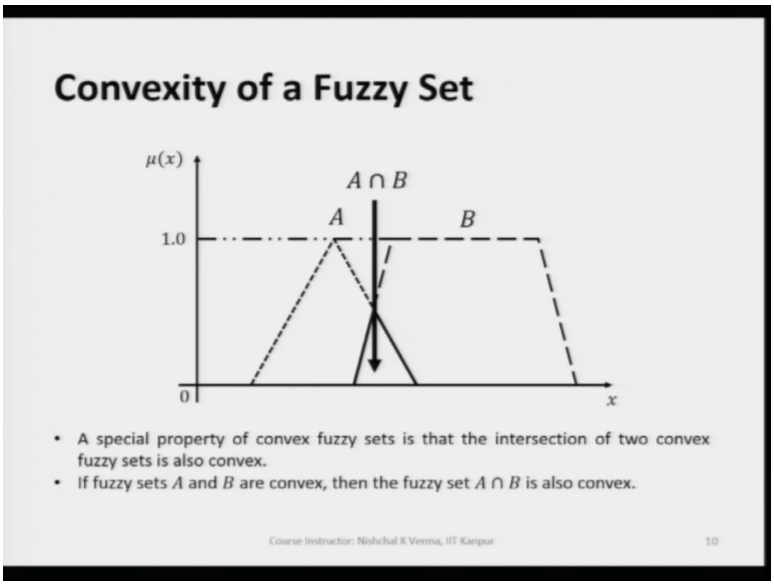
(Refer Slide Time: 11:40)



Just by looking at any set we can also comment on the convexity of the fuzzy set like if we have any non convex fuzzy set. We will see that we have the fuzzy sets like this like the fuzzy set here D fuzzy set is a non convex fuzzy set whereas, fuzzy set A, B, C are convex fuzzy sets.

So, it means what the this fuzzy set D is not monotonically increasing or decreasing whereas, fuzzy sets A, B, C are monotonically increasing or decreasing fuzzy sets. So, that is how we can clearly by just looking at the fuzzy sets we can clearly make the make the distinction.

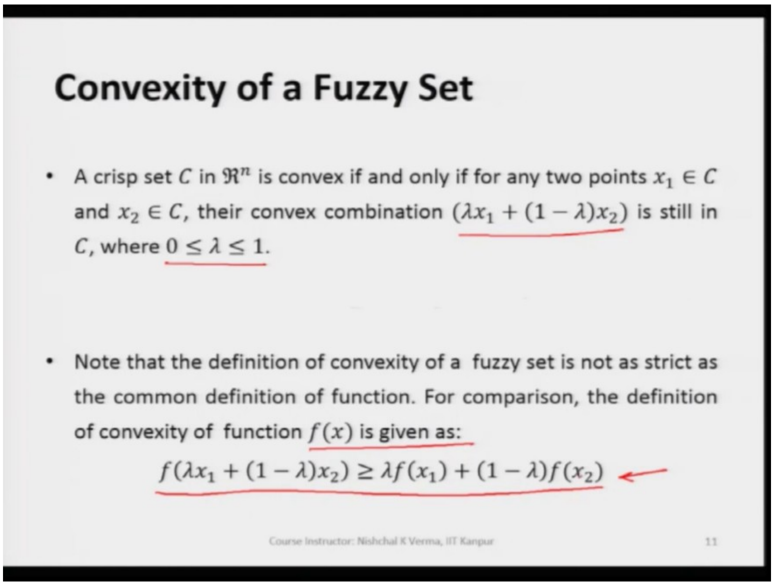
(Refer Slide Time: 12:42)



Here we have a very important property of the you know convex fuzzy sets like if we have two fuzzy sets; two convex fuzzy sets let us say A and B. So, if we take intersection of these two fuzzy sets, which is coming out as this fuzzy sets. This is the fuzzy set which is which we find by $A \cap B$.

So, this fuzzy set will also be convex fuzzy sets. So, intersection of these two fuzzy sets how we will be getting this will discuss this in coming lectures. So, in detail, but at this point we need to know that if we have two convex fuzzy sets A and B, if we take intersection of these two fuzzy sets that will also be a convex fuzzy set.

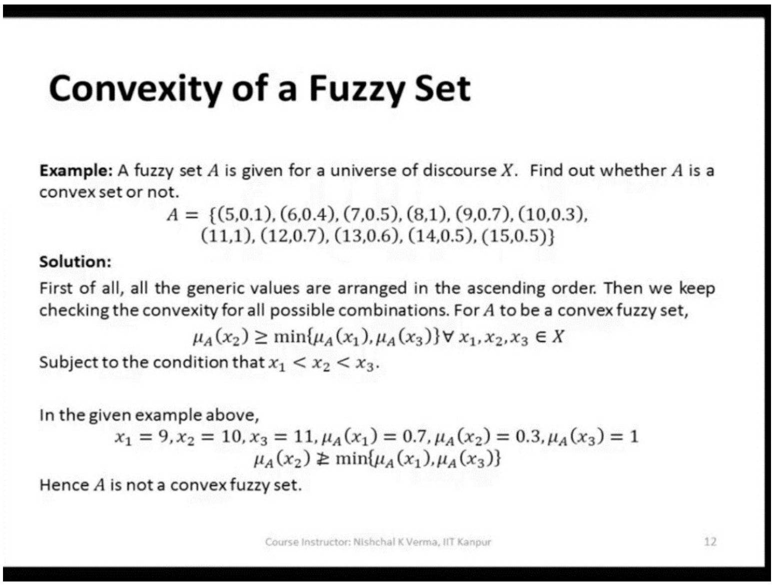
(Refer Slide Time: 13:50)



So, we can also say here that a crisp set C that is defined in R^n is convex if an only if for any two points x_1 that belongs to c and x_2 that belonged to c their convex combination here is still in C. Whereas I have already mentioned about lambda. So, λ will always be like this like λ can take any value from 0 to 1.

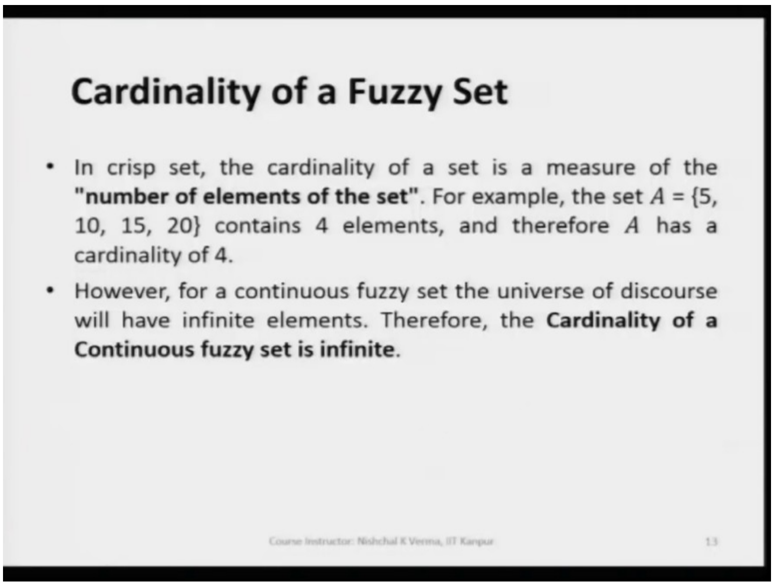
Note that the definition of convexity of a fuzzy set is not as strict as the common definition of function for comparison the definition of convexity of function *fx* is given as, so here we have this condition as which checks the convexity of any function *fx* and the convexity of a fuzzy set that we have just discussed is coming from this relation this condition.

(Refer Slide Time: 15:19)



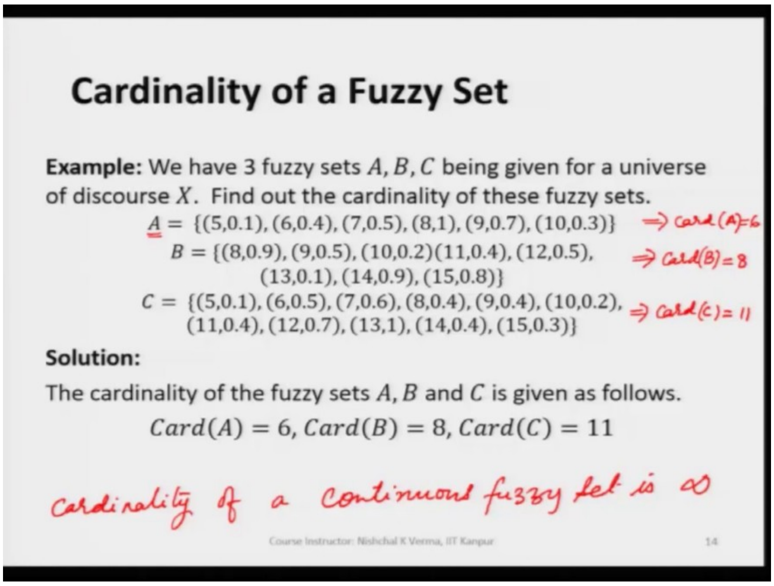
So, let us now take an example here to better understand the convexity of a fuzzy set. So, if we take a discrete fuzzy set A whose universe of discourse here is X and if we are interested in checking whether this fuzzy set is convex or not. So, here if we are interested in checking we can choose any two points of this of the fuzzy set.

(Refer Slide Time: 15:59)



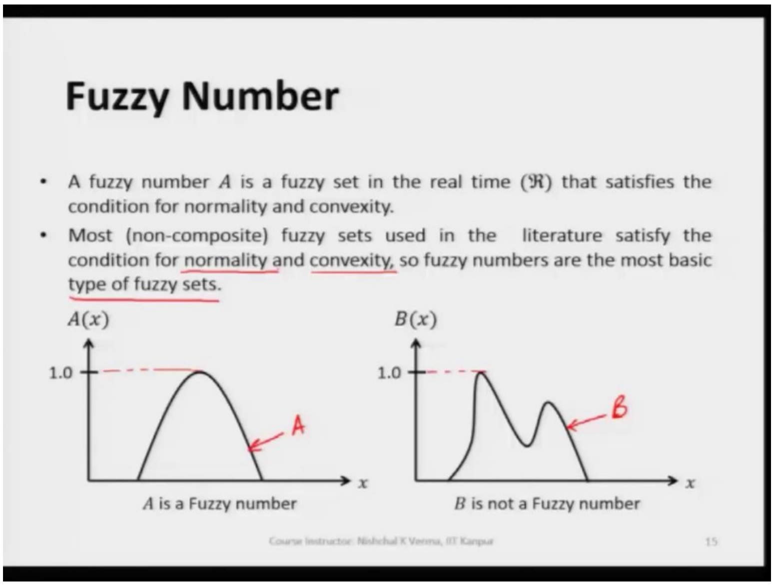
Okay, so now let us go to the cardinality of a fuzzy set. So, we all know. How do we define how we do how do we know the cardinality of a fuzzy set of a crisp set, cardinality of a fuzzy cardinality of a set first. So, what is the cardinality of a crisp set? Cardinality of crisp set is simply the number of elements that are present in the set. So, if we take an example here as A set A here so, set$A$is let us say $A=\{5,10,15,20\}$. So, this means here the cardinality of a fuzzy set is 4 because here we have only 4 elements; however, for a continuous fuzzy set the universe of discourse will have infinite elements. So, the cardinality of continuous fuzzy set is infinite.

(Refer Slide Time: 17:24)



So, it means what, it means if we have a fuzzy set which is discrete. Let us say in this example we have a fuzzy set A. Which is a discrete fuzzy set and this has 1, 2, 3, 4, 5, 6 6 elements. So, the cardinality of the fuzzy set here will be 6. Similarly the cardinality of the fuzzy set B here will be 1, 2, 3, 4, 5, 6, 7, 8. So, the cardinality of the fuzzy set B will be 8. And similarly we have the cardinality of a fuzzy sets C 11 because it has 11 elements. So, this is how the cardinality of discrete fuzzy set we can quickly find.

(Refer Slide Time: 18:23)



Now, if we have a continuous fuzzy set. So, since in a continuous fuzzy sets we have in the fuzzy set we can have infinite elements present, infinite membership infinite generic variable values present. So, that's why we can say the cardinality of a continuous fuzzy set is always infinite. So, that's how we define the cardinality of a fuzzy set. Now we come to a term fuzzy number.

What is a fuzzy number? Like crisp number we have fuzzy number. So, fuzzy number A is a fuzzy set, A is simply a fuzzy set which follows two conditions. So, first condition is the condition of normalized normality it means this fuzzy set should be a normal fuzzy set. And then it should also follow the condition of convexity. So, this means that if A is a fuzzy number fuzzy set any fuzzy set A is a fuzzy number. Then this fuzzy set should be this fuzzy set A should be the normal fuzzy set number 1 and number 2 this A fuzzy set should follow the condition of convexity.

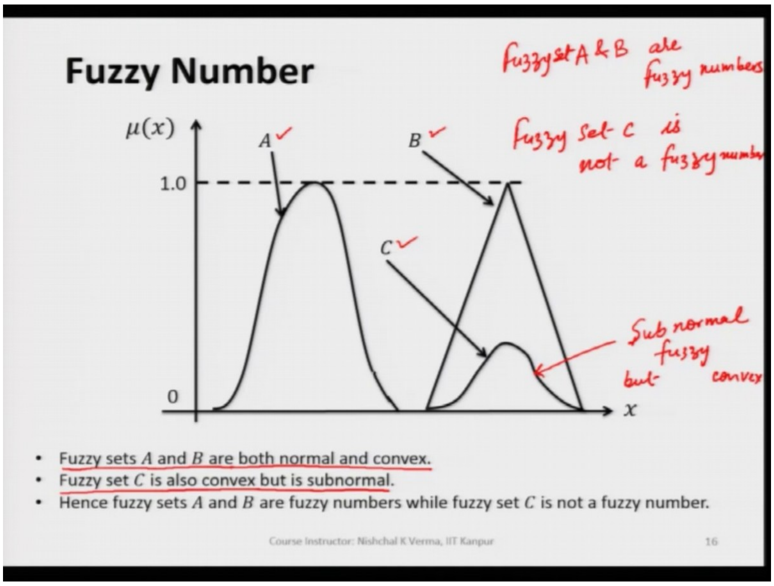
It means the this fuzzy set A should be a normal fuzzy set and a convex fuzzy set. So, these two conditions must be satisfied before we say that this fuzzy set is a fuzzy number. Most non composite fuzzy sets used in literatures satisfy the condition of normality and convexity. So, as I mentioned the fuzzy numbers are the most basic types of type of fuzzy sets.

So, we see here A is a fuzzy set and since this fuzzy set is a normal fuzzy set because it its core is nonempty. And at the same time this is monotonically decreasing or increasing. So, this fuzzy set follows the criteria of convexity, so since this fuzzy set is normal fuzzy sets and a convex fuzzy sets. So, this means this fuzzy set is qualified to be called as a fuzzy number.

Now, if we take another example here we have a B fuzzy set. So, we can clearly see here by just looking at it we can clearly say that this fuzzy set is normal fuzzy set that is fine. So, this is this says this one of the values are maybe we can say the core of this fuzzy set is non empty. So, we can say this fuzzy set is normal fuzzy set.

However this fuzzy set is non convex fuzzy set. So, since this convexity criteria is not satisfied means this fuzzy set is not a convex fuzzy set. So, B can be said as B is not a fuzzy number.

(Refer Slide Time: 22:33)



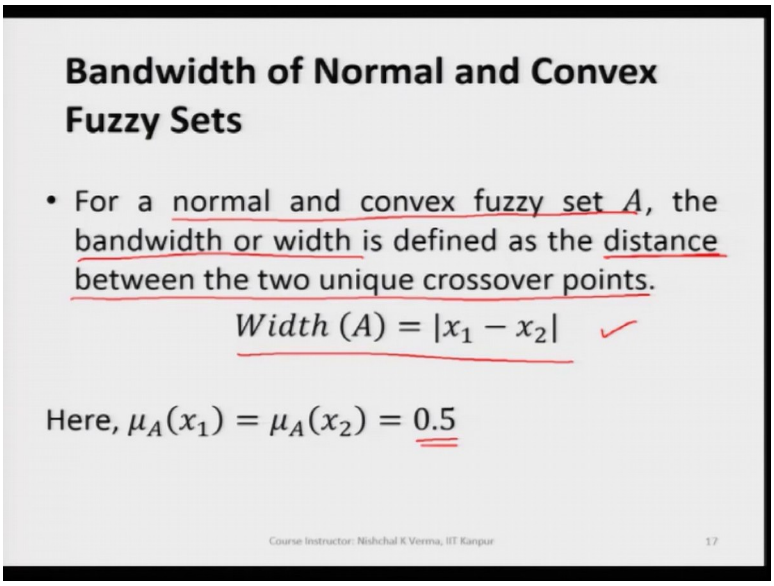
So, here we have another diagram we have three fuzzy sets A, B, C and these three fuzzy sets if we look at we see A fuzzy number very clearly because this is normal and convex. If we look at B fuzzy set B is again a normal fuzzy set and convex fuzzy set. So, B can be a fuzzy number while if we talk of C, C is a subnormal fuzzy set core of the c fuzzy set is empty.

So, because none of the generic variable values have has the corresponding membership value equal to 1. So, core of the fuzzy set C is empty. So, this means this is a subnormal sub normal fuzzy set. Although this satisfies the criteria of convexity. So, this subnormal, but non convex. So, that is why C is not a fuzzy set.

So, fuzzy sets here fuzzy sets A and B are both normal and convex whereas, C is convex, but subnormal. So, this way we can say A fuzzy set, fuzzy set A and B are fuzzy numbers, whereas fuzzy set C is fuzzy set C is not a fuzzy number. So, this way we understand whether a fuzzy set is a fuzzy number or not.

So, let me repeat here that fuzzy number should follow two criteria. One is the it should be normal and then it should be the should follow the condition of convexity. So, it should be convex fuzzy set. So, if n is fuzzy set which is a normal fuzzy set and a convex fuzzy set we can say that this fuzzy set is a fuzzy number.

(Refer Slide Time: 26:01)

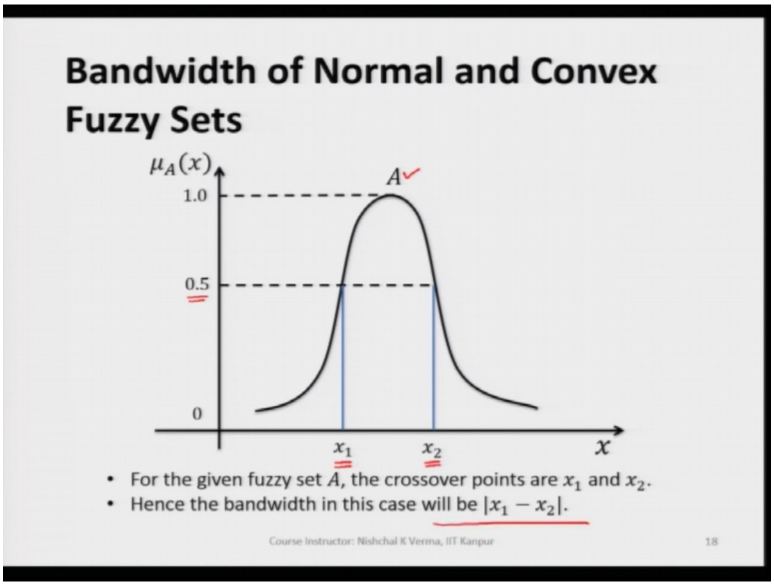


Now, another term that is bandwidth of normal and convex fuzzy set. So, bandwidth here is defined by this expression, so for a normal and convex fuzzy set. So, please understand that this bandwidth is defined only for a normal and convex fuzzy sets. So, it means that the bandwidth is always found for a fuzzy number.

So, the bandwidth or width many times we use width also for bandwidth. So, either of these two words are used. So, bandwidth our width here is defined as the distance between the two unique crossover points and we know what are the crossover points of a fuzzy set. Crossover points are basically the points whose corresponding membership values are equal to 0.5.

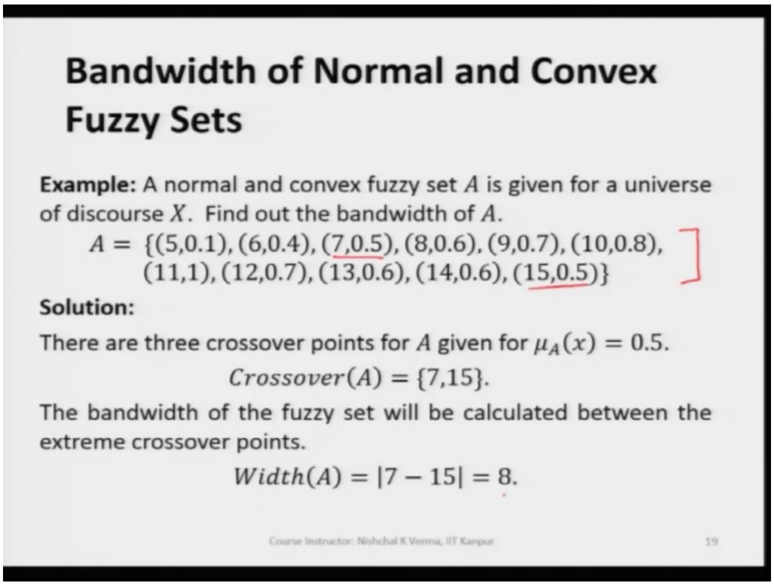
So, this way we find the bandwidth of a fuzzy set and of course, as I already mentioned that the this fuzzy set will be a fuzzy number because this follows two criteria. The first criteria is this fuzzy set is normal and the other one is this fuzzy set is a convex fuzzy set.

(Refer Slide Time: 27:50)



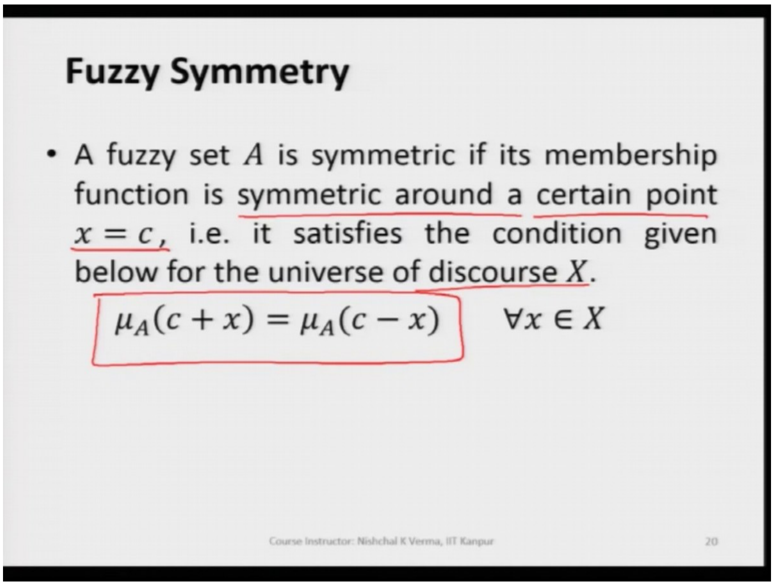
So, here we have an example and with this we can understand better. So, here we have a fuzzy set A and we have the x_1 and x_2, 2 points and these rest these respective membership values are 0.5. So, if we take the distance between these two and these this distance will be called as the bandwidth.

(Refer Slide Time: 28:25)



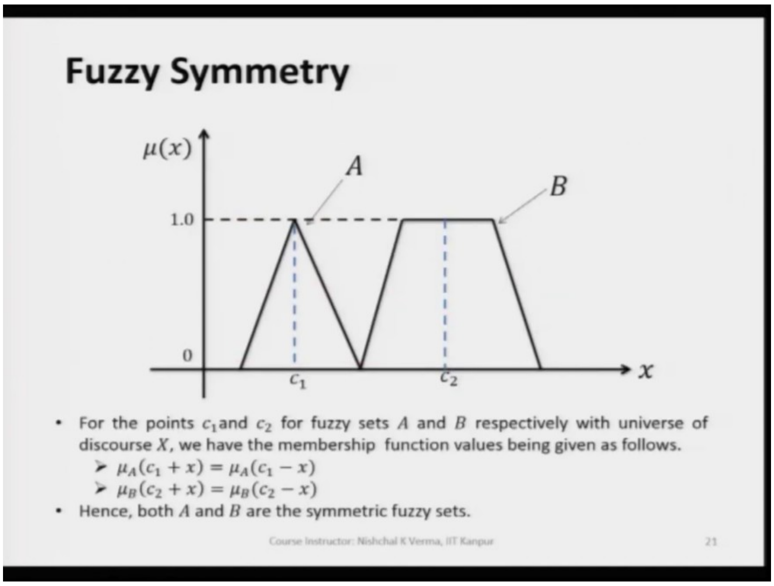
And then if we take discrete fuzzy set. So, with this example we can understand the bandwidth of discrete fuzzy set. So, if we take this fuzzy set here and with this we first find the points for which the respective membership values are 0.5. So, we will find in this kind of fuzzy set. The fuzzy set which is convex and normal. So, we will find only two points. So, we find here we have two points and if we take the distance between these two will get the bandwidth.

(Refer Slide Time: 29:19)



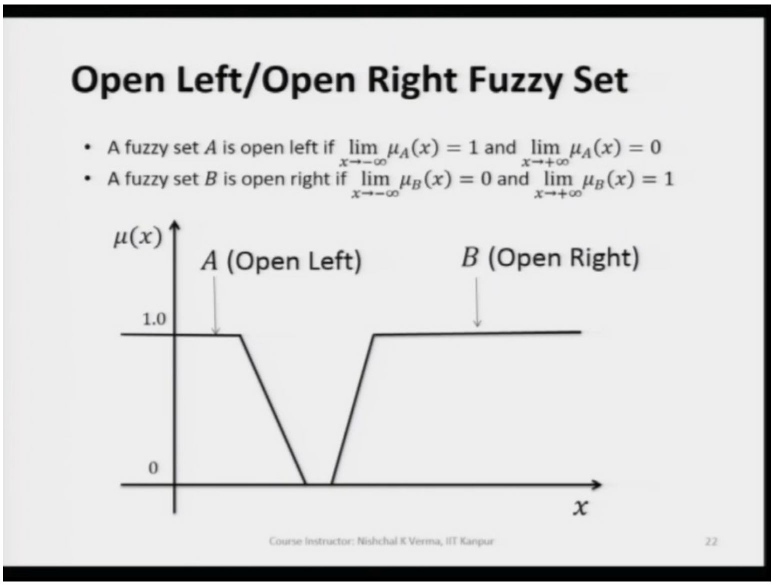
Now, for fuzzy symmetry. So, fuzzy symmetry of a fuzzy set is defined as that like a fuzzy set is called symmetric, if its membership function is symmetric around a certain point x is equal to c. It satisfies the condition given below for the universe of discourse X. So, this means that if this condition is satisfied and then we can say the fuzzy set is a symmetric fuzzy set. And this point c should be any point that is taken from the universe of discourse.

(Refer Slide Time: 30:12)



So, let's now understand this more clearly by taking this example. So, if we have two fuzzy sets here A and B. And if we choose c_1 as the point for checking the symmetry. So, if we take any point c_1 and if this condition is satisfied like $\mu_A(c+x)=\mu_A(c-x)$ then we can say the this fuzzy set is symmetric. Similarly, for B we can check and these two will be found as a symmetric fuzzy sets.

(Refer Slide Time: 31:02)

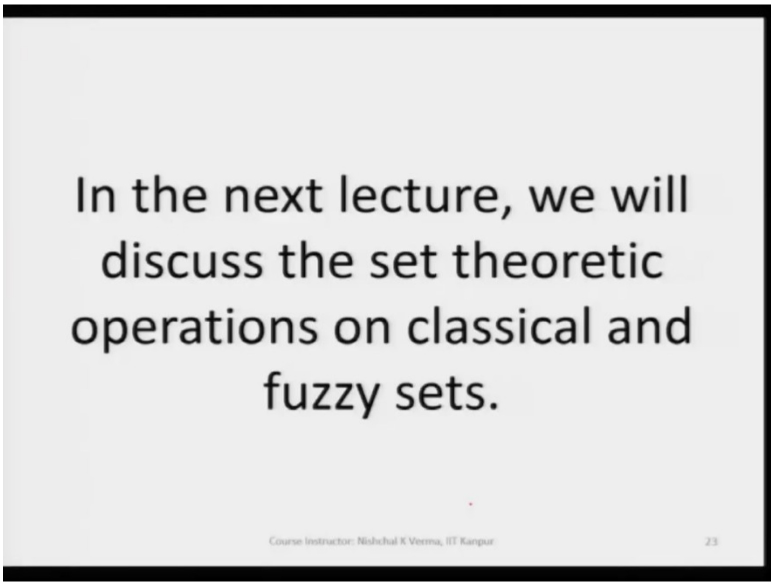


Another termed that we use here is open left, open right fuzzy set. So, if we take a fuzzy set A and this will be called open left if we if this satisfies the condition that is limit of

$\lim_{x \to -\infty} \mu_A(x)=1$. And $\lim_{x \to +\infty} \mu_A(x)=0$. So, then this will be called as open left. Similarly a fuzzy set here this is B and this is called as open right because the $\lim_{x \to -\infty} \mu_A(x)=0$ and $\lim_{x \to +\infty} \mu_A(x)=1$.

So, this means that if we have fuzzy sets which are open on the right side are attaining the value of the membership is close to 1 or equal to 1 and the right side is called as open right. And if it is left side it is called open left. So, with this most of the terms that are used in fuzzy systems theory are covered.

(Refer Slide Time: 32:52)



And in the next lecture we will discuss the set theoretic operations on classical and fuzzy sets thank you.