Fuzzy Sets, Logic and Systems and Applications
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Lecture - 60
TSK Fuzzy Model

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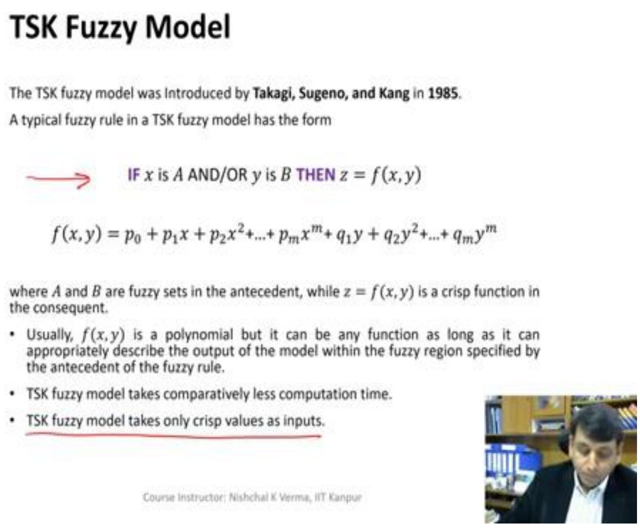

Hi, welcome to the lecture number 60 of Fuzzy Sets, Logic and Systems and Applications. Today in this lecture I will discuss Takagi Sugeno and Kang fuzzy model, this model is in short known as TSK fuzzy model or TS fuzzy model. So, let us first understand the TS fuzzy model or TSK fuzzy model as it was introduced by Takagi Sugeno and Kang in 1985. And a typical fuzzy rule in a TSK fuzzy model has the form of IF x is A and then we have some connective either AND or OR and then next antecedent y is B.

So, here we have the premise and then we have the consequent. So, the fuzzy rule for TSK fuzzy model is of the form of it has the premise part and the consequent part. So, premise part remains as it is or as it was in Mamdani, Larsen, Tsukamoto. So, here also the same means the premise part is fuzzy. It may have either single antecedent or multiple antecedent. Whereas when it comes to the consequent part here there is a significant difference. What is this difference? Difference is this.

So, in TSK fuzzy model we have the output in terms of a polynomial, the polynomial is here and this polynomial is of the form of $f\ (x, y)$ means function of x, y if x, y are the inputs. So, let us say we have two inputs here first input is x, the second input is y. So, $f(x, y) = p_0 + p_1 x + p_2 x^2 + \cdots + p_m x^m + q_1 y + q_2 y^2 + \cdots + q_m y^m$.

So, here in this polynomial we have basically we have a constant which is p_0 and then we have the x terms. So, we have first degree terms of x and y like $p_1 x$, $q_1 y$. Then we have second degree terms $p_2 x^2$, $q_2 y^2$ and so on. So, here in this polynomial we have the mx degree or m degree terms. So, in general we are writing $f(x, y)$ as the m degree polynomial. So, the rule that is used here is a two input rule or I would say two antecedent rule, whereas, in general it can be any number of it can have any number of antecedents.

So, what we have to remember here is that in TSK fuzzy model the consequent part has the output which is always in the form of a polynomial and this polynomial is in the terms of the inputs that are fed in the premise part. So, here says x and y are the inputs which you can see in the premise part here. So, the polynomial is always a function of x and y which is nothing but the function of the generic variables x and y.

Now, here this is very important to note that a TSK fuzzy model takes comparatively lesser computation time because here the z can be obtained very quickly just by substituting the values of x and y. So, this takes very less time of computation and then TSK fuzzy model takes only crisp values as the input. So, please note here this is very important point like in Mamdani model, Tsukamoto model, Larsen model all of the models we have had the inputs in the form of fuzzy or crisp, but here in TSK fuzzy model we always have the inputs that means, the x and y only crisp values.

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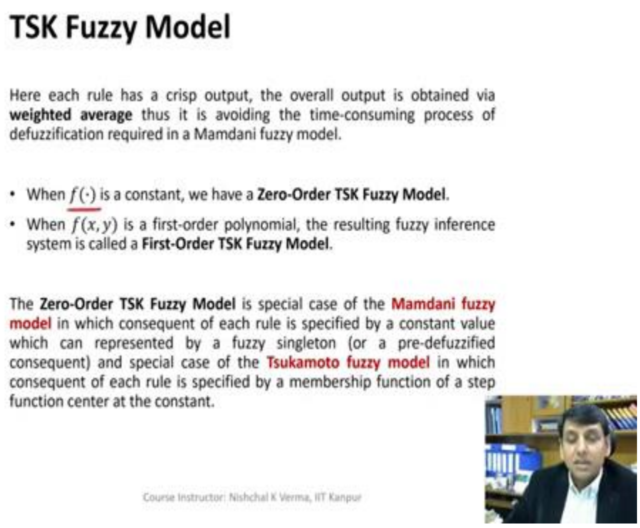

So, then we have the each rule has a crisp output in terms of the $f(x, y)$ as I have just explained and then since every rule is generating some output. So, the overall output of the TSK fuzzy model which is characterized by multiple rules will be the weighted average of these outputs.

So, here we can have f the function the polynomial as a constant and if we have such a case then we see we say this as the zero order TSK fuzzy model we will discuss this in detail in coming slides. We have explained this and then when y when $f(x, y)$ is a first order polynomial the resulting fuzzy inference system is called for first order TSK fuzzy model.

The zero order TSK fuzzy model is the special case of the mamdani fuzzy model in which consequent part of each rule is a specified by a constant value which can be represented by a fuzzy singleton as we already have seen in the beginning of this course we understood what is a singleton fuzzy singleton. So, constant can also be represented in terms of fuzzy singleton and then the special case of Tsukamoto fuzzy model in which the consequent of each rule is a specified by a membership function of a step function centered at the constant.

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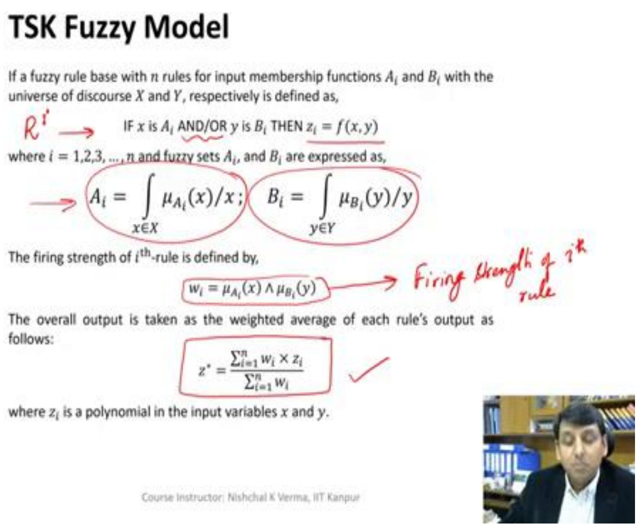

So, that way the zero order TSK fuzzy model can be understood and here in zero order TSK fuzzy model if we go ahead to understand we see that the $f(x, y)$ is a constant value that is p_0 here. Then so since we have let us say the i^{th} rule in TSK model, let us say we have a set of rules and i^{th} rule is here is i^{th} rule.

So, i^{th} rule will have the rule of this kind if x is A_i, x is the input A_i is the fuzzy region where this x is falling into and then we have the connective here AND or OR, and then we have another input y and which is again falling into B_i, i here signifies that the fuzzy region which is relevant for i^{th} rule and then we have the premise part which is z_i for i^{th} rule the output for i^{th} rule and this is nothing but a polynomial which is $f(x, y)$.

So, here we may have a TSK fuzzy model which can have n number of fuzzy rule small n number of fuzzy rules. So, if this is the case then for A_i to be a fuzzy set here B_i again to be a fuzzy set continuous fuzzy sets and if this is the case then for certain input x and y, we can have the w_i which is nothing, but the firing strength of the particular i^{th} rule. So, I can write here the firing strengths of i^{th} rule and this we can obtain by either taking min of $\mu_{A_i}(x)$ and $\mu_{B_i}(y)$ or we can multiply these two if we are interested in the product.

So, here since these firing strengths are of each rules are known then we can go ahead for the final output by taking the weighted average of the each rule. So, this way we find the final output of the of the TSK fuzzy model.

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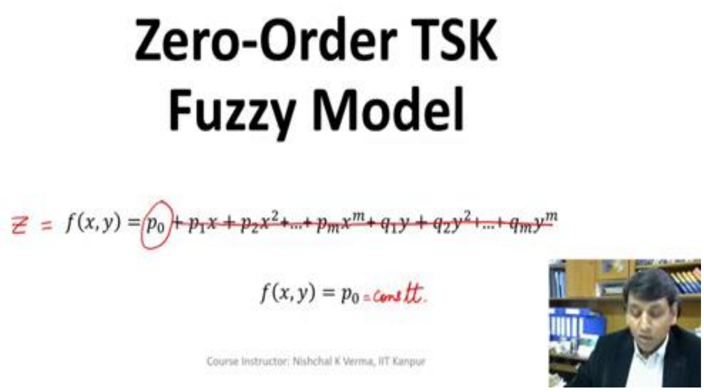

So, let us now start with first zero order TSK fuzzy model. So, as I have already mentioned that the consequent part here in TSK fuzzy model the fuzzy rules will have the consequent part of this kind. If the output basically the z will be defined in terms of $f(x,y)$ as I have already mentioned this x and y will be the corresponding generic variable generic input variables and when we talk of zero order zero order TSK fuzzy model then it means we will have only the constant that is p_0. So, here we have the p_0 only.

So, $f\ x,y$ or I can say $f(x,y)$ will be equal to only the constant part that means, the p_0. So, this is the constant part. So, no x term, no y term. So, this means that the zero order or zero order TSK fuzzy model output will not depend on the generic input variables here in this case x and y.

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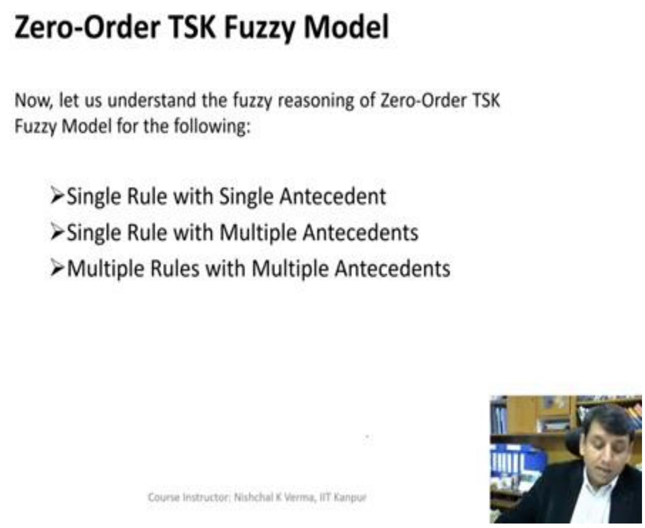

So, let us now go for the fuzzy reasoning of zero order TSK fuzzy model and we have three cases as we have had earlier for other fuzzy models.

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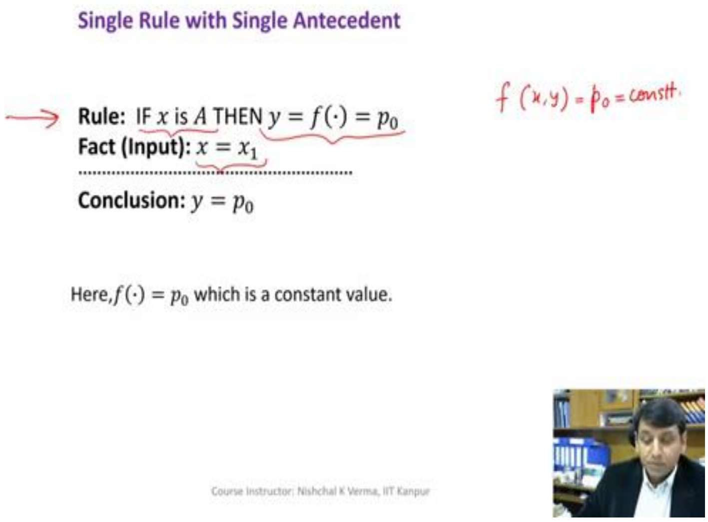

So, we have the first case where the single rule with single antecedent type of TSK fuzzy model we have and let us now first go with this and understand as to how we are going to get the output computed if we have a TSK fuzzy model with single rule with single antecedent.

So, as I have already mentioned that we can have only the crisp input. So, here x is equal to x_1 we are supplying. So, the fact or the input we can have as x is equal to x_1 and here is the rule we have only one rule. So, only one rule is mentioned here and rule has the single premise, single antecedent. So, single antecedent is here x is a and then we have the consequent part and consequent part is $f(x, y)$ which is the p_0 here and which is a constant value means it is not depending upon the input generic variables that means, here in this case we have x.

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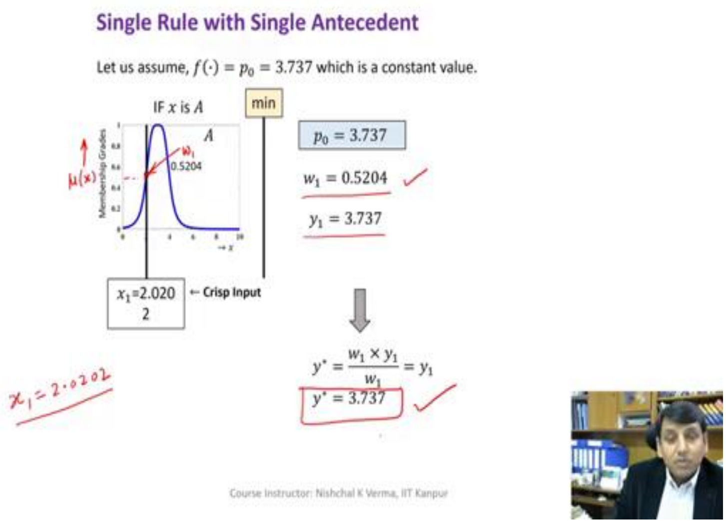

So, let us move ahead and see how can we get the output computed with respect to the certain input that we feed to the output. So, let us assume that we have we have x_1 which is the input variable input generic variable here 2.0202. Let us say we have this value and we are feeding this value to this model to the TSK fuzzy model characterized by the single rule with single antecedent rule fuzzy rule which is mentioned here.

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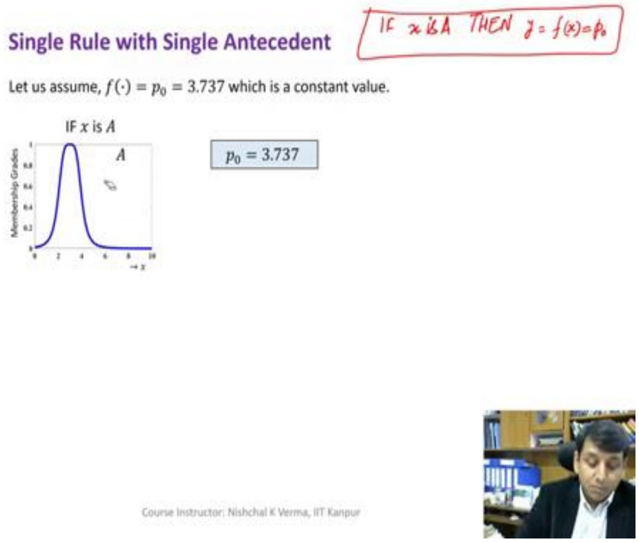

If x is A so x is here. So, I can write here the rule that is characterizing is characterizing this model is if x is a then the output here in this case let us say y. So, y is $f(x, y)$ or maybe since we are using y. So, I can write here z. So, z is equal to the output is p_0 or if you use y it is better. So, let us say y which is the function of fx because we have only x in the premise part as a single antecedent.

So, here we have a constant and constant is p_0. So, this is what is the rule which is characterizing, which is representing the fuzzy the TSK fuzzy model. So, if this is the case now and please note that we have A and p_0 both are known for this model when we say we have a depth we have a model it means all these parameters of A and p_0 are known. So, let us now apply the input here.

So, when we apply the input we see that corresponding to x_1 we have the membership value here which is coming out to be 0.5204. So, here. So, this is the intersection point and this I can write as w_1. So, since we have only one antecedent min is not applicable here. So, the same value will be transferred to the output here for the weighted average. So, w_1 is known now how we will have the y_1 corresponding to this rule.

So, let us say y_1 if we are supplying x_1 corresponding output is y_1. So, y_1 since the, so, y_1 will be 3.737 which is same as p 0 because this output is not depending upon the input. So, we have the y_1 and now the final output will be the same even when we do the weighted average. So, you see here we have done the weighted average since we have single rule.

So, the weighted average is going to remain the same. So, this way the output corresponding to x_1 is equal to 2.0020 will be 3.737 all right. So, now the next case here for the zero order TSK model would be where we can have a single rule, but with the multiple antecedent.

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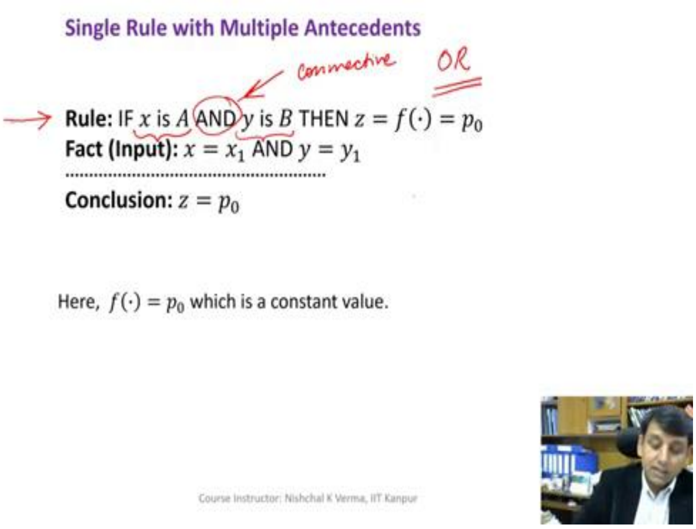

So, let us now go ahead with this case here. So, here also we have you see the zero order and zero order means the output is a constant the every the consequent part is a constant consequent part means the output of every rule is a constant and which is not depending upon the input generic variables.

So, here also we have p_0 so, but what is the difference here is that we have two antecedents one is the x is A and the other one is here, the other one is y is B and these two antecedents can be joined by a connective here we have AND. So, this is the connective. Please note that this connective can be OR as well. So, we can have any other connective. So, here in this case we have AND connective. So, now, since we have two antecedents. So obviously, here we have two input generic variables x and y.

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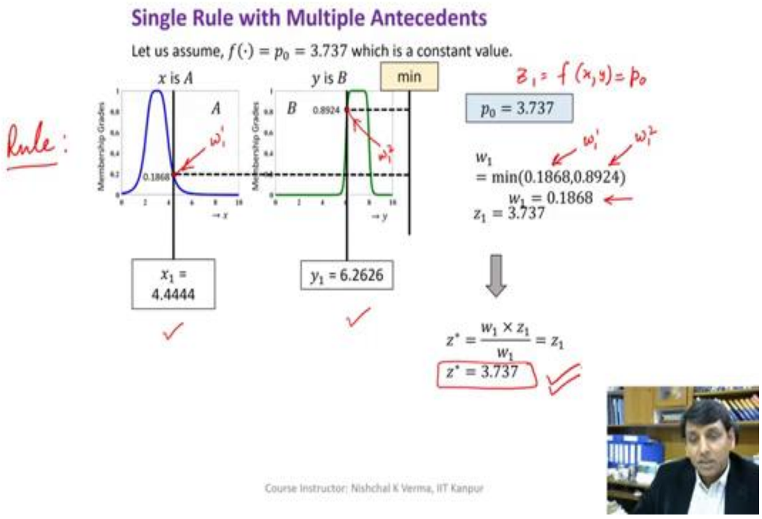

Now when we have the input and we are now applying this input and we are interested in finding the corresponding output, now let us see how to proceed with this. So, let us assume that we have x_1 the input value which is a crisp value x_1 is equal to point a 4.444 and y is equal to y_1 is equal to 6.2626. So, let us assume these two values to crisp values as the input values.

So, this is arbitrary value you can have any other value and please remember that we take only those values which are going to intersect the fuzzy sets A and B. Why because otherwise these inputs these fuzzy reasons may not be relevant, and this rule not be applicable. So, this is the rule in this case here where we have if x is A and y is B. So, in this case we have the output which is p_0. So, here also if we apply this these two inputs.

So, let us see now how to get the w_1^1 and then w_1^2 as the intersection points corresponding to x_1 and y_1 respectively. And here we are taking min why we are taking min because we have the connective AND. So, connective of the connective in between the two antecedents we have AND. So, that is why we are taking min here if we would have or connective then we would have taken max. So, for and connective we could have also taken gone for product. So, it is up to us what we want or what as what is needed to be what is the preference.

So, if we take the minimum of here the w_1^1 and then w_1^2. So, when we take minimum we are going to get the w_1 is equal to here w_1 is equal to 0.1868 which is the lowest value and since we have w_1 we have z_1 here and z_1 is nothing but the here the z_1 is $f(x, y)$ and which is nothing, but p_0. So, we can very easily say that z_1 is p_0 and z_1 since z_1 is p_0 we have now z_1 is equal to 3.737. So, we now know the w_1 and z_1.

So, now since we have only one fuzzy rule here only one rule that is in this case we have. So, the weighted average is going to give us the same output here because we have only one rule. Had had it being many rules or more than one rules then the output would have been accordingly computed. So, let us now go for the third case where we have multiple rules with multiple antecedents.

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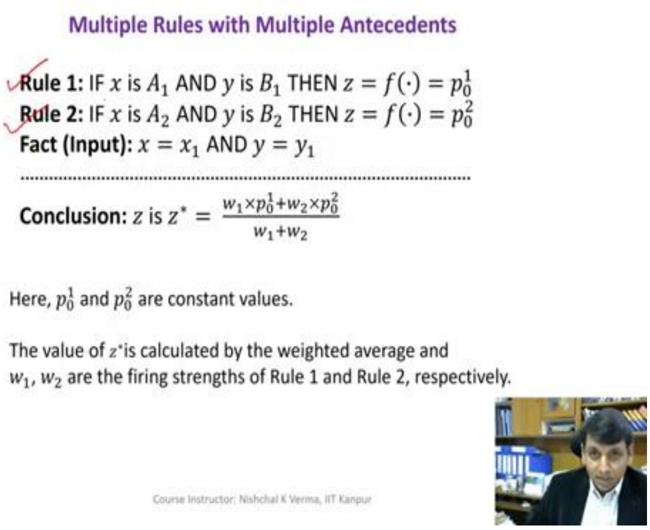

So, here you see for simplicity we have two rules first rule second rule and then we have multiple antecedents here we have only two antecedents we have taken two antecedents for simplicity. So, otherwise we could have taken any number of antecedents rest other things remain the same.

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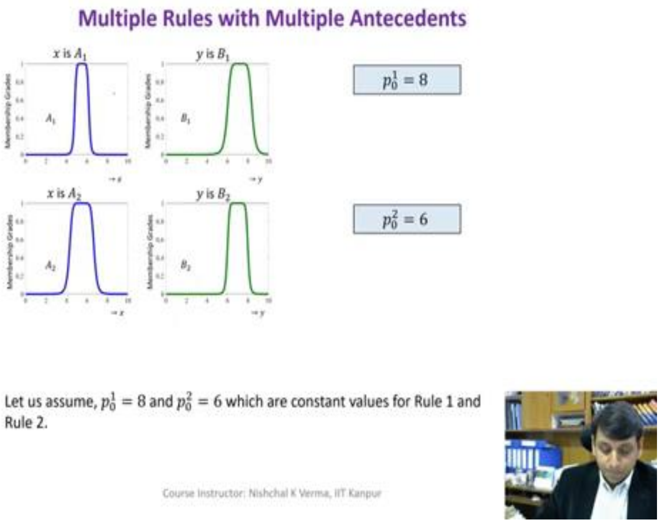

So, we have two antecedents and the connective is and. So, let us now proceed with the this model having two rules and two antecedents.

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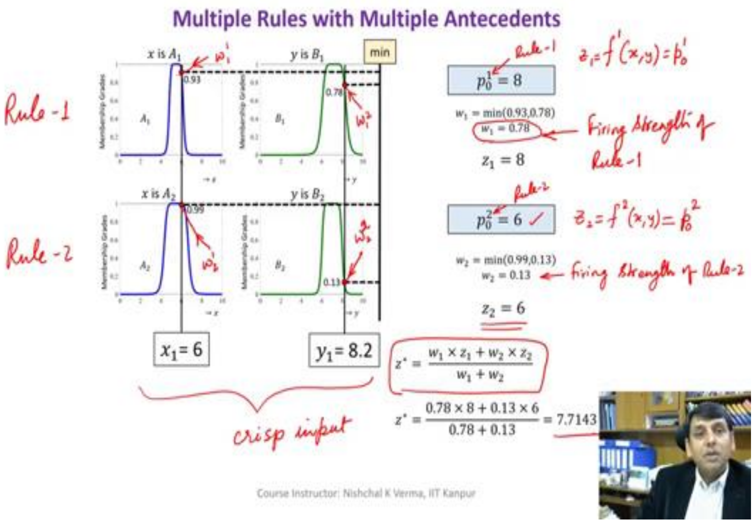

So, let us apply. So, we have the rule number 1 here and we have the rule number 2 here. And we are now giving the input to the model. So, and please remember that this model this input is the crisp input you can see. So, any arbitrary input which falls within the fuzzy region can be fuzzy regions of A_1 and B_1 can be taken A_1, B_1, A_2, B_2 can be taken or else it may be given as the unknown input suitably.

So, for x_1 is equal to 6 y_1 is equal 8.2. Let us now find first using first rule the w_1^1 which is the intersection point. So, intersection point let us call this as w_1^1 and this is coming out to be corresponding to x_1 is equal to 6 it is cutting A_1 at 0.93 membership value. So, this becomes our w_1^1.

Similarly, let us now find the corresponding to y_1 8.2 value the intersection point on B_1. So, let us call this as w_1^2, so this is our w_1^2. Now, we have w_1^1 w_1^2 both now let us take min because the connective is AND. So, when we take min our value is going to be w_1 which is also called as the rule strength or I or I can write here or I can say here the firing is strengths of rule 1.

So, firing a strength of rule 1 is w_1 is also called weight of the rule. So, this is 0.78. Now p_0^1 we already have as 8 this is a constant value. So, we we write it by p_0 and p_0 is the constant value and 1 we are writing here for rule number 1 for rule 1. So, that is why we have also used one similarly here for rule number 2, p_0^2all right.

So, now we have obtained w_1 and z_1. So, z_1 we have very quickly found this is x, y which is p_0^1, here in this case let us say this is here in this case z_1. So, z_1 is p_0^1 similarly now on the same lines when we use rule number 2 we find the intersection point that is let us say w_2^1 for the first second rule first antecedent. And similarly here for y_1 is equal to 8.2 we write here w_2^2. So, w_2^2 means second rule second antecedents now both of these values are 0.99 and 0.13.

And when we take min of the two we are getting the firing a strength firing a strength of rule number 2. So, we have firing a strength of rule number 2 which is coming out to be w_2 is equal to 0.13 and second rule gives us p_0^2. So, p_0^2 is giving us 6 already. So, z_2 will become 6 here. So, I can write here that our z_1 z_2 basically is $f^2(x, y)$ is equal to p_0^2. So, this way we get z_2 is equal to 6. So, now, we have w_1 z_1, w_2 z_2.

Now, we can very easily find the weighted average. So, let us find the weighted average of the outputs of each rule and when we use this formula for weighted average the z is star which is the final outcome the weighted average of the output. We are getting 7.7143. So, this way we can find the complete the final output corresponding to x_1 is equal to 6, y_1 is equal to 8.2 using the TSK fuzzy model, which is characterized by 2 rules with 2 antecedents and this comes under the category of multiple rules with multiple antecedents.

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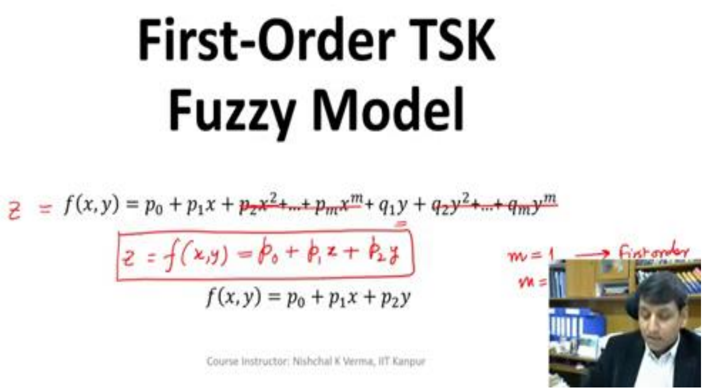

Now, let us move to the first order TSK fuzzy model. So, as I have already mentioned the ordered basically is defined here with this with the if when we talk of first order TSK model. It means the input generic variable at least the maximum the maximum power of these input cannot exceed 1. So, this means what? So, the output of each rule will be of this form only. So, p_0 the constant can be there p_0 and then $p_1\ x$ this $p_2\ x$ cannot be there, similarly no higher terms cannot be there no higher terms can be there or the higher terms cannot be higher terms of the input generic variables are not there.

Similarly, for y also only since this is first order TSK fuzzy models only y will be there y raise to power 1. So, more than 1 cannot be there. So, what is coming out to be here is that $f(x,y)$ which is the form of the output of each rule. So, $f(x,y)$ let us say for 2 input generic variables. So, we can have $p_0 + p_1x$ and then we write here q_1y, but since there is no other variable. So, I can write here p_2y. So, that is how this is of this form. So, first order TSK fuzzy model will have the output the consequent part, that means the output of this form for 2 inputs if we have only 1 input then; obviously, this y term will go away.

So, along with the constant we will have one more term and this term will be the one more term for the single antecedent, but similarly if we have multiple antecedents we will have multiple antecedent, but the power of the degree of I mean the power of each generic variables the input generic variables cannot exceed more than 1.

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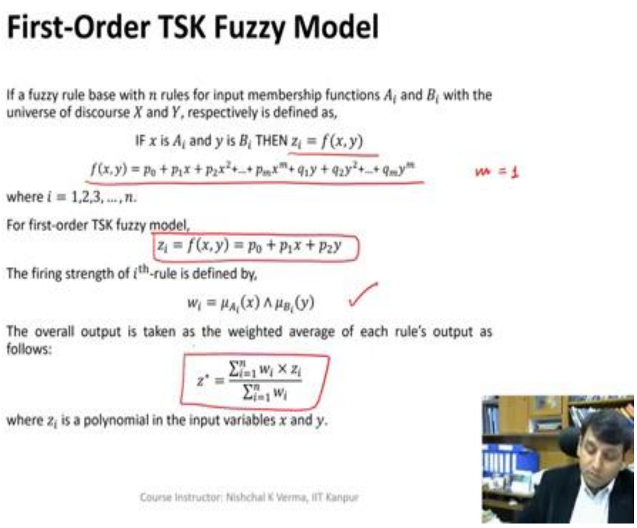

So, let us quickly go ahead and understand. So, here this is mentioned this is the general form of the polynomial of the output here first order TSK fuzzy model, but here m cannot be, m can be only 1.

So, m value of m can be only 1. So, I can write here the when we say first order TSK fuzzy model it means m is equal to 1. The value of m is equal to 1. Similarly, when we say 0 this is for first order, similarly m is equal to 0 for 0 order. So, m is equal to 0 0 means 0 order what does this mean this means that only constant will be there. So, now, as I mentioned then the output will be like this similarly all others terms.

And conditions will remain the same means we have the min if it is needed for the connective AND or we will take max in case of the OR connective and similarly we will have the weighted average we have already done.

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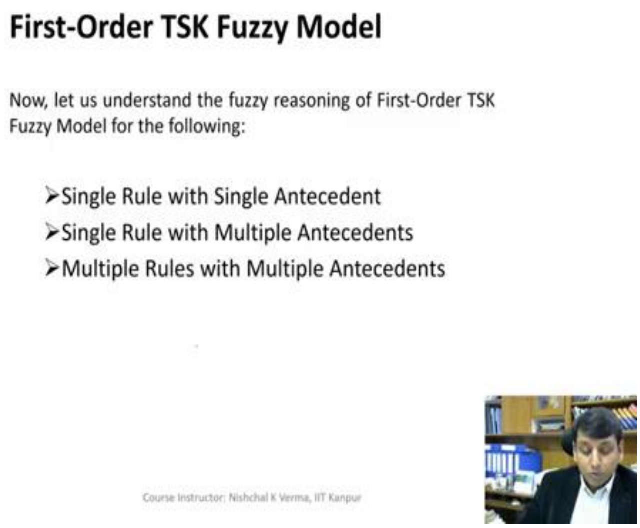

So, this way we proceed and now let us under this first order TSK fuzzy model let us discuss all those three cases we just have discussed. So, the first case again is the single rule with single antecedent.

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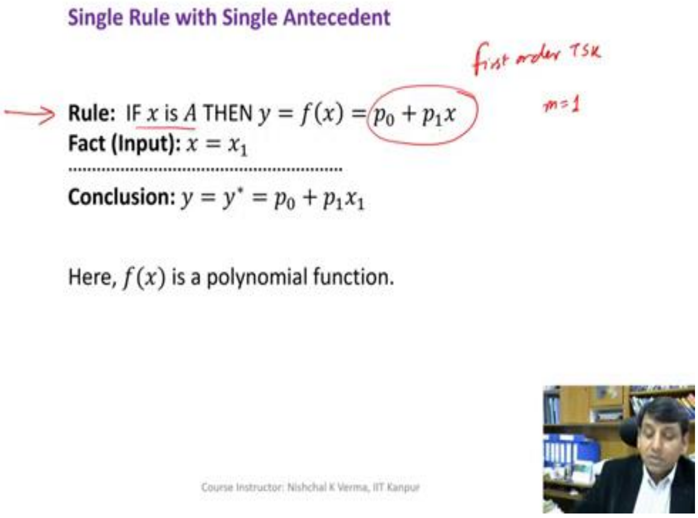

So, when we have a single rule with single antecedent we have the single antecedent here and please note that here for the first order first order TSK, first order TSK we have m is equal to 1 only this means that we have $f(x) = p_0 + p_1 x$.

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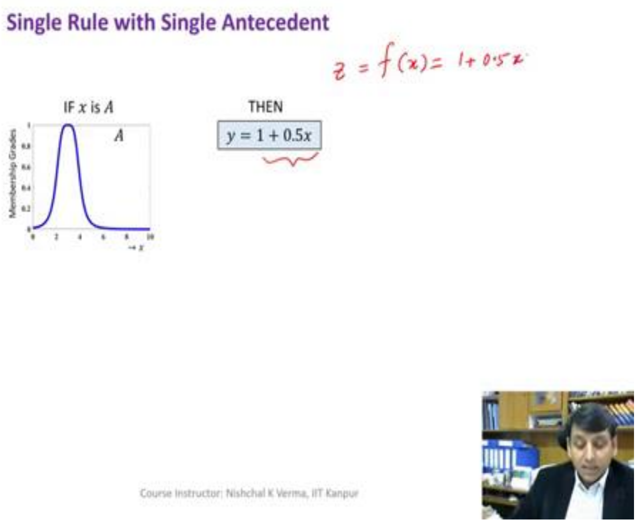

And now, if we have the rule of this kind let us say we have a TSK fuzzy model which has the polynomial the $f(x)$ like this is let us say $z = 1 + 0.5x$.

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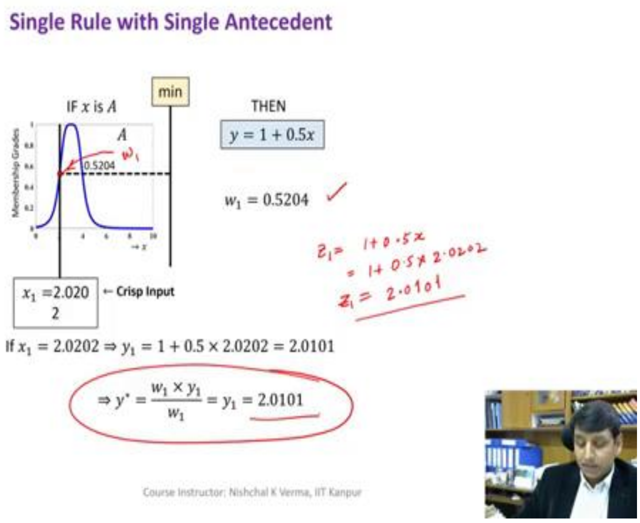

So, if this is the case then let us apply some input and let us apply the crisp input $x_1 = 2.020$. So, this is the same input which we have applied in the previous case previous category that was the zeroth order.

And that was again for single rule with single antecedent. So, we get the w_1 here and since there is no connective only 1 antecedent is there single antecedent is there. So, this becomes the w_1. So, the this becomes the w_1. So, this is the weight of the rule or firing strength of the rule so w is known. And then our $z_1 = 1 + 0.5x$ we already know because we have applied this.

So, $1 + 0.5 \times 2.0202$. So, this way we get the total here is 2.0101. So, we have now z_1 and since we have only 1 rule we have single rule. So, the weighted average is not needed or even if you go for this that is going to remain the same. So, this way we get the output of the TSK fuzzy model which is characterized by a single rule with single antecedent and when we apply the input x_1 ,2.0202 we are getting the output here we are getting the output 2.0101.

Alright so now, let us go to the second case of first order TSK fuzzy model where we have a single rule, but with multiple antecedent.

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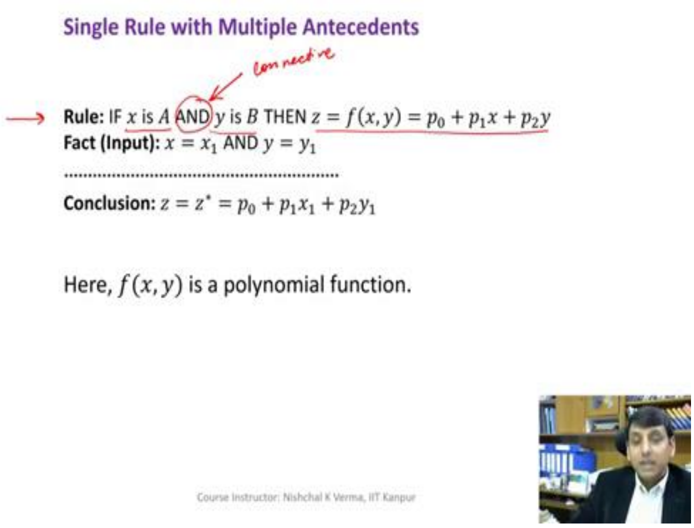

So, here also for simplicity we have taken only two antecedents x is A and y is B, but these two antecedents are connected by the connective AND and again I would like to mention that this connective can be any connective can be OR as well or, but or whatever. So, here since we have AND, AND will have the composition of min. So, we will use min if you would have.

Or here then max would apply as the composition. So, AND is the connective here. So, rule says if x is A the first antecedent and then we have the connective and then we have the second antecedent y is B. So, then here the output is of the same type which we have just discussed and this category is the first order TSK fuzzy model.

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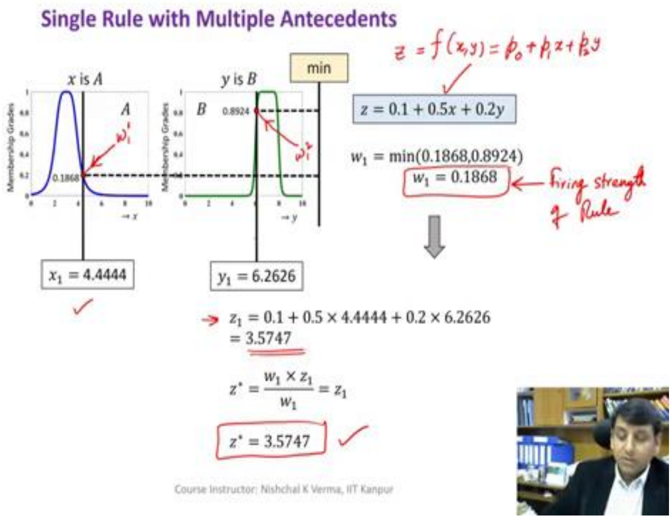

Now when we and before this I would like to mention that the here for simplicity we have taken the 2 antecedents, but we can have any number of antecedents now let us go ahead and apply some crisp inputs x_1 and y_1 and since we have here the single rule. So, let us apply this to a TSK model which is characterized by a single rule with 2 antecedents x is A, y is B.

So, we see that here we are applying the same inputs as we have done in the zero order tsk. So, x_1 is equal to 4.4444 and y_1 is equal to 6.2626. So, we see the corresponding points of intersection and this is designated as the w_1^1 and then this is also designated as w_1^2. Now, within these we take the min because we use the connective AND if we would have used or connective then here you would have taken max. So, this gives us the again the firing a strengths of rule is also called weight.

So, we have the z_1 because we have the first order polynomial equation here which is the output and please note that since we have here two antecedents. So, that is why the z is in terms of x and y. So, we see here the x, y which is nothing, but the output let us say z and this is of this form $p_0 + p_1x + p_2y$. So, this is how we are getting the z calculated for this.

So, and this way this is nothing, but the z_1. So, here what is z_1, z_1 is this. So, z_1 corresponding to x_1 y_1 and p_1 is known p_0 is known p_0 is known p_1 known p_2 known.

So, and w_1 is also known. So, we can quickly get z_1 corresponding to x_1 and y_1. And since we have again here the single rule. So, the output is going to remain the same. So, the weighted average even if we take here is going to be 3.5747. So, this is equal to z_1 now the third case is multiple rules with multiple antecedent.

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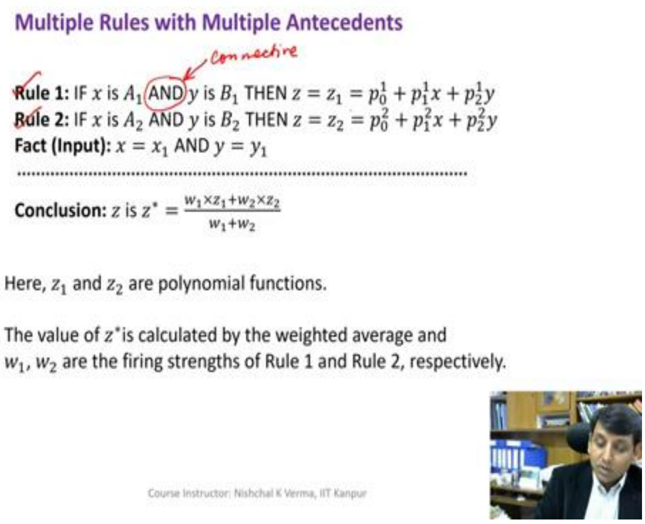

So, let us now take two rules under this case for simplicity rule number 1, rule number 2 and both the rules have 2 antecedents again for the simplicity and is the connective as I have mentioned.

So, here the output also remains the same as we have seen in this the previous cases of first order TSK fuzzy model the only difference that we have here is because we have 2 rules. So, we have differentiated we have a written z_1 and z_2 for the corresponding rules as the outputs.

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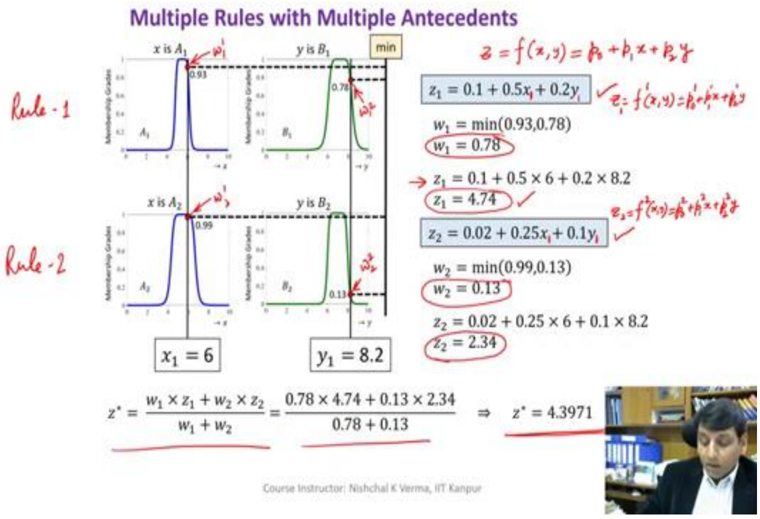

Now, let us use these two rules rule number 1 and rule number 2 to generate the outputs corresponding to the inputs, crisp inputs supplied. So, let us apply the input x_1 and y_1, x_1 is equal to 6 y_1 is equal to 8.2 and let us see the corresponding outputs. So, corresponding to rule number 1 the w_1 is the points of intersection corresponding to x_1 the first antecedent first input.

So, we find here the w_1^1 and then we have here in rule number 1 the points of intersection here for the second antecedent for y_1 is equal to 8.2 is w_1^2. Similarly when we use rule 2 we get this as this intersection point corresponding to x_1 is equal to 6 as w_2^1 here for corresponding to y_1 in rule number 2 for the second antecedent we get the point of intersection as the membership value and which represents the w_2^2.

Now, let us use rule number one output here. So, let me first write the z is of the form of the output is of the form of x, y and then we have $p_0 + p_1x + p_2y$. And since the z_1 is given for the first rule, z_2 is given for the second rule. So now, if we use the x_1 and y_1 this is x_1, y_1. So, if we use x_1, y_1 values which is the input set of inputs which is the input x_1, y_1 so then we get z_1. So, we see that we get the z_1 here which is 4.74. So, when we use x_1 is equal to 6 and y_1 is equal to 8.2 using the first rule output we get z_1 is equal to 4.74 here.

So, similarly now we use rule number 2 and find the z_2. So, here on the same lines when we use the input here. So, x_1 and y_1 we use as the input. So, z_2 for the same x_1, y_1 we are getting as the z_2 is equal to here 2.34 can see here and how do we get this we already have

$z_2 = 0.02 + 0.25x_1 + 0.1y_1$. So, please understand here and that this z 1 is like this, so z_1 is here $z_1 = f^1(x, y)$.

And this is of this form $p_0^1 + p_1^1 x + p_2^1 y$ similarly our z_1 z_2 is of this form our z_2 is $f^2(x, y) = p_0^2 + p_1^2 x + p_2^2 y$. So, here we have z_1, z_2 and we also have w_1 and w_2 by taking the min of the corresponding weights corresponding membership values. Now since we have 2 rules here and corresponding to each rule we have its outputs.

Now, we take the weighted average of it to get the final output. So, final output we use here the weighted we take the weighted average and this way we get z is equal to 4.3971. So, this way if we have a TSK fuzzy model which has which is characterized by the set of rules set of multiple rules with multiple antecedents then we will proceed with this kind of.

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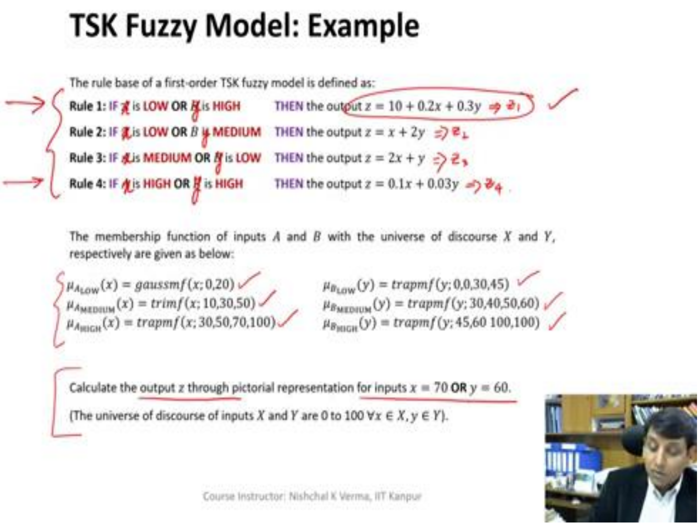

And then here let us take an example to understand the TSK fuzzy model better and this example is basically a first order TSK fuzzy model and which is having four rules you see here.

So, we have rule number 1 we have rule number 2, 3, 4. So, all the rules have the outputs that are different. So, I can write here this is the z_1, this is z_2, this is z_3 this is giving us z_1 , z_2, z_3 and so on this giving us z_4. So, when we have a TSK fuzzy model and this is characterized by 4 rules and we see that each and every rule is coming under the category of the first order TSK fuzzy model. So, and then we see that the fuzzy regions that are mentioned here in the rule A, B they are basically having three fuzzy regions they are having three fuzzy regions.

And all and each fuzzy regions are represented by the fuzzy sets which are represented which are characterized by the membership functions $\mu_{A_{low}}, \mu_{A_{MEDIUM}}, \mu_{A_{HIGH}}$ and $\mu_{A_{LOW}}$ is Gaussian, $\mu_{A_{MEDIUM}}$ is you know triangular, $\mu_{A_{HIGH}}$ is trapezoidal.

Similarly, the input in the antecedent part is divided into three fuzzy regions and every fuzzy region is defined by or represented by fuzzy set LOW, MEDIUM, HIGH and all these fuzzy sets are characterized by the membership functions $\mu_{B_{LOW}}$, $\mu_{B_{MEDIUM}}, \mu_{B_{HIGH}}$ and you can see the μ_B is trapezoidal, $\mu_{B_{MEDIUM}}, \mu_{B_{LOW}}$ is trapezoidal, $\mu_{B_{MEDIUM}}$ is also trapezoidal, $\mu_{B_{HIGH}}$ is also trapezoidal.

So, what is the what is that which we need to do is here we have to find the we have to compute calculate the output for the inputs x is equal to 70 and please understand here the connective that is mentioned here is OR. So, here also you see the connective instead of AND we have OR. So, if we have this kind of situation we know what to do. So, we have the inputs x is equal to 70 or y is 60.

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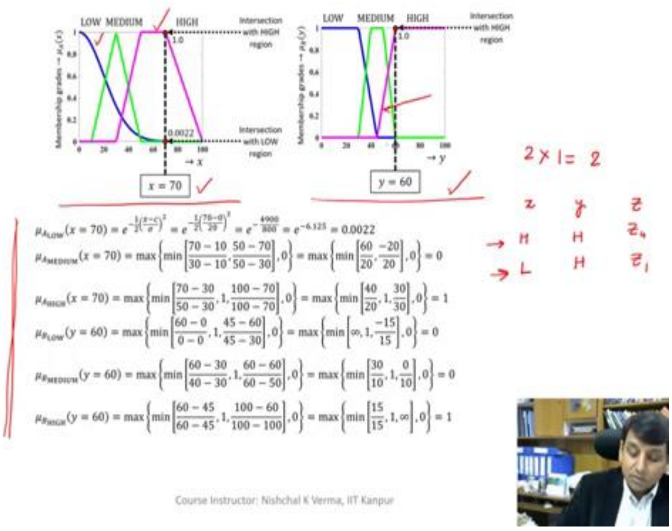

So, let us now proceed with this. So, first of all what we do we plot all these membership functions and these membership functions basically designate the fuzzy sets LOW, MEDIUM, HIGH and similarly for the output as well. Now, both the inputs so first input here and then second input here means the first input means first input variable for first input generic variable, second input generic variable y here and both the inputs are now plotted and we have the membership functions here.

And now, our task is to find the intersection points. So first of all corresponding to this input the input that is given. So, our input that is given is x is equal to 70 or y is equal to 60. So, corresponding to x is equal to 60 when we plot this we see that here this x is equal to 60 falls under two fuzzy regions.

First fuzzy region is the this one the HIGH region which is basically the HIGH fuzzy set and the other one is this fuzzy region this is a Gaussian this is represented by Gaussian. So, LOW fuzzy region is also coming into picture and LOW fuzzy region here is very less because, this is represented by a Gaussian fuzzy set Gaussian fuzzy membership function. So, there are two regions which are relevant for x is equal to 70 similarly.

Now, let us check for the y is equal to 60. So, for y is equal to 60 we see that only one fuzzy region is relevant that is high. So, for y is equal to 60 we have only HIGH fuzzy set which is relevant. So, we see that we have two points of intersection in the first antecedent, that means the first input generic variable and the y is equal to 60, we have only one fuzzy region which is relevant. So, 2 into 1 is going to give us 2 combinations what are those 2 combinations first is HIGH-HIGH, HIGH and then HIGH.

So, both the inputs basically. So, when the input is falling in the I am writing here as the when x is falling in x is falling in HIGH and then y is falling in HIGH then what is going to be the z the output. So, this we will get by inspecting the rules that are given to us the fuzzy rules that are given to us, but here we have two combinations. So, another combination is when x is falling in LOW and y is falling in HIGH because here only the y is falling only in high. So, now, for these 2 combinations only we need z.

So, let us check here we have 4 rules. So, first is the when the input is falling in the x is falling in LOW this is basically x this is when the input is this is y, this is y this is x, x, y, y. So, when x is LOW OR y is HIGH. So, this combination is this.

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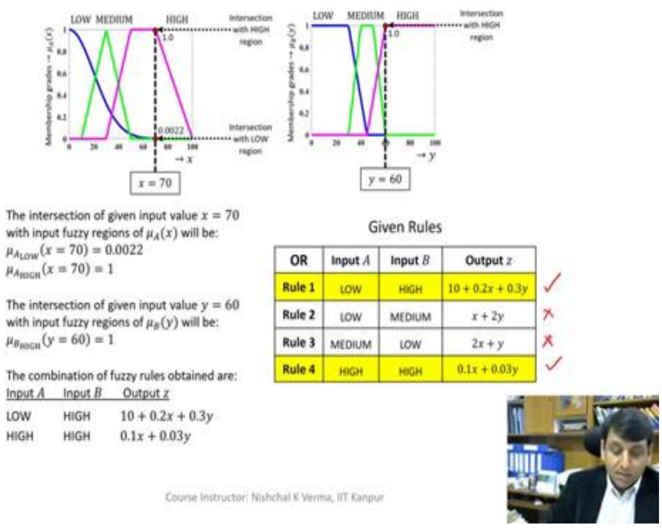

So, for this the output will be applicable is this z_1. So, we can say the rule number 1, that means the z_1 will apply and z_1 is defined by the 10 plus the output is equal to $10 + 0.2x + 0.3y$ similarly the other case HIGH HIGH.

So, when x is HIGH here the fourth ones. So, this is up this is relevant and this is a relevant means rule number 1 and rule number 4 are relevant. So, HIGH-HIGH. So, output four here will be mentioned. So, here we will write the z_4. So, this way we see that no other rule is applicable because we get only 2 combinations and this can be written like this as I just explained. So, rule number 1 is applicable and rule number 4 is applicable out of all the 4 rules given this is not applicable this is not applicable. Now take these 2 rules and find the output of each rule corresponding the inputs that are being supplied.

So, and then whatever output that we get we take the weighted average of it and that is going to be the final output. So, let us go ahead and do that.

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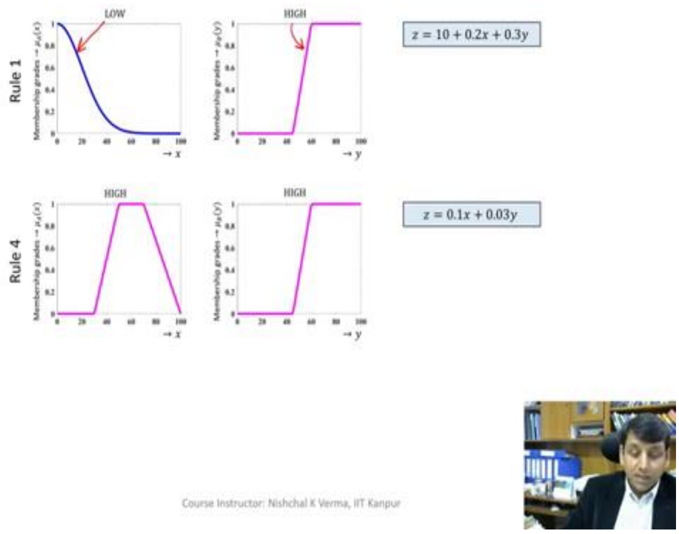

So, here as I have already mentioned that rule number 4 are rule number 1 and rule number 4 are applicable. So, let us take rule number 1. So, rule number 1 is already given if you have any confusion you can refer here. So, rule number 1 is given x is LOW y is HIGH. So, LOW we already know. So, LOW is here this fuzzy set is LOW for LOW.

So, we take this fuzzy set here LOW is for low. So, we have to plot it and then similarly for HIGH we can see from there. So, HIGH is here in rule number 1 we have LOW and HIGH combination in rule number 4 we have HIGH-HIGH combination. So, the membership function are the fuzzy set for HIGH-HIGH we can obtain from here if you have any doubt it is shown here the HIGH is here this is HIGH. So, this can be very easily plotted. So, let us proceed with this. So, let us take rule number 1 and apply the input.

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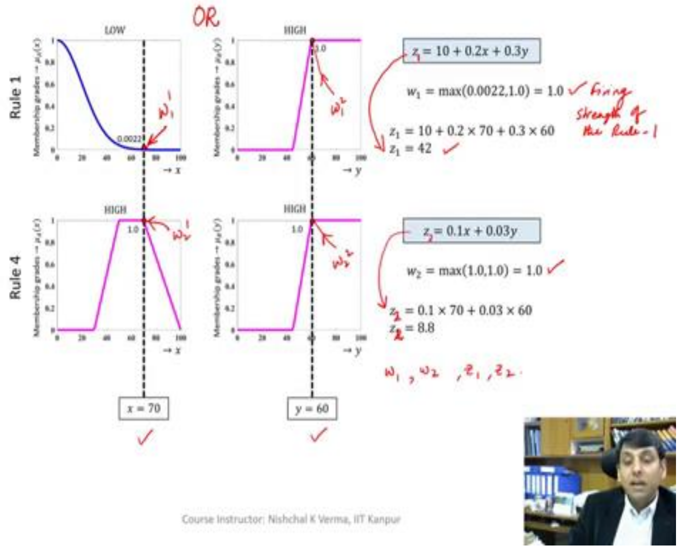

So, we apply the input here x is equal to 70, y is equal to 60. So, corresponding to x is equal to 60 we get here the intersection point let us call this as w_1^1. Similarly corresponding to y 60, y is equal to 60 for rule number 1, let us call this as w_1^2, w_1^2. So, w_1^1 is 0.0022, w_1^2 is 1. And since here we have the connective OR that we have given that we have been given. So, the we use instead of min we use max.

So, w_1 will be the maximum of w_1^1 and w_1^2 and which is coming out to be 1. So, now we have the rule strength the firing we have the firing strength of the rule w_1 and similarly I can write here the firing is strength of the rule 1, alright so and z is already given. So, by just substituting the value of x and y we get z_1 here. So, z_1 is 42 if we substitute the value of x and y. So, this is z_1 and similarly when it comes to the rule number 4 we find here the point of intersection as w_2^1 and here w_2^2.

And here also we take max of these 2. So, both of these are 1, 1. So, we take max and max is again going to be 1 and this is the output which is z_2. So, z_2 by just substituting this we are going to get here the these is z_2 and when we substitute the values of x and y here we are going to get z_2 is equal to 8.8. So, now, we have what we have w_1 we have w_2 we have z_1 we have z_2. Now we can very easily get the weighted average of these outputs corresponding to the rule number 1 and rule number 4 see here.

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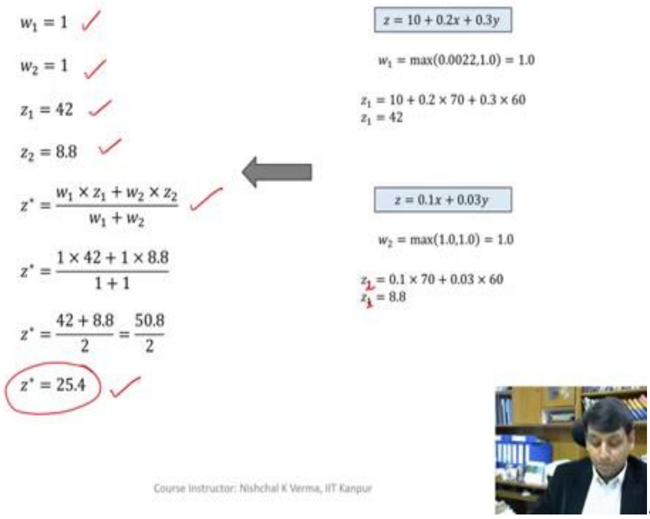

So, we have w_1 is equal to 1 you can see here and then w_2 is equal to 1, z_1 is equal to 42, z_2 is equal to 8.8 and when we take the weighted average we are getting the weighted average as 25.4. So, this means that corresponding to the input x is equal to 70 and y is equal to 60 we are getting the overall output as 25.4 when we use the TSK fuzzy model with multiple antecedent and multiple rules.

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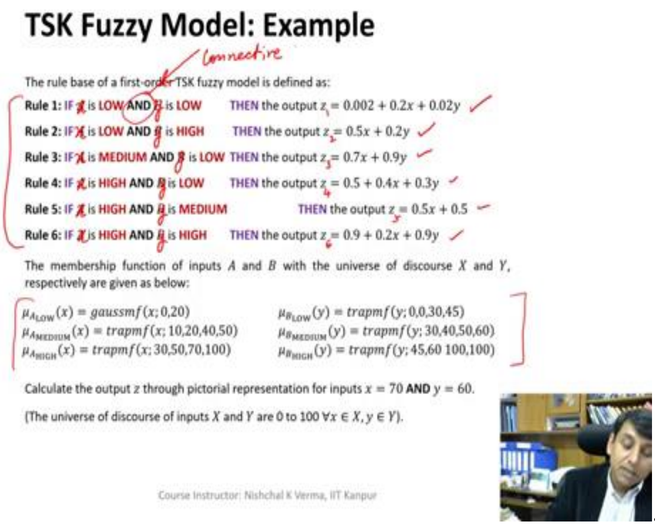

So, this way we quickly get to know as to how we can manage to get the final output corresponding to the inputs that are given for any TSK fuzzy model. So, let us take another

TSK fuzzy model which is having 6 rule here 6 rules we have. So, any fuzzy model which is characterized by 6 fuzzy rules and we have the rule number 1 as x_1 is a we have first rule as x is LOW y is LOW then the output is this let us say this is z_1.

And then the second rule here is x is LOW y is HIGH then the z_2 is given by this expression. Similarly x is MEDIUM, then y is LOW, and then this is z_3 means x is HIGH and then y is LOW then the output here is for the first fourth rule is z_4. Similarly rule number 5 as x is HIGH y is MEDIUM then the output is z_5. Rule number 6 has x is HIGH y is HIGH then the output z_6 is given by this expression, and the corresponding fuzzy regions that are given for LOW, MEDIUM, HIGH for x input variable is given here and similarly for the y input variable the corresponding LOW, MEDIUM, HIGH fuzzy regions are characterized by these membership functions.

Now, then task is here to find the corresponding output to x is equal to, the input x is equal to 70 and y is equal to 60. So, here the input remains the same, but interesting thing that we see here is that the connective is AND instead of OR. So, here we have this connective. So, if we have this kind of example, this kind of problem. So, in the previous one we saw that we have had or as the connective here we have and so obviously we are going to take composition as min.

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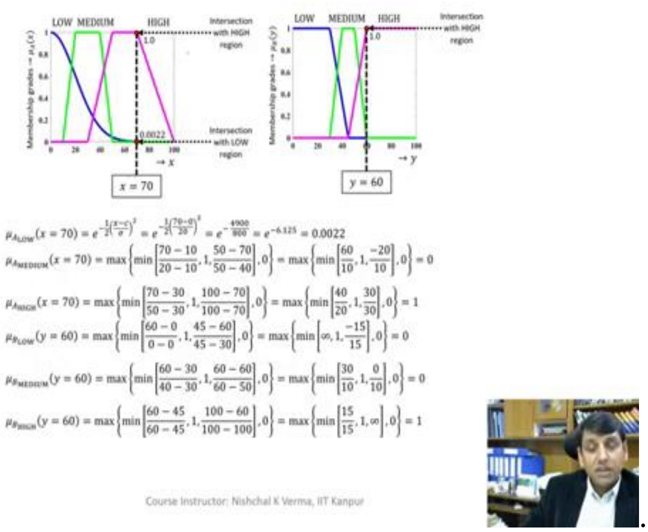

So, let us proceed with this we have already seen as to how we can get this plotted. So, here also for x is equal to 70 we have two intersection points first is the for LOW and second is for HIGH. Similarly the y for y is equal to 60 only one intersection points. So, the combinations again will remain same as the previous example.

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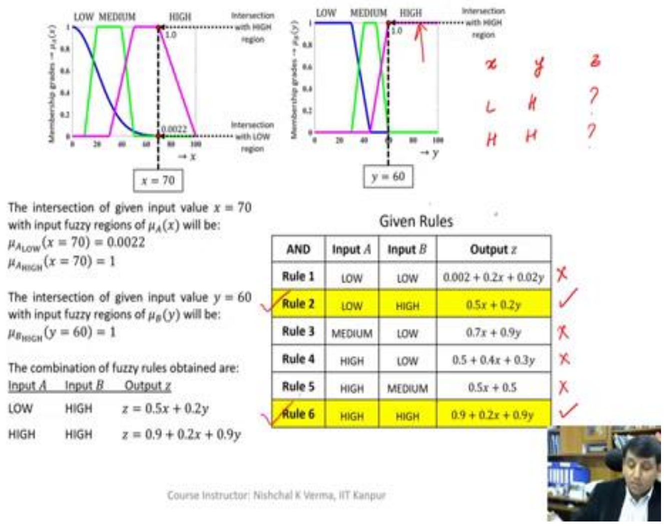

Now, here we have to look for the applicable rules. So, since we have 6 rules that are given for the fuzzy model for the TSK fuzzy model. So, we have only two combinations that is LOW-HIGH, HIGH-HIGH. So, rule number 2 will apply and 6 will apply this rule will not apply because the combination wise we LOW-LOW does not exist only LOW-HIGH exist here.

So, the rule number three MEDIUM-LOW does not exist HIGH-LOW does not exist HIGH-MEDIUM does not exist and HIGH-HIGH exists. How do we get this I have already explained in the previous example, but I can do it again. So, we see that x is equal to 70 cuts HIGH fuzzy region here HIGH fuzzy region here and then LOW fuzzy region which is characterized by the membership Gaussian membership function.

So, x and y if we write and then z. So, x is cutting two fuzzy regions LOW and HIGH and y is cutting only one fuzzy region here the HIGH for others it is 0. So, we can write L-H and H-H. Because HIGH is for both the cases and this way we have to let look for the corresponding z and this we can find by looking at the set of rules and we have here two rules that are applicable and then with this we will proceed same as we have done in the previous example.

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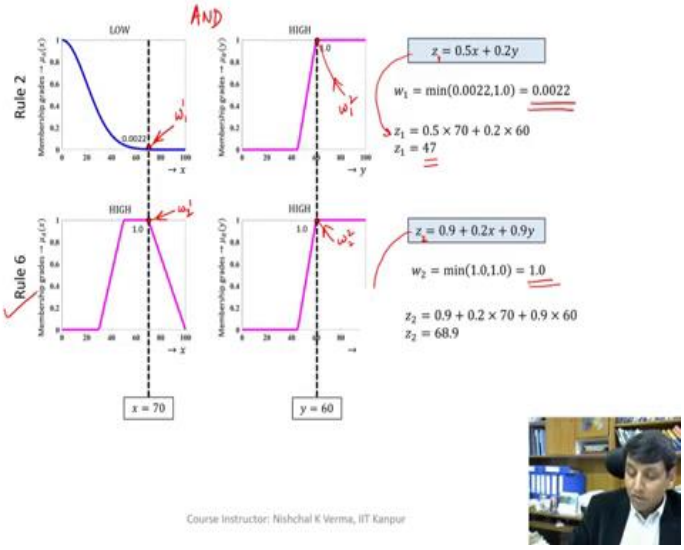

So now, we see that here we get when we take 2 rules. So, we have to take only 2 rules other rules rest of the rules will just discard. So, because here rule number 2 and rule number 6 are only applicable. So, we will start with rule number 2 rule number 2 is here. Now we apply the input x is equal to 70. So, w_1^1 let us say similarly here this is w_1^2 for the second antecedent y is equal to 60 and then corresponding output here is z_1.

And this z_1 by just substituting the value of x and y we can have here this is coming out to be 47 and now when it comes to w_1 will have min why min because here the connective is AND. So, AND connective is used. So, that is why we are taking min here. So, if we take min of the two values w_1^1 and w_1^2 we get 0.0022. So, we have here w_1 and z_1 now similarly when we apply the rule 6 for the input x is equal to 70, y is equal to 60 we have z_2 like this z_2 is equal to this is already given.

And when we use x and y the values of x and y we get z_2 here computed like the 68.9 and the w_2 is here w_2^1 and w_2^1 and w_2^2. So, both are 1 here. So, even if we take min is going to be the same. So, w_2 is the firing strength of the second rule which is 1.

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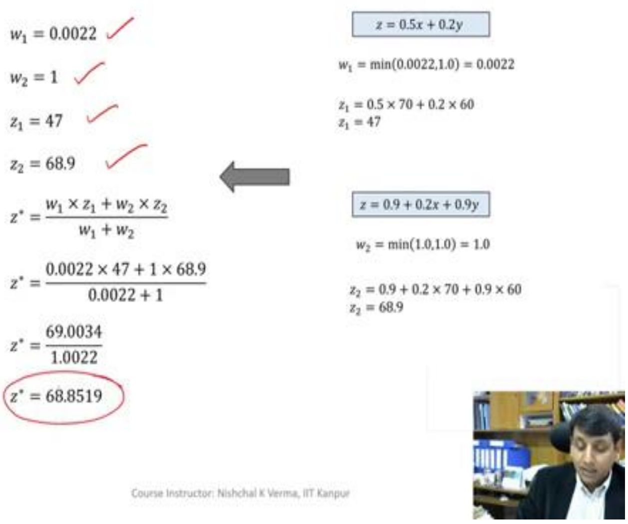

So, now we use these values here. So, w_1 w_2, z_1 z_2.

And then the weighted average is coming out to be 68.8519. So, this way here we can say that the corresponding to x is equal to 70 and y is equal to 60 we are getting the output of the TSK fuzzy model as 68.8519. So, I think we have understood now as to how we can compute the outcomes of the TSK fuzzy model in various cases for various inputs and with different kinds of sets of the rules and with this I would like to stop here.

Thank you very much.