Fuzzy Sets, Logic and Systems and Applications
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Lecture – 08
Nomenclatures used in Fuzzy Set Theory

So, welcome to the lecture number 8 of Fuzzy Sets, Logic and Systems and Applications. In this nomenclature, some of the terms that we have already discussed remaining terms, remaining nomenclatures that we will be discussing in this lecture.

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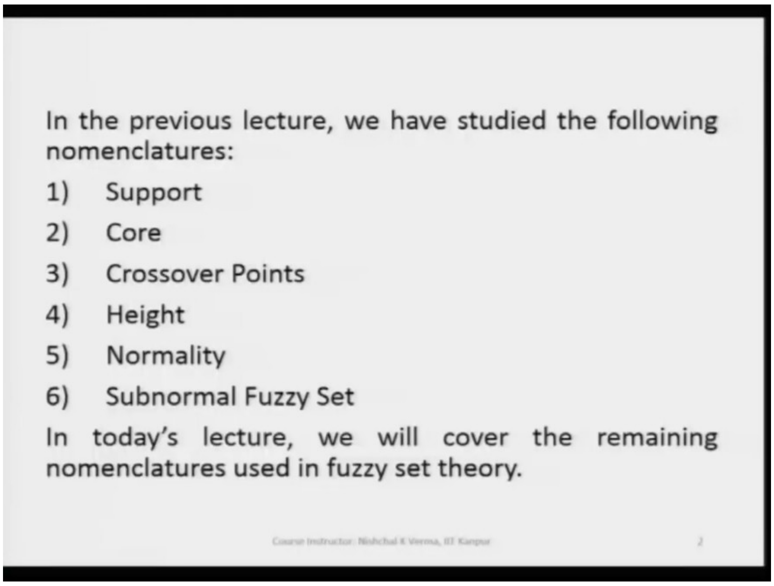

And, as I mentioned; the support, core, crossover points, height, normality, subnormal fuzzy sets, all these terms in the nomenclatures have already been covered. And in today's lecture, we will discuss the remaining terms in the nomenclatures used in the fuzzy set theory.

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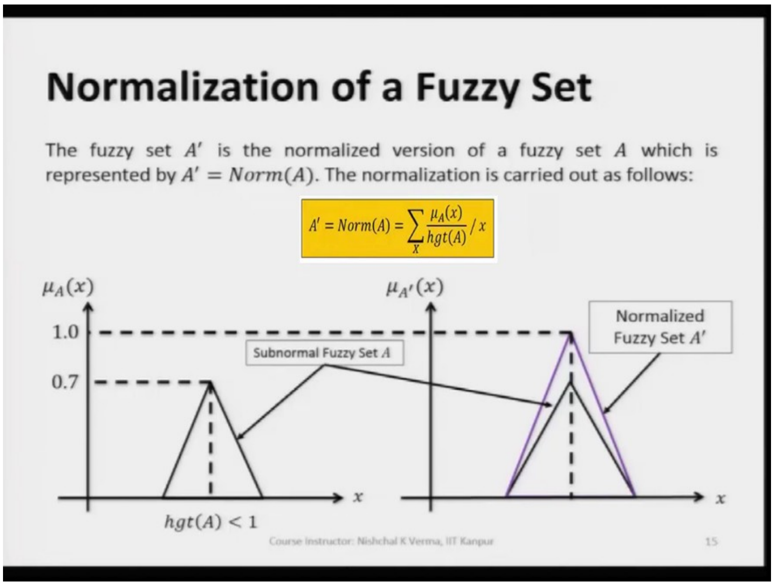

Now, by now we understand what is a normal fuzzy set. So, normal fuzzy set is a fuzzy set, which whose height is 1. So, if we have fuzzy sets whose height is not 1, let us say and we would like to increase its height up to 1 or in other words if you say that; we would like to normalize a fuzzy set A.

So, let us see how we can do that. So, if we have a fuzzy set A and A is a subnormal fuzzy set. So, if we have here; if we have a fuzzy set A which is subnormal, this fuzzy set is subnormal. And, if we would like to normalize this fuzzy set; it means, we would like to increase the height of this fuzzy set up to 1. So, here we can better understand this normalization by taking a examples and one of the examples is here.

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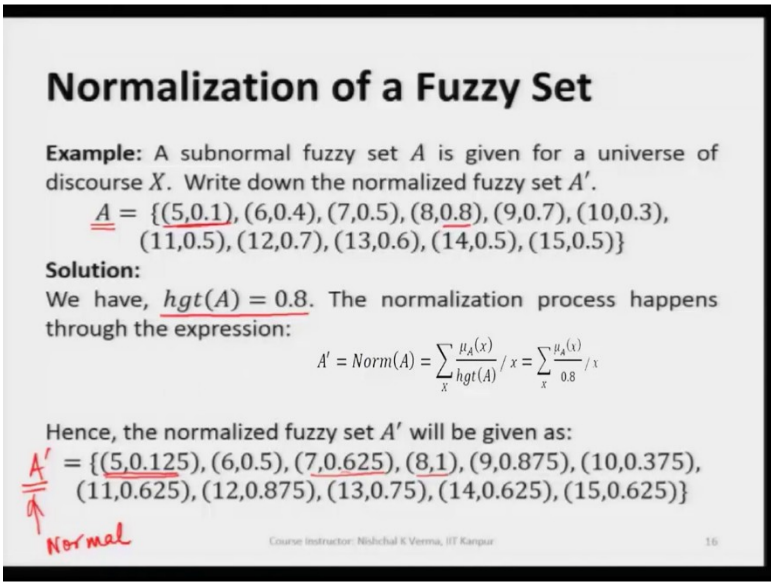

So, if we take fuzzy set here A, which is subnormal fuzzy set. We see here, this is subnormal fuzzy set and so, when we say subnormal means, none of the elements here present in this fuzzy set has its corresponding membership value 1. So, we can clearly see here that this fuzzy set is this A fuzzy set is a subnormal fuzzy set. Now, we would like to normalize this fuzzy set. So, we first find the height of this fuzzy set and height of this fuzzy set is 0.8 because, the maximum of all the membership values present in this fuzzy set is coming out to be 0.8, which is here.

So, we have the height of a fuzzy set now and then simply we divide this fuzzy set. So, when we say we divide this fuzzy set, it means we divide the corresponding membership values. So, this has to be noted here that like the first element in the fuzzy set A here and the corresponding first element here in the fuzzy set A dash, which is a normalized fuzzy set. So, the membership, only the membership value is normalized. So, we need to understand that the generic variable value which is 5 here remains as it is. So, 5 remains as it is, but the corresponding value is normalized corresponding membership value is normalized.

So, how is it normalized here is. So earlier, the 5 has its corresponding membership value 0.1. Now, if we divide it by 0.8, this becomes 0.125. So, this way this the corresponding membership value is changed. And similarly, 6 will have 0.5 and then 7 will have 0.625. And similarly, all other points will have its corresponding membership values normalized. So, clearly here, if we see here 8 will have in $A^{'}$ 8 will have its membership value 1, which earlier was 0.8 in the A membership A fuzzy set, which was a subnormal fuzzy set.

So, now we see that the A dash is the normal fuzzy set; normal fuzzy set. And normalization process we understood very clearly. So, when we say here, A divided by height of A it means that, we are dividing the respective membership values, the member the generic variable values are never normalized.

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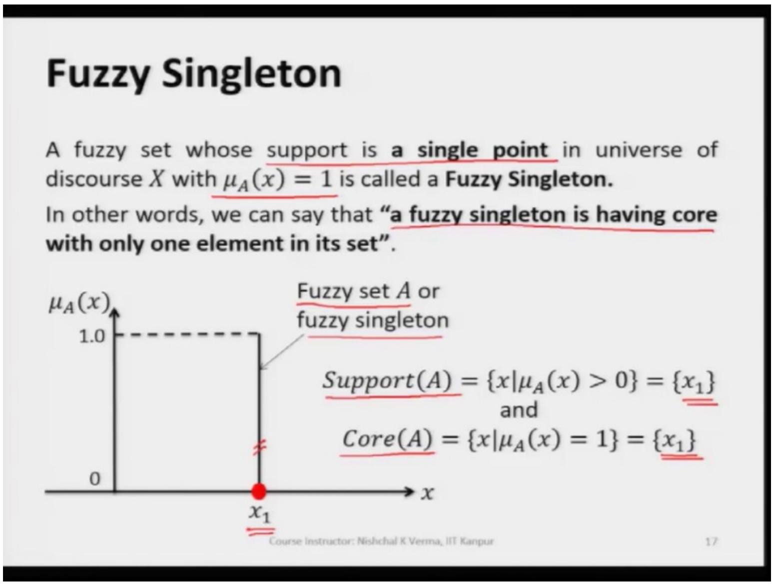

Now, coming to fuzzy singleton. So, what is a fuzzy singleton in fuzzy systems theory is a fuzzy set whose support is a single point and of course, in the universe of discourse capital X, and so support is a single point and then, there is another condition the single point should have its membership value equal to 1.

So, if this condition is satisfied, we call the corresponding fuzzy set, the fuzzy set for which this condition is satisfied is a fuzzy singleton. In other words, a fuzzy singleton is having core with only one element in its set, what does it mean? It means that, if I have a fuzzy set and if we try to find the, if we are interested to know whether this fuzzy set is a fuzzy singleton or not, then we will quickly try to find the support or I would say a core. So, if this core is a single value, then the core is the single value and the support is also a single value. So, it means what? It means that, a fuzzy set which has a single element. So, the support is a single value and the core is also a single value, but if the support is a single value it should have its mu equal to 1.

So this way here, if we see that, if we have a fuzzy set A which will be a fuzzy singleton when the corresponding x here, so if we take a support here, support here will be only one value which is x_1 and this at this x_1 will have its corresponding membership value 1. So, support so you see here, the support of A is nothing but x_1 and of course, the support has a single element and the core is also a single element. So, in both the cases we have x_1 only. So, this x_1 this kind of fuzzy set is called a fuzzy singleton.

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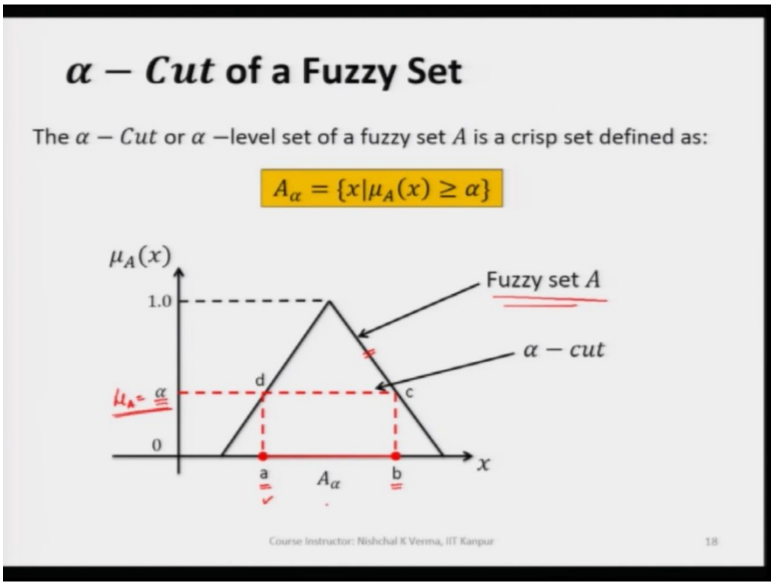

Now, let us understand the $\alpha - cut$ of a Fuzzy Set in a fuzzy systems theory. So, alpha cut or alpha level set of a fuzzy set is a crisp set that is defined as

$$A_\alpha = \{x \vee \mu_A(x) \geq \alpha\}$$

Alpha can be any value in between 0 to 1, it can be 0 also. So, let us now try to understand, what is alpha cut of a fuzzy set?

So, if we have a fuzzy set A here, this there is a fuzzy set A, and if we are interested to find the $\alpha - cut$ of this fuzzy set. So, if we choose to have my α like this, this is nothing but the membership value alpha. So, the corresponding to this, we try to find the generic variable values. So, which is here in this case we have a and then here if we extend this we will have another point here, which is another point of the generic variable b. So, we will get two generic variable values which are here a and b.

So, a and b will correspond to the values, the generic variable values at which we have membership value α. So, if we take α we have to see whether the membership value is alpha or not. But, this α as I mentioned can be any value in from 0 up to 1. And please understand that, when we talk of $\alpha-cut$, so we are not interested in only quality, but we are interested in, we will have collection all the generic variable values for which, the their corresponding membership values are more than $r=\alpha$.

So, in this case all the points in between even in including a and b will be included. So, that is why, we are showing this all these $\alpha-cut$. So, all these values the collection of all the values in between right from a to b, and please understand that, a has been showed by solid dot and b also has been shown by a solid dot. So, it means what? It has this line will include all those points, all the points for which we have the membership values corresponding membership values either α or more than α.

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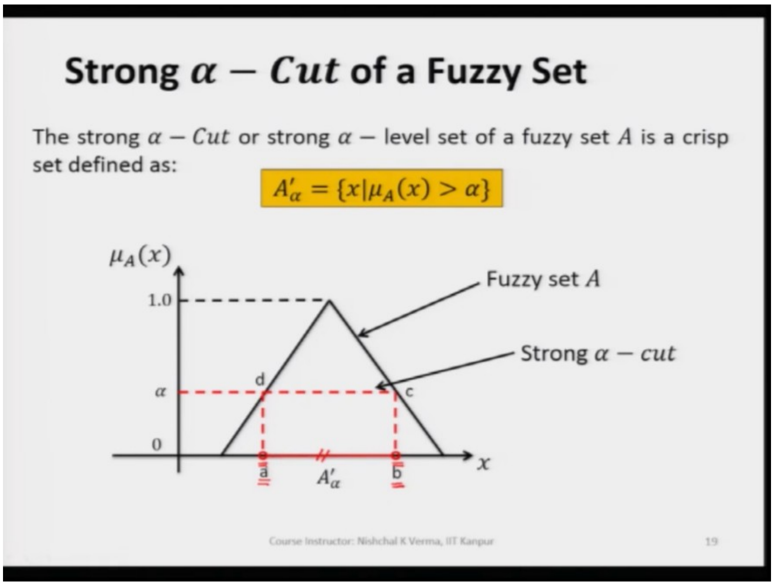

So, this we this way we understand $\alpha-cut$. Now, if you are, we have another nomenclature, which is another nomenclature that we use here, is a strong alpha. So, if we are interested in finding out a strong alpha cut. So, everything remains same except the quality goes away. So, if we are interested in a strong alpha cut, so a strong $\alpha-cut$ is represented by $A_\alpha^{'}$. So, $A_\alpha^{'}$ is equal to collection of all the generic variable values for which we have the corresponding membership values more than α. So, in this case here, if we take the same example as we took in the $\alpha-cut$ case. So, instead of this solid dot here we will have the hollow dot.

What does this mean, that we are excluding the quality condition means this the generic variable value a and b will not be in included because, at this point here α is exactly equal to the membership value is exactly equal to α. So, we will take only those values of the generic variable for which, the membership value is more than α. So, more than α will we will be getting by this line. So, because of that the terminals of a and b will be hollow it means that, this points will not be included; the start of a will not be included because, here the alpha is the membership value is α and here at b also the membership value is exactly equal to α.

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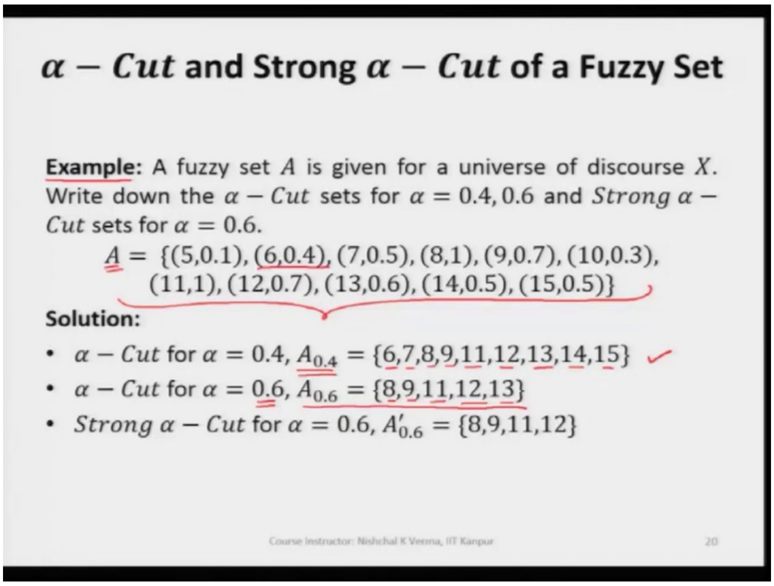

So, let us now take an example here, to better understand alpha cut and strong $\alpha - cut$ of a fuzzy set. So, here is an example. So, if we take a fuzzy set A for universe of discourse X and if we are interested in finding out a alpha cut and this alpha let us say is 0.4 here, and it could be any value in between as I mentioned that right from 0 to 1. So, let's now try to find the $\alpha - cut$ of this fuzzy set A and the alpha the value of alpha is equal to 0.4. So, we can represent as I mentioned this *alpha cut* by A subscript 0.4. So, let's now try to see here in the fuzzy set A.

So, we see here this point, the fuzzy set, so 6 we can we will have 6 and then 7 and then 8, 6, 7, 8 and then 9 and then we will have 11, 12, 13, 14, 15. So, all are all these are the generic variable values for which, the corresponding membership values are either 0.4 or more than 0.4. So, this way we can write A 0.4. Now, if we change the value of alpha to 0.6. So, my A

0.6 cut of A will be collection of 8, 9,11,12,13. So, this way we will be able to get the alpha cut of a fuzzy set.

Now, when we are interested in making this alpha cut a strong means strong alpha cut. So, for alpha is equal to 0.6. We will have collection of all those points for which, their corresponding membership value is more than or greater than 0.6. So, we will not be taking those values for which alpha is equal to 0.6 we will be taking only those values only those generic variable values for which alpha for which the membership value is more than 0.6. So, we will exclude in the previous case we have included those values for which the corresponding values of the membership was equal to 0.6 here in this case we will include.

So, this way A strong $\alpha-cut$ a strong $\alpha=¿0.6$ cut will be collection of $A_{0.6}{}'8,9,11,12$. So, this way we will be able to find out the $\alpha-cut$ and a $A_\alpha{}'$of a fuzzy set.

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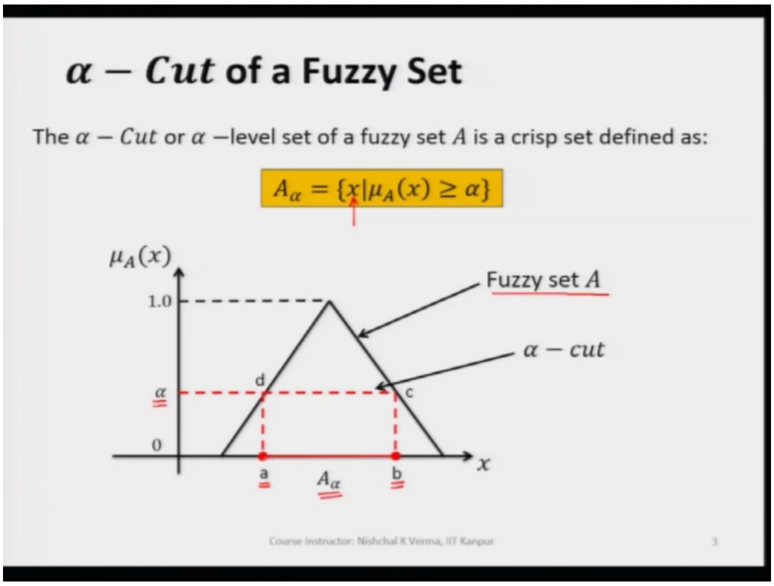

Let me go through alpha cut and a strong alpha cut little bit again. So, alpha cut of a fuzzy set that we had already seen is defined by $A_\alpha=\{x \vee \mu_A(x)\geq\alpha$. So, what does exactly this mean is that we have to collect all the generic variable values of a fuzzy set for which, the respective or the corresponding membership values are either equal to alpha or more than alpha and this is called alpha cut of a fuzzy set.

So, let us now understand $\alpha-cut$ more clearly with this diagram. So here, we have a fuzzy set A and if we are interested in finding the $\alpha-cut$ of this fuzzy set that is A, so we will first look for the value alpha and alpha will always be value which is, which can be either 0 also, but normally more than 0 and it can go up to 1. So, let's say our alpha is some value here which is in between 0 and 1 and then we will look for the corresponding generic variable value which is a here. So, this will be the generic variable value for which, the membership value is equal to α.

So, this will be the start and then we will take up all the values of x that is the generic variable. So, all the values of x for which the α is either the membership value is either equal to α or more than α, so for this fuzzy set A we clearly see that, we get another point b here which also has the corresponding membership value α. So, in between and including a and b, we get all the values of the generic variable for which we have the respective or the corresponding membership values either greater than alpha or equal to alpha. So, that makes the sense here as to include all the values of the generic variable right from a up to b.

So, since this is a continuous fuzzy set and therefore we can have infinite number of such values of generic variable. So, that is why this has been shown by a line and if we look at the terminal of this a b the line ab the terminals are the solid dots. So, solid dot would indicate here that the a is also included point a is also included and b is also included in the $\alpha-cut$ of the fuzzy set. So, this has to be no noted here that, $\alpha-cut$ is a set of all the values of generic variable and this will not contain the generic the membership their corresponding membership values

So, only the $\alpha-cut$ will contain only $\alpha-cut$ is the set which will contain only the generic variable values.

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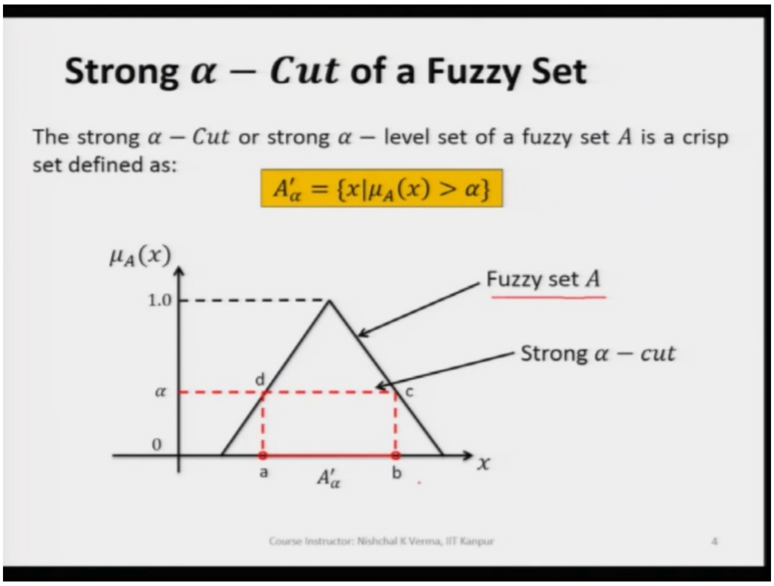

Now, if we talk of a strong $\alpha - cut$ of a fuzzy set. So, here we make a little change and that is the equality, that is the equality is not there. So, this means that if we are interested in finding a strong $\alpha - cut$ of a fuzzy set. First of all, we define strong $\alpha - cut$ of a fuzzy set by $A_\alpha^{'}$. So, dash is the difference here as compared to the $\alpha - cut$. So, if we talk of strong $\alpha - cut$ the this will be represented by the a, the a of $A_\alpha^{'}$ and this will be equal to the collection of all the points of the generic variable values for which their respective membership values are greater than α.

So, this is to be noted that here the membership values the respective membership values are greater than alpha, here equality sign is not there. So, this is the difference in between the $\alpha - cut$ strong $\alpha - cut$. So, if we take a fuzzy set here in this example we have fuzzy set A and we are interested in finding out strong $\alpha - cut$ of a fuzzy set. So, please understand that since the equality sign is not there only the membership values which are greater than α are included. So, that is why as we have already discussed in the $\alpha - cut$, the line a and b, a and b here will be the strong $\alpha - cut$. But, the terminals of this line a and b means the start of this line at a and end of this line b will be represented by a hollow circle.

It means the point a is not included and b is not included. So, it is not included because exactly at the point a the generic variable value point a, we have the membership value the corresponding membership value α. So, that is why it is not included because equality sign is not there. So, we are only considering those values of generic variables for which their respective membership values are greater than α. So, now I think this is very clear difference in between the $\alpha - cut$ and the strong $\alpha - cut$ of a fuzzy set.

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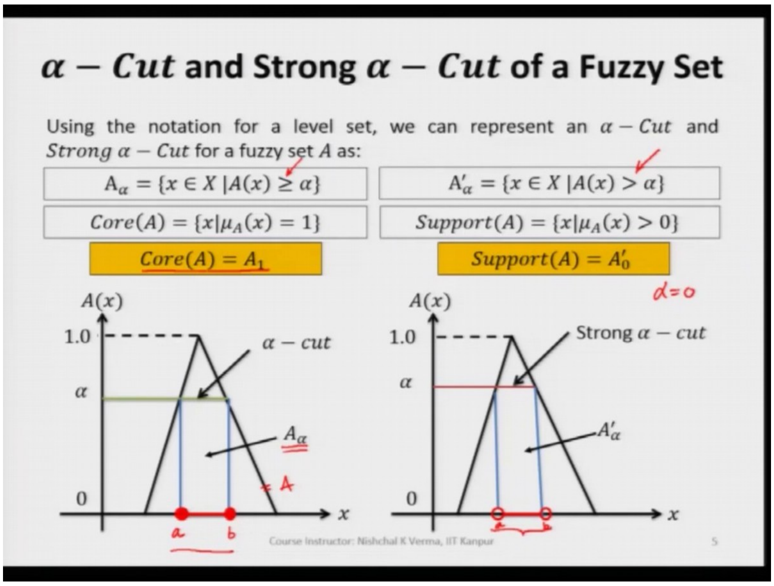

Here also, we see the clear difference in between $\alpha - cut$ and strong $\alpha - cut$. So, we see how is it defined the quality sign is here see the equality and greater than, but in alpha cut here we see only the greater than sign is there. So, this way we understand this very clearly. Now, there is another way of understanding this like if we are interested in a one, a one means the one cut of a fuzzy set A, but this has to be a normal fuzzy set because α is equal to 1 has been taken here. So, if we take an example here and we have a fuzzy set let us say A this is a fuzzy set A and if we are interested in alpha cut, this will be the $alpha\ cut$ here this can be small a this can be b. So, $\alpha - cut$ is this line a and b.

And strong alpha cut as I have already explained this a here and b here will be the strong alpha cut. So, we also have if we are interested in finding A_1. So, A_1 means the one cut of the fuzzy set. So, one cut of the fuzzy set A. So, this will have all those points this will have the collection of all those points for which, the, their corresponding membership values are equal to 1. So, obviously, the core is also the same. So, we can say that a core of a is equal to α 1 is one cut of a fuzzy set A. Similarly, the support of a can also be defined in terms of the strict $\alpha - cut$. So, if we take the value of α here as 0.

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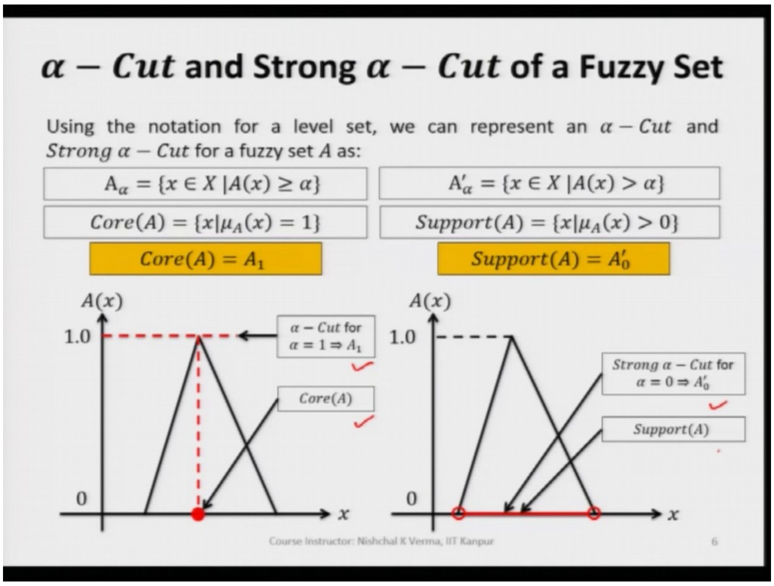

Alright so, now this is exactly the same thing was explained and through diagram you can understand it little bit more clearly.

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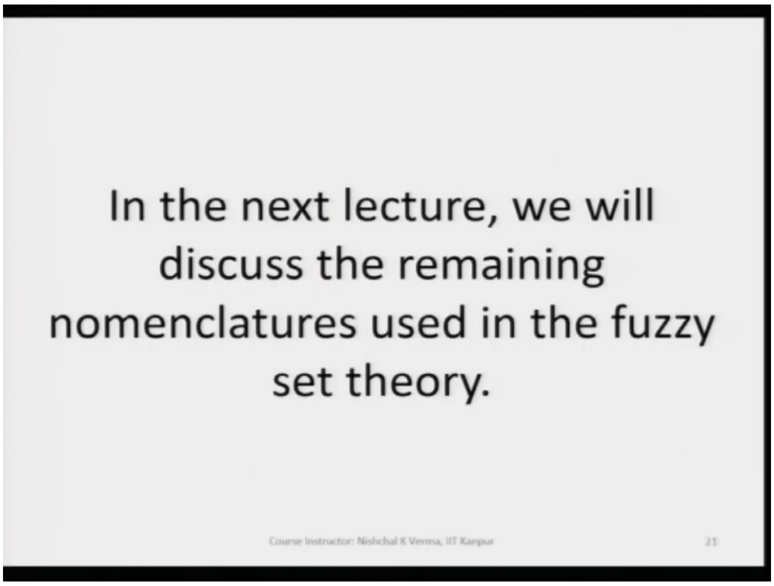

So, at this point we would like to stop for this lecture and the in the next lecture, we will be discussing the remaining nomenclatures used in the fuzzy set theory. So, it means we will be you know moving ahead to discuss remaining nomenclatures.

Thank you.