Fuzzy Sets, Logic and Systems and Applications
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Lecture - 07
Nomenclatures used in Fuzzy Set Theory

Welcome to lecture number 7 of Fuzzy Sets, Logic and Systems and Application. So, in this lecture, Nomenclatures used in Fuzzy Set Theory will be covered.

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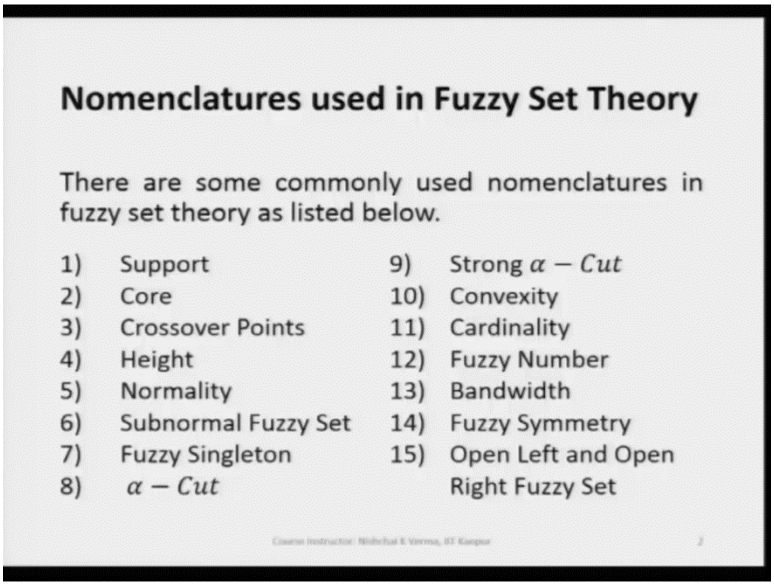

So, here are few nomenclatures that are listed. So, we have here 15 nomenclatures that are commonly used in fuzzy set theory. And in this lecture, we will be discussing support, core, cross over points, height, normality, sub normal fuzzy set.

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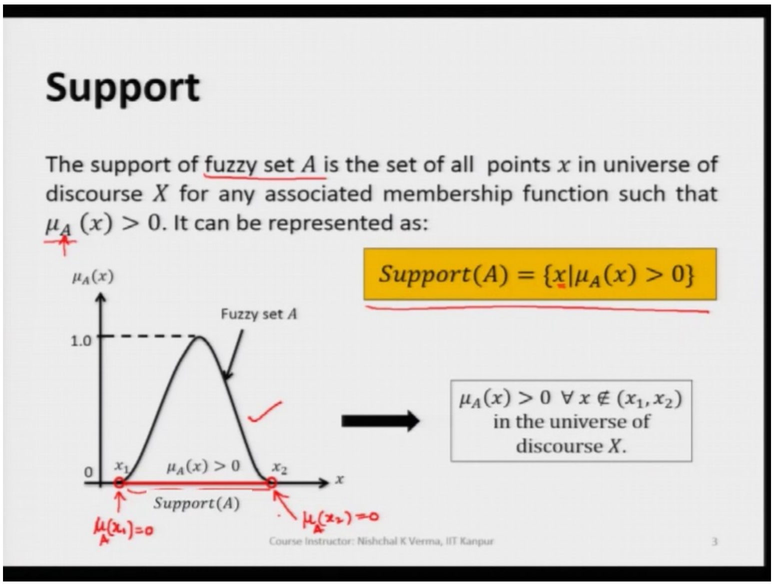

So, let's now discuss the nomenclatures used in fuzzy set theory one by one. So, the first one is support. So, what is a support? So, the support of a fuzzy set A is the set of all the points small x in universe of discourse capital X for any associated membership function such that $\mu(x)$.

So, since we are talking of a fuzzy set A. So, that is why we are saying here the μ_A. So, this A represents the fuzzy set; the name of the fuzzy set, so $\mu_A(x)$. So, $\mu_A(x)>0$. So, support of A is written here.

$$Support(A)=\{x \vee \mu_A(x)>0\}$$

So, it is very clear that; if I have a fuzzy set let us say here, so this fuzzy set has here for the value of x_1, the mu x_1 is 0. See here the $\mu(x¿¿1)=0¿$. And then at x_2, $\mu(x¿¿2)=0¿$. So, and I will be writing A here and A here, because we have taken fuzzy set A; and this μ corresponds to the fuzzy set A, so that is why μ_A has been written. So, now, this two points x_1 and x_2 will be excluded from all the values of x here, which has the membership values more than 1. So, this point will not be x_1 and x_2 point will not be covered in the support of a fuzzy set.

So, if we have a continuous fuzzy set here as it is shown here, so we will have all those points; because since this is a continuous point, we will have as per as the number of elements are concerned, we may have the infinite number of points. So, that is why it is shown by a line here. So, we can clearly represent this line as the support of a fuzzy set A and this fuzzy set is a continuous fuzzy set.

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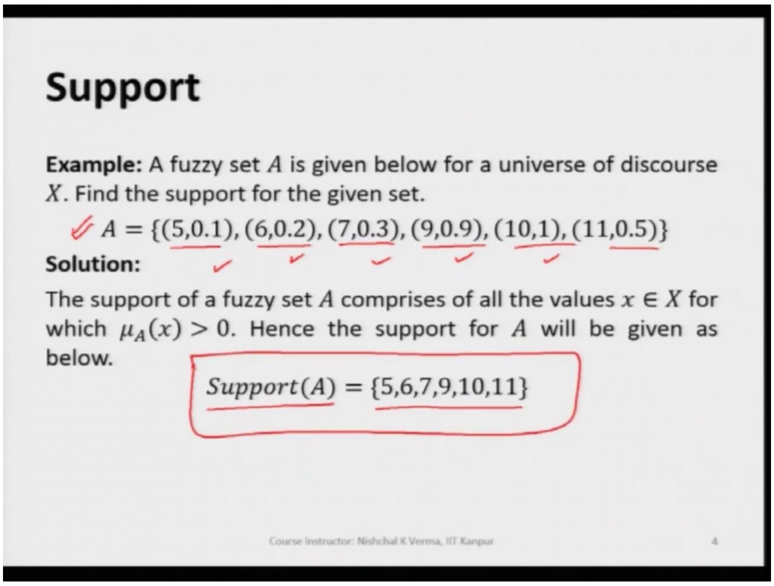

So, let us now take an example here where the fuzzy set A is a discrete fuzzy set. So, if you take a fuzzy set here as A is equal to this

$$A=\{(5,0.1),(6,0.2),(7,0.3),(9,0.9),(10,1),(11,0.5)\}$$

, we have a set fuzzy set which includes all these points; where the first element here is the generic variable value and then here the second element is the second the element of this pair the value and then the corresponding membership value.

Similarly, we have 1, 2, 3, 4, 5, 6. So, in this discrete fuzzy set we have six elements. And so, we look at this fuzzy set. It is very clear that all these generic variable values $5,6,7,9,10,11$, they have their corresponding membership values all greater than 0. And as per the definition if we are interested in finding the support of a fuzzy set, especially here; this fuzzy set is a discrete fuzzy set.

So, we can write the support of a fuzzy set A for this case will be the collection of all the points for which the corresponding membership values ¿0. And we can clearly see here, all the points all the 6 points here; first, second, third, fourth, fifth, sixth. All this points, all the generic variables have their corresponding membership values ¿1. So, it means that, the support of A will have a set of all those generic variable values.

So, it means if we would like to write the support of fuzzy set; specially the discrete fuzzy set, then we will have here for this case, for this discrete fuzzy set will have a set of points 5, 6, 7, 9, 10, 11. So, this way we will write the support of a fuzzy set like this

$Support(A)=\{5,6,7,9,10,11\}$,

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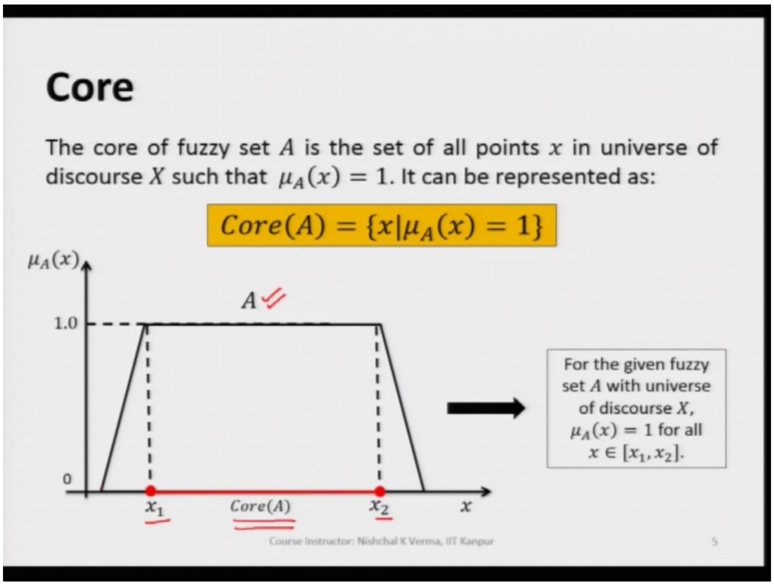

Now, let us discuss course the core of a fuzzy set. So, the core of a fuzzy set is the set of all the points in the universe of discourse such that $\mu_A(x)$ ¿1.

So, this can be represented as if you would like to write the core of a fuzzy set and this will be equal to the collection of all the points or the set of all the points for which the corresponding membership values are 1. We can very clearly understand the core of a fuzzy set like this. If we take a fuzzy set A, this is fuzzy set here and we would like to find the core of this fuzzy set, we'll take the collection of all the, we'll take the set of all the points here for which corresponding membership values are 1.

So, we can clearly see here for this membership function which is a trapezoidal membership function and of course, this is a continuous fuzzy set. So, we'll we can say that this line x_1 till x_2, we have for each and every point which is coming in between we'll have the membership their corresponding membership values equal to 1. So, this line will represent the core of fuzzy set; here for this fuzzy set.

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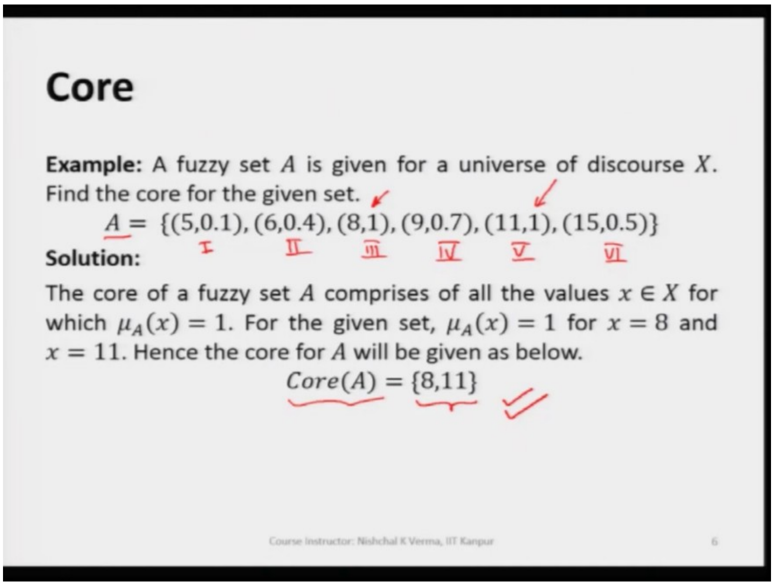

So, let us now take an example to better understand the core of a fuzzy set, especially for the discrete fuzzy set here. So, let us take this example fuzzy set A here is given by A see here and this fuzzy set has I, II, III, IV, V, VI points. And this points basically are the fuzzy points; fuzzy point means it has two values. This is a pair, the first value as we have seen in the previous case also previous example also. The first element here is the generic variable value and then the second element here the pair the second element of this pair is the membership value, corresponding membership value.

So, if we look at this fuzzy set, we have six elements, six points and if we see we find there are two points. There are two generic variable values for which the corresponding membership values are 1. So, one is here; the first one is here and then the second one is here. So, as per the definition, core of a fuzzy set is collection of all the points, all the generic variable values for which the corresponding membership values are 1.

So, that is why here if we apply that will find 8 and 11 and we can write here as a set. We can we can write core of a fizzy set, core of a discrete fuzzy set is collection of the generic variable values which are 8 and 11. So, this is very simple very clear. So, for discrete fuzzy set we if we are interested in finding core of a fuzzy set we can very easily collect those values those generic variable values for which the membership; the corresponding membership value is 1.

Rest generic variable values we just leave aside. So, we only take the generic variable values here and this is very important here to be noted that when we are writing either support of fuzzy set core of a fuzzy set, we only write the generic variable the collect the collection of generic variable values. We do not write the corresponding membership values also along with the generic variable values.

So, we only write the values of x and that is how we get the support of a fuzzy set and core of a fuzzy set.

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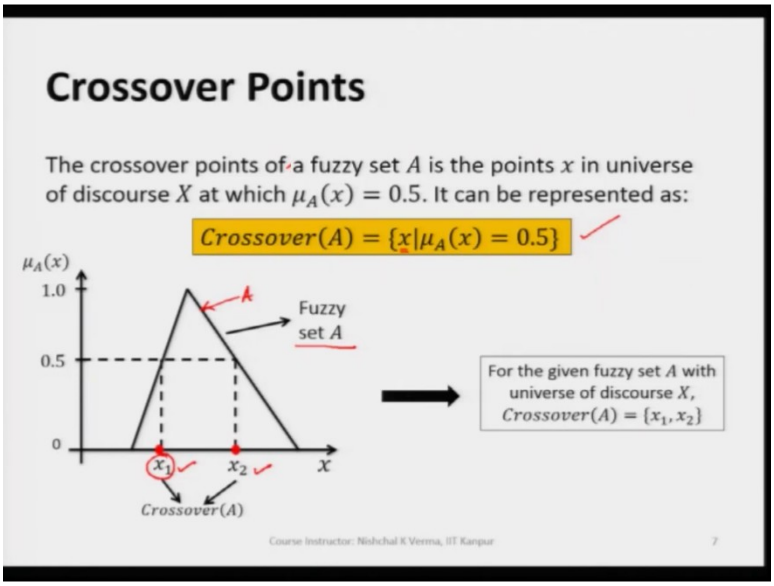

So, now let us move to another nomenclature that is used in fuzzy systems theory; the crossover points. So, the crossover points of a fuzzy set A is the points x in universe of discourse capital X at which $\mu_A(x)$ ¿0.5. It can be represented as the crossover of A (Refer time: 14:00) and this crossover of A will give us a set of all the values of x and as we know this x is nothing, but the generic variable values. And, these values will be only those values for which $\mu_A(x)=$¿ 0.5; means the half.

So, if we take an example here to understand the crossover of a fuzzy set A here. So, if we have a fuzzy set A. This is a fuzzy set A, is A fuzzy set and we try to find the point corresponding to which a membership value is 0.5.

So, x_1 is the point here corresponding to which the membership value is half or 0.5, so x_1. For this fuzzy set, we have two values x_1 and x_2 for which we see here the membership value is 0.5.

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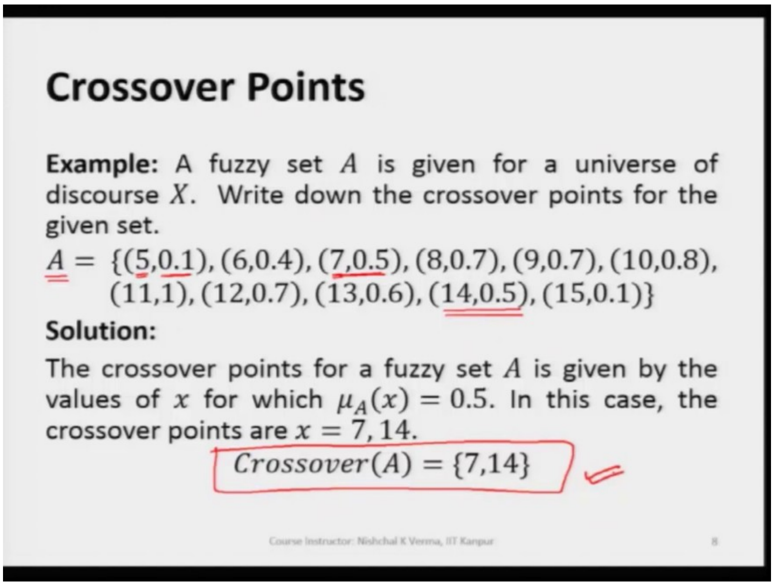

Similarly, here if we would like to understand better, we can refer to this example for crossover points.

So, if we have a fuzzy set A like this. Here we have a fuzzy set and in this fuzzy set we have one, two, three, four, five, six, seven, eight, nine, ten, eleven elements or eleven pairs. So, if you are interested in finding out crossover points for this fuzzy sets, we only look for the generic variable values for which the corresponding membership values are 0.5. So, in this fuzzy set this is a discrete fuzzy set obviously.

So, we see here in this fuzzy set 7 is the generic variable value for which the membership value is 0.5. So, this is the first generic variable value that is we have found and then let's now look at some other value for which we have the membership value of membership value half. So, we see here 14 is another point for which the membership value 0.5. So, this way we find two points; 7 and 14 and their corresponding values are their corresponding membership values are 0.5.

So, that is how we write here the crossover of a fuzzy set.

$$Crossover(A) = \{7, 14\}$$

So, this way we get crossover of a discrete fuzzy set.

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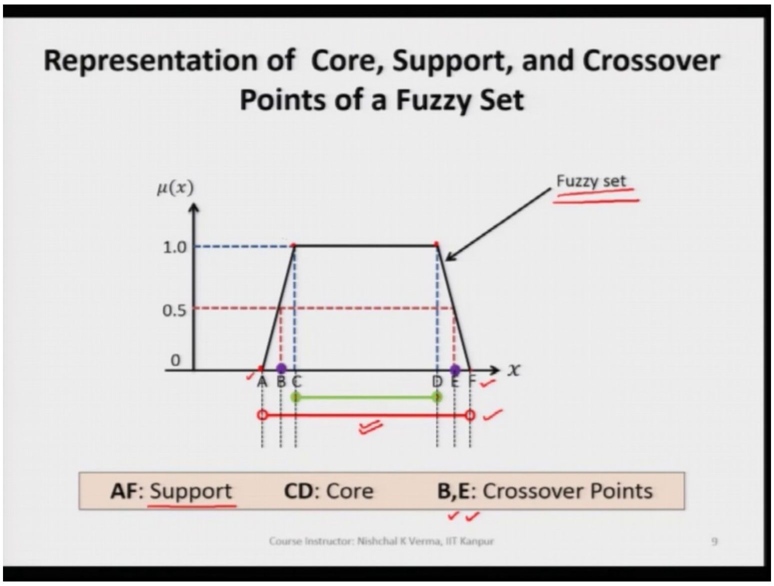

Now, here is representation of core support and crossover points of a fuzzy set. So, if we take a fuzzy set, let us say some fuzzy set which is trapezoidal fuzzy set here. If you are interested in core of this fuzzy set, we can clearly see here that as per the definition; core of the fuzzy is represented by the line CD here. So, if we look at this point here.

So, right from C we have all the generic variable values corresponding to which the membership values are 1. So, that is how, and this goes still C till D. So, this way CD will represent the core of a fuzzy set. And this fuzzy set here is the trapezoidal fuzzy set which is continuous fuzzy set and if you are interested in finding support of the fuzzy set.

So, support of a fuzzy set is as per definition, it its collection it gives us the collection of all the generic variable values for which their respective membership values are more than 0. So, this will start right from 0 plus. Right from 0 plus means; the this will start from A. A is a point where we have the membership value, the A is the point let us say if it is x_A.

So, this x_A will have it is corresponding membership value 0, but x_1+ϵ will have it's, it's corresponding membership value more than 0. So, just after A, all the points and just before F, all the points of the generic variable will be included in the support of a fuzzy set. So, this way if you are interested in finding the fuzzy set, we can say the support of the fuzzy set is represented by the this line the red line. And please a please look at the ends of the A and F, I mean this line. So, we see here the hollow circle. So, we see a hollow circles at both ends; it means that the generic variable value for which the mu x is 0 is not, these are not included at both the ends.

So, this way we clearly understand what is a support of a fuzzy set and then when we are interested in finding the crossover of this fuzzy set of course, we will find the points on the fuzzy sets. The generic variable values on this in this fuzzy set for which the membership values are 0.5.

So, please see here clearly that we have B point and E point for which the corresponding membership values are 0.5. So, that is how the crossover points are B and E. So, this way we here make the clear distinction among the core support and a crossover points of a fuzzy set.

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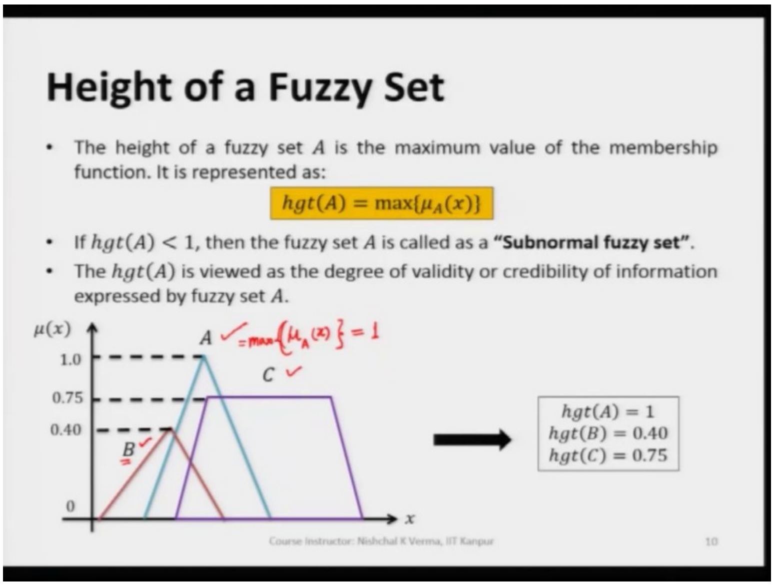

Now, let us discuss the height of a fuzzy set. So, how can we find height of a fuzzy set? So, height of a fuzzy set first of all is defined by $hgt(A)$ and this

$hgt(A)=1 hgt(B)=0.40 hgt(C)=0.75$

So, A is represented by the blue line and B is represented by the red line red color and C is represented by the violet color.

So, if we talk of A; so we clearly see that the highest value of membership here goes till 1. So, if we take all the membership values corresponding all the generic variable values, we will find the max of the membership values of this fuzzy set A. So, I can write it like this like.

If I have $\mu_A(x)$ here and if I take this by the middle bracket like this and if I take max of these, I am going to get 1. Similarly, we talk of height of the fuzzy set B here, for B fuzzy set the height of the B fuzzy set will be 0.4 because if we will take all the corresponding membership values and take the *max* of this will be getting 0.4.

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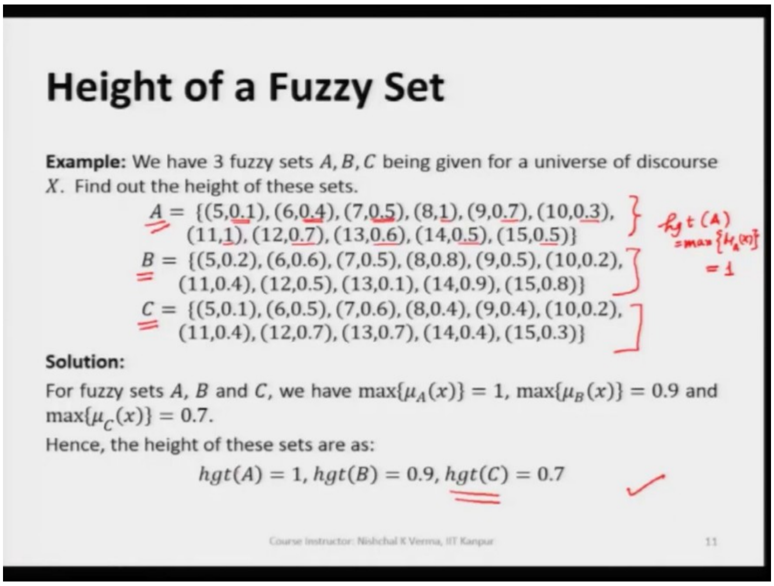

Discrete fuzzy set can be understood by this example.

So, if we take a fuzzy set A here. So, this is fuzzy set A. So, if we look at all the values of the membership here; membership values. So, we see that these are the values of these are corresponding membership values and if we collect these values and take max of this, we are going to get *max* of the *max* of $\mu(x)$, $\mu_A(x)$ and this will be equal to 1 for the fuzzy set A. So, this way the height of the fuzzy set here will be this; we can write as $hgt(A)$. So, $hgt(A)=1$. Similarly, what will be the height of fuzzy set B? So, by looking at the fuzzy set here we can clearly see the highest value of membership that it has it contains is 0.9 in this membership, in this fuzzy set.

So, the height of fuzzy set B will be 0.9. Similarly, for B for C fuzzy set, the height of C height of C fuzzy set will be 0.7 in this example; because the highest value of the membership, the maximum value of membership values or I can say if we take max of all the membership values that it contains will be 0.7.

So, this way we clearly find the height of the fuzzy set.

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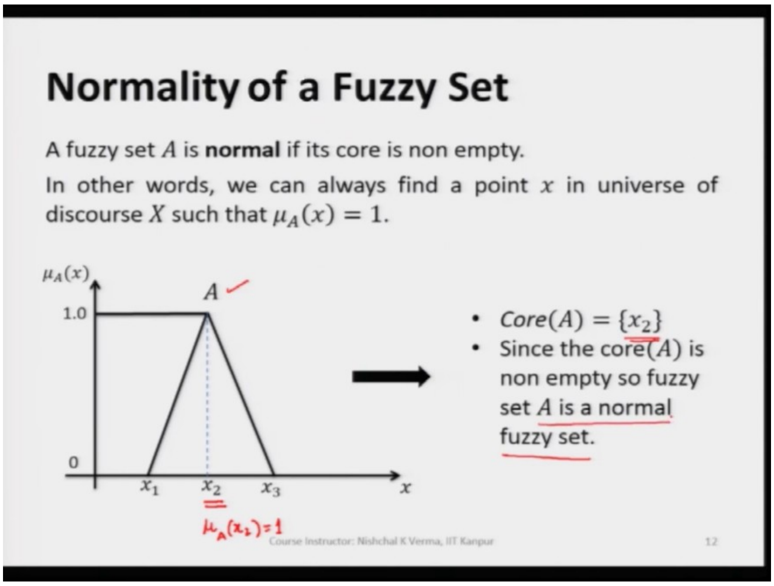

Now, let us look at the let us now discuss the normality of a fuzzy set is nothing but it is defined as a fuzzy set A is normal if it is core is non empty. So, this can be defined in other words as we can find we can always find a point x in universe of discourse capital X such that $\mu_A(x)=1$.

So, what does this mean here is that if we have a fuzzy set A let us say and if this fuzzy set A has at least one point, one generic variable value for which or it's corresponding membership value is 1. So, at least we should have a membership value is equal to 1. So, in other words we can understand this as that if the height of the of a fuzzy set can go up to 1, the fuzzy set will be a normal fuzzy set.

So, we can see here in this diagram, we have a fuzzy set a here this is the fuzzy set. And if you see here x_2 is the point, x_2 is the generic variable value for which we have the membership the member ship value 1. So, if we have such case where we are getting a point at least a point like this we can say the fuzzy set is a normal fuzzy set.

So, there may be case when we may not be able to find any such point for which the corresponding membership value is 1. So, this kind of membership, this kind of fuzzy set is referred to as a sub normal fuzzy set. So, here corresponding to this fuzzy set A, the core of A is $x_2$and since the core of A is non empty of course, because we are getting at least a point x_2 for this fuzzy set.

So, we can say the core of A is non empty. So, this fuzzy set is a normal fuzzy set. There may be a case where core of a fuzzy set empty. So, such a fuzzy set is referred to as sub normal fuzzy set.

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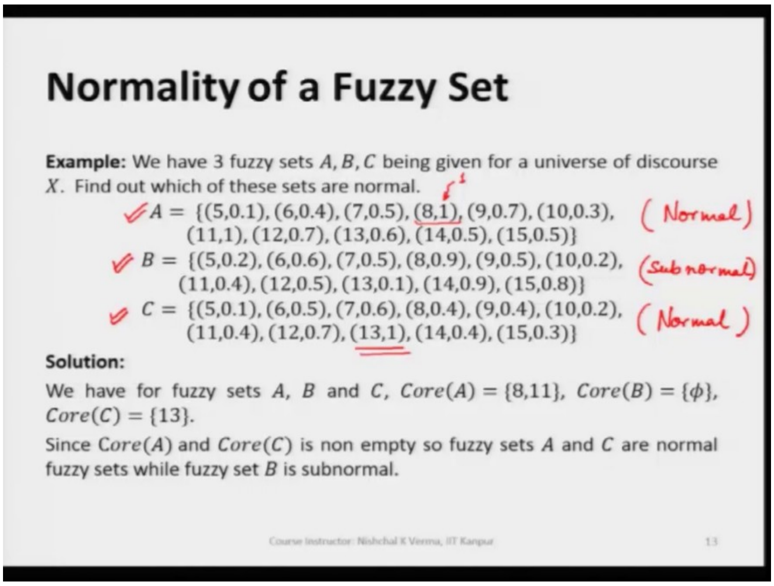

So, let us now take these examples here. A, B of discrete fuzzy sets A, B, C and we see here that if we take a fuzzy set A and we clearly see that we have in this fuzzy set; we get a point, we get a fuzzy point where we get the μ corresponding μ the corresponding membership value 1.

So, it means that we have at least one point present one fuzzy point present one point present for which we have the corresponding membership value 1. And please understand when we say at least it means if we have multiple such points present then also this fuzzy set will be a fuzzy set; a normal fuzzy set. So, at least means at least one such point is present for which the corresponding membership value is 1. So, A is a normal fuzzy set, A is a normal fuzzy set this is a normal fuzzy set. Now what about B fuzzy set? So, let us now check all the points here for fuzzy set B is a B fuzzy set.

So, in B fuzzy set we if we see here in the fuzzy set, all the points. So, we do not find any such a point such a membership such a generic variable value for which we get the membership value 1. So, we can say this fuzzy set is not normal or we can say the B fuzzy set is subnormal. Similarly, now let us check a fuzzy set C.

So, just by going through the all these points here which are in the in this set, fuzzy set. So, we see that we get here this point, so the generic variable value 13 has it is corresponding membership value 1. And, so at least we have one such a point present, such generic variable value present for which we have mu of the membership value is 1.

So, that is why we can say C is normal fuzzy set. So, this way we can clearly check all the normality of fuzzy set or normality in a fuzzy we can check.

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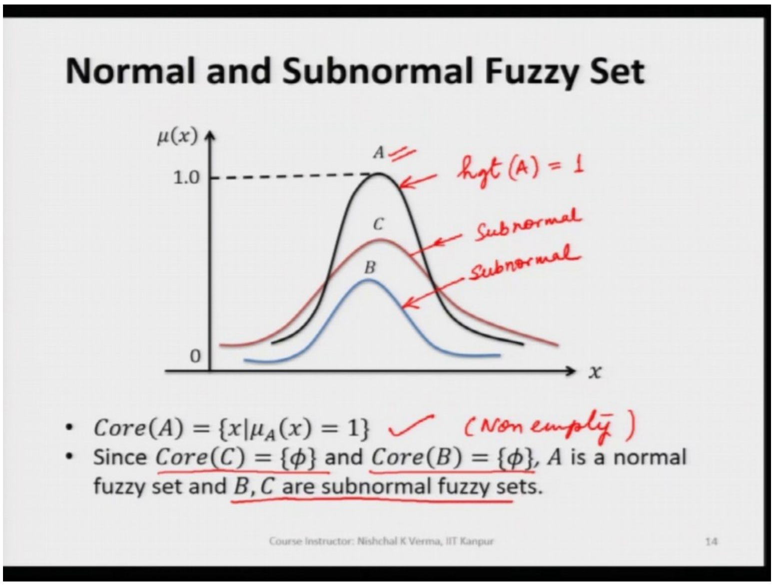

Now, we have a diagram here and this figure has the fuzzy sets A, B, C. So, let us now clearly check whether A is a normal fuzzy set or a sub normal fuzzy set. So, A, B or C all these are normal fuzzy sets are subnormal fuzzy sets. So, if we see A here, A reaches up to A is the height of fuzzy set A has or is 1. So, we can say here the height of a fuzzy set A is 1. So, it means fuzzy set A reaches up to 1. So, it means it is membership value the highest membership value, the maximum membership maximum of all the membership values is 1.

So, that is why we can say the core of this fuzzy set is non empty and if the core of this membership core of this fuzzy set is non empty; it means that this fuzzy set is a normal fuzzy set. Whereas, if we see here will not be getting any point any generic variable value for which we get $\mu(x)=1$. So, this will be a sub normal fuzzy set. Similarly, here C also will be the sub normal fuzzy set.

So, only A is a normal fuzzy set, right. So, because the core of A which is written here. So, core of A; this is nonempty non empty. And if we talk of C, so since core of C is empty here and core of B is also an empty set. So, that is why this B and C both the sets are subnormal fuzzy sets.

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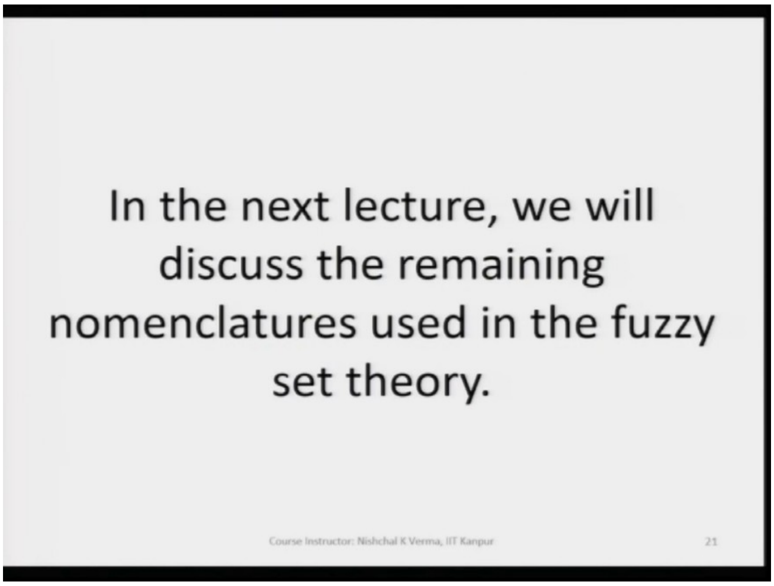

At this point I would like to stop for this lecture and in the next lecture we will be discussing the remaining nomenclature used in the fuzzy set theory. So, it means we will be you know moving ahead to discuss remaining nomenclature.

Thank you.