Fuzzy Sets, Logic and Systems and Applications
Prof. Nishchal K. Verma
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

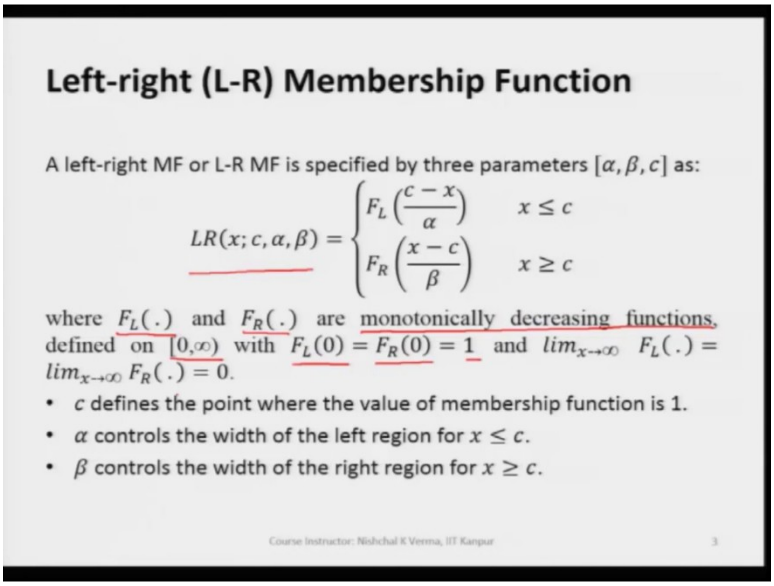
Lecture - 06
Membership Functions

So, welcome to lecture number 6 of Fuzzy Sets Logic and Systems and Applications. So, in this lecture we will be discussing fuzzy Membership Functions and these membership functions are basically single dimensional membership functions. So, in the last lecture, while discussing the fuzzy membership functions, we had already seen these commonly employed membership functions.

(Refer Slide Time: 00:45)

And the you can see the names of these membership functions. So, out of these 10 membership functions we have already covered 5 membership functions. So, remaining 5 membership functions that is from left-right membership function, we will cover in this lecture.

(Refer Slide Time: 01:23)



So, let us begin with this membership function; that is left-right membership function. Left-right membership function is defined here by this function. So, you see here, the left-right membership function can simply be written as or defined as

$$LR(x; c, \alpha, \beta) = \begin{cases} F_L\left(\frac{c-x}{\alpha}\right) x \le c \\ F_R\left(\frac{x-c}{\beta}\right) x \le c \end{cases}$$

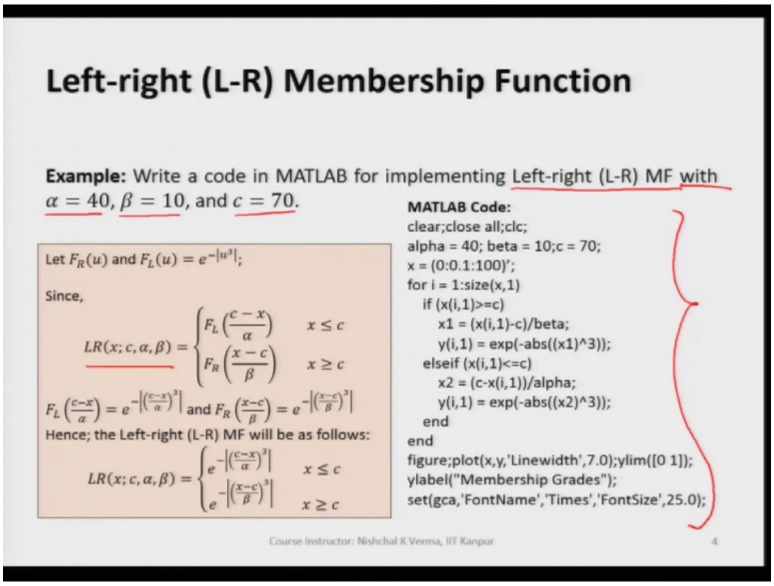
So, if we see here F_L and F_R and these are just the functions. So, $F_L$function and F_R function you see here F_L function and then F_R functions and these F_L, F_R functions are monotonically decreasing functions, so this has to be noted. And these are defined on strict interval here with F_L; when we have F_L of 0 and this will be equal to 1. And if we have F_R also, so F_R of 0 will also be equal to 1.

So, we can also write like this as it is written. So, F_L of 0 is equal to F_R of 0 equal to 0. So, what does this mean here is that, both the functions when we have taken of 0 this will be equal to 1. And F_L when $x \to \infty$ means when x is approaching infinite this F_L will be equal to 1 and so not only F_L the F_R also.

So, both the F_L and F_R $x \to \infty$ is going to give you 0. Here these parameter c, alpha, beta these three functions basically, these are the parameters which determine this shape of the left right membership function.

So, c here defines the point, c this c defines the point where the value of membership function is 1. And alpha, this alpha controls the width of the left region of left region for x less than or equal to c; beta here, this beta in this left right function controls the width of the right region for x more than or equal to c, so this way we define a left right membership function.

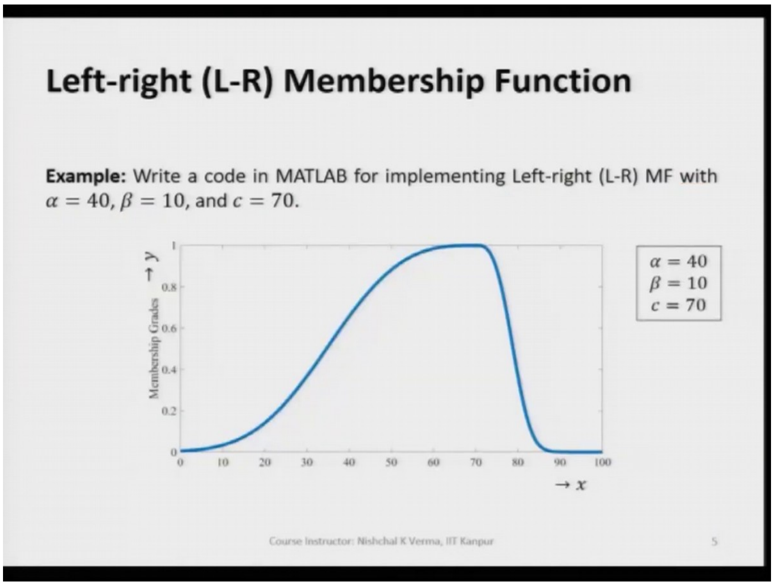
(Refer Slide Time: 05:00)



So, let us take an example here to understand this better. So, if we write a code in MATLAB for implementing left right membership function, with $\alpha = 40$, $\beta = 10$ and $c = 70$. So, these three parameters if we substitute in this function and based on this function left-right function we will be able to plot left-right membership function by using this MATLAB code.

So, this MATLAB code you can simply write or you can copy and you can take this two MATLAB. And you can run and if you run this MATLAB code you will see that with these parameters you are going to get this shape.

(Refer Slide Time: 06:04)



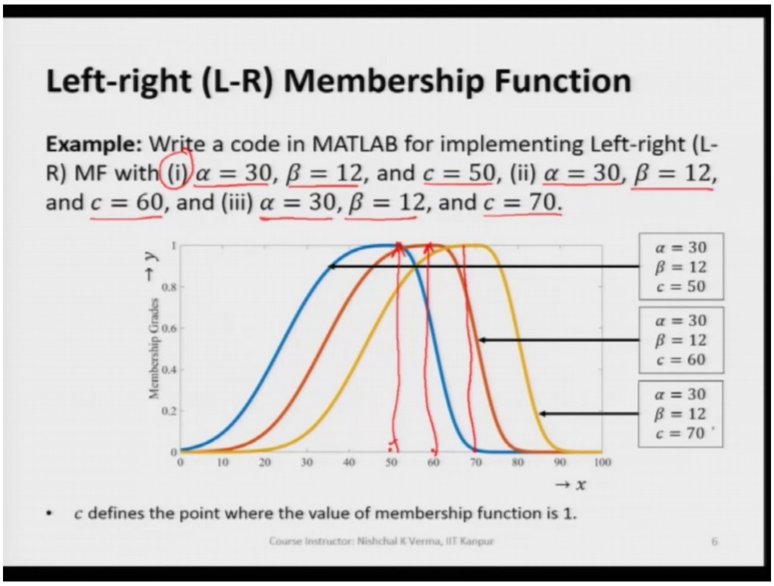
So, when you have $\alpha = 40$, $\beta = 10$, $c = 70$, this kind of shape you are going to get. So, please note that the $c = 70$ is the point here at here at which the membership function has its value 1. So, in other words we can say at $c = 70$ the function has reached at the highest level and that is 1.

And let me go back to the previous slide here and we see here that as I have already spoken about F_R here and F_L; so, F_R and F_L both are the functions. So, in this case apart from these parameters α, β and c we also take we also choose a suitable monotonically decreasing functions. So, here in this case F_R and F_L has been have been chosen as

$$F_R(u) = e^{-¿u^3 \vee ¿¿} \text{ and } F_L(u) = e^{-¿u^3 \vee ¿¿}$$

So, if we have substituted this function these parameters. And we have taken the function included in this we'll be able to get the plot here as, you see after getting these MATLAB code executed.

(Refer Slide Time: 07:51)



So, let us take another example here to understand this better. Here in this example we have three cases for left-right membership function plot. So, the first case here you see this first case, in which we have α=30, β=12 and c=50. So, if we take this value α=30, you see it means α=30, β=12 and c=50.

So, c=50; means this is the point 50 is a point where this particular plot will have it's, this particular plot will have its highest value. So, if we see here the plot for these values and let me make it clear here that here also we have used the same monotonically decreasing function.

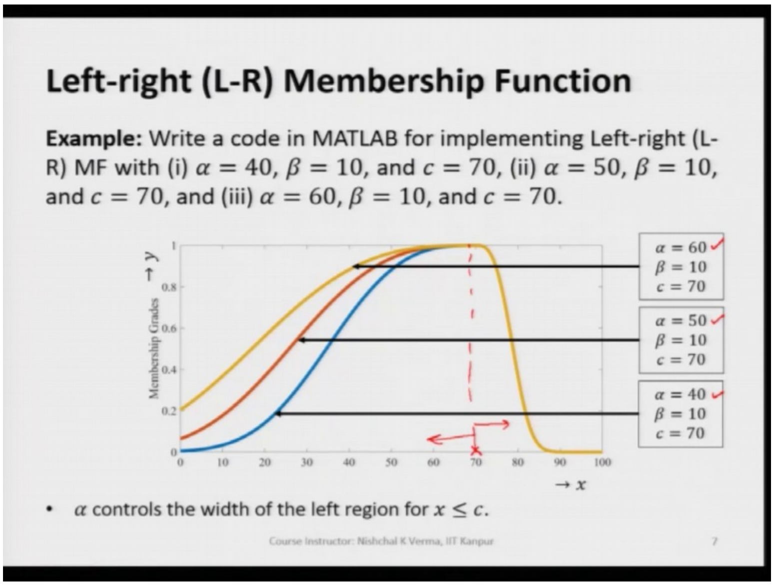
So, but we are only varying these alpha the values of alpha, beta and c. So, these are the three cases the first case is where α=30, β=12, c=50. And the second case we have α=30 and β=12 and c=6 0. And the third case here is α=30, β=12 and c=7 0.

So, we can clearly see here that, what we are wearing here is the c only. So, in all the three cases we are changing only the value of c; it means we are shifting the point where we have the membership functions value 1. So, let me go back to the function definition. Here, you see here it is clearly written that c defines the point where the value of membership function is 1.

So, in this example, in this example this, this c is changing in one case we have 50, in the other case we have 60 and then the third one we have 70. So, we can clearly see that for the first membership function, which is shown in blue. So, we see that here the membership value is 1; we see here. And the second case we have its membership value 1. So, this is the other membership function the second membership function the second case. And the third case here we have the membership value 1. So, this way we see that, the c is the parameter which is changing in all the three cases rest two parameters are not changing at all.

So, as we clearly know that the α is responsible for change in the left half of the region about c. And β is responsible for the change of the shape the right half of the c. So, we can clearly see once again I would like to tell you that these two values are same alpha, beta are same in all the three cases only the c is changing.

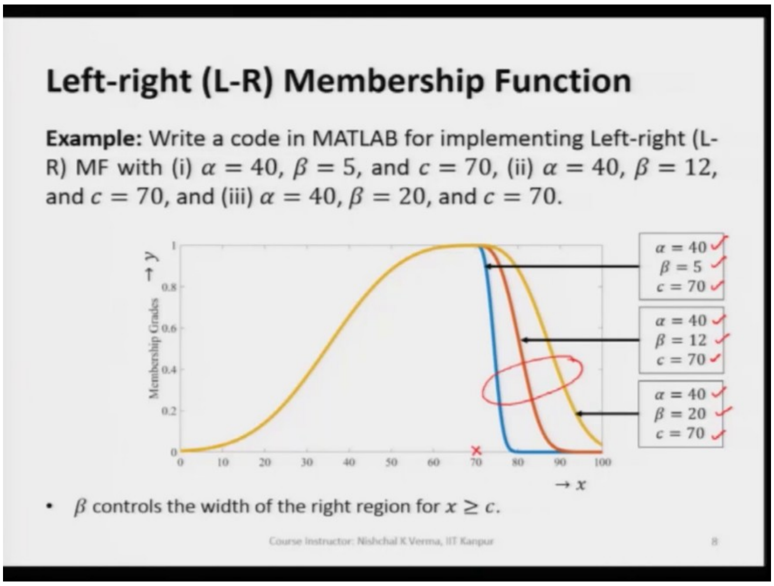
(Refer Slide Time: 12:08)



So, let us take another example, here where you see we have again three cases where we are only changing alpha. So, values are alpha are changing see here see in one case we have 60, the second case we have 50 and the third case we have 40. And other two parameters in all the cases are same like beta, c are same in all the three cases. So, if we see here c, since c the value of c remain same in all the three cases.

So, let us see where the c is $c = 70$; so, you see 70 is this point this point and around this I mean this side is left side of this c and this side is right side of this c. So, if you see here as I already mentioned that, when we vary when we change alpha. So, we see clearly that left side of the curve is changing. So, this is responsible for the change in the shape of the left side on the left side.

(Refer Slide Time: 13:26)

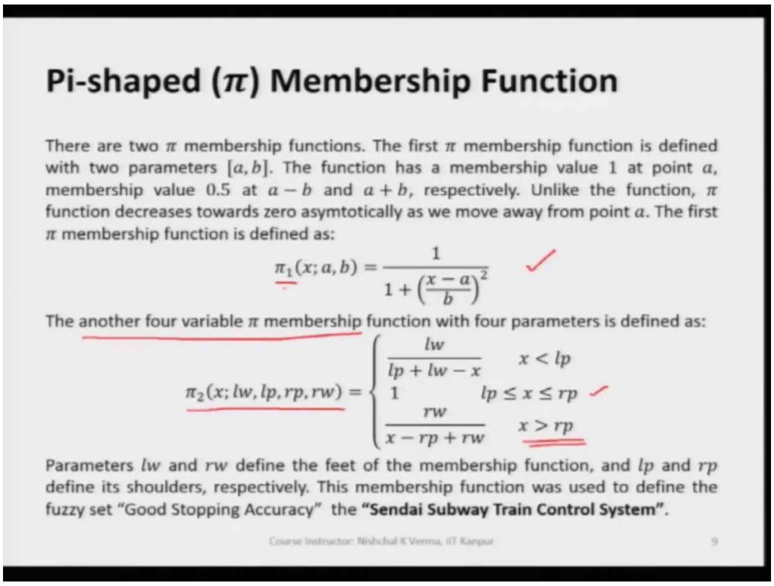


And if we change in the other example we see here that, c remain same in all the three examples and α remains same see here 40. So, in all three cases alpha, c remains the same only β is changing. So, in one case we have β as 5 and the second case we have β 12 and the third case we have β 20.

So, beta is changing as 5, 12, 20. So, we see that c is here c is the point c is basically I can say it can be regarded as a center around which you know we have the left side on the left side change β is responsible right side change the α is responsible.

So, c here is at 70 and if we see very clearly by changing the values of β, we see that this side is the shape of curve is changing. The plot we can clearly see that the blue is for $beta=5$ and a red is for $\beta=12$ and yellow is for $\beta=20$. So, this way we can control the shape of the left right membership function by changing alpha beta and c suitably.

(Refer Slide Time: 15:10)



Now we can go to the other membership function, which is a called pi shaped membership function. So, pi membership function there are two types of pi membership functions; there are two functions; basically, first function here is represented by π_1. So, π_1 function is having only two parameters.

$$\pi_1(x; a, b) = \frac{1}{1 + \left(\frac{x-a}{b}\right)^2}$$

So, this pi membership function has only a and b; that means, two parameters and these two parameters actually control the shape of π membership function. So, and then, here we have another four variabled π membership function.

So, but this four variabled π membership function has four parameters. So, this pi membership function is represented by,

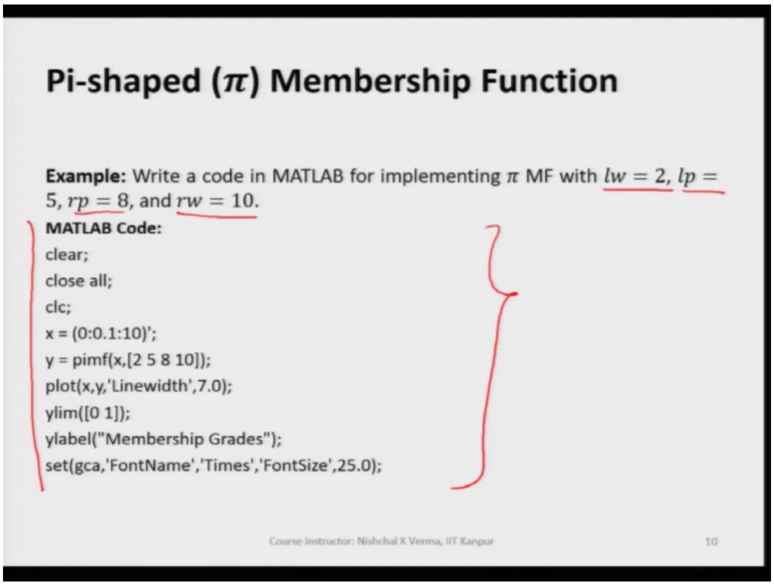
$$\pi_2(x; lw, lp, rp, rw) = \begin{cases} \frac{lw}{lp + lw - x} & x < lp \\ 1 & lp \le x \le rp \\ \frac{rw}{x - rp + rw} & x > rp \end{cases}$$

So, this way we have two kinds of pi membership function. And here in the second case so, first case is represented by first type of pi membership function is represented by π_1 which is clearly mentioned here. And the other kind of pi other type of pi membership function here is represented by π_2. So, π_1 is only two variable membership function and this function basically we can say this function has membership value at point a membership value 0.5 at a minus b; which is mentioned over here and a plus b respectively.

So, unlike the function pi, function decreases towards 0 asymptotically, as we move away from point a, so this is here you can see represented by π_1. Now, in the second case which is represented by second type of pi membership function which is represented by π_2; this is characterized by four parameters lw, lp, rp, rw.

Parameter lw and rw define the feet of the membership function and lp rp define its shoulders respectively. So, this is very interesting membership function which was used in the controller fuzzy controller, which was very popular fuzzy controller used for the sendai subway train control system.

(Refer Slide Time: 19:34)

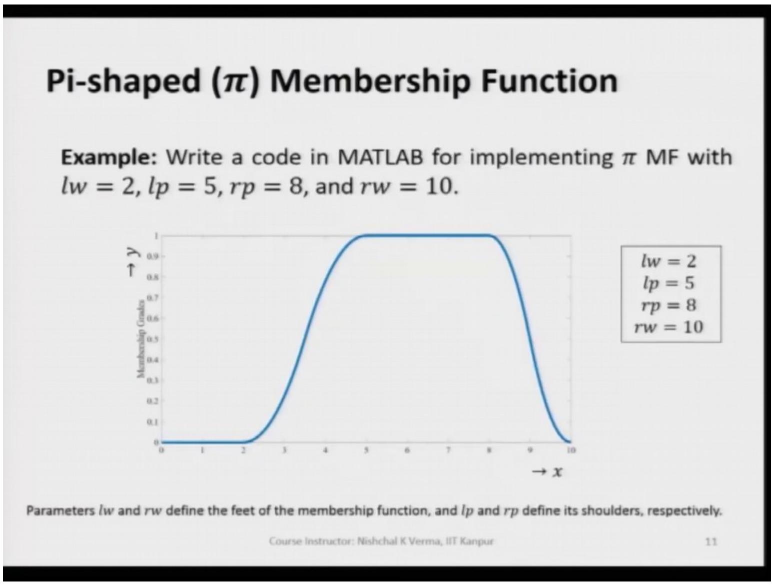


So, here we have the MATLAB code which you can use for plotting pi shaped membership function. And this has been given with an example a case where we have lw, lp, rp, rw values given as 2, 5, 8, 10 respectively.

So, we can clearly see here that this pi shaped membership function is a second type membership function which requires four parameters. So, we see here the MATLAB code.

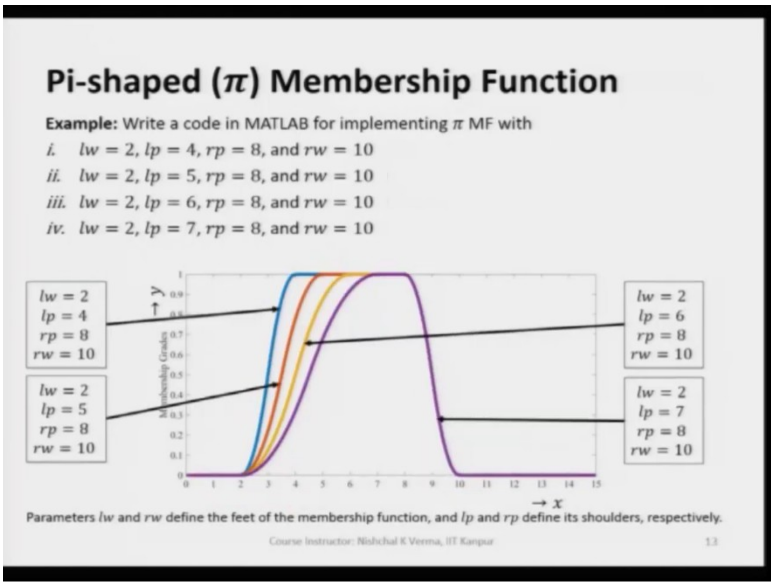
So, if we can if we write this code in the MATLAB and run this code in the MATLAB we get this kind of plot.

(Refer Slide Time: 20:30)



So, we see here the shape of this plot is pi type. So, we see here as I mentioned the lw, lp, rp, rw values. And I had already explained these parameters as this lw and r w defined the feet. So, lw defined the left feet, r w defined the right feet, whereas l p defined the defined its left shoulder and r p defines its right shoulder.

(Refer Slide Time: 21:15)



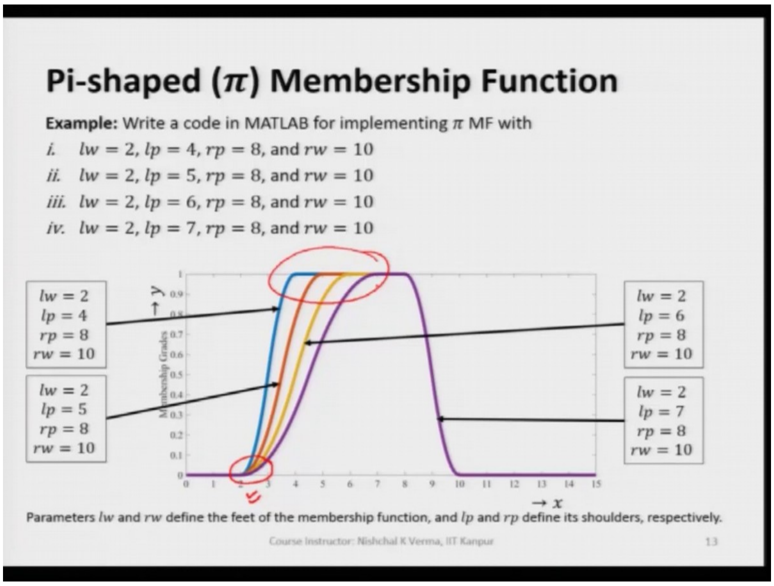
So, now let us understand this pi shape membership function better. And for in this example we have taken four cases to better understand these parameters which are controlling the shape of the pi membership function.

So, we have taken first case here where $lw=0.5$, $lp=5$, $rp=7$, $rw=10$. And I, I already mentioned that l w is controlling left feet lp is controlling left shoulder, r p is controlling right shoulder, r w is controlling right feet. So, we have taken four cases here. So, in first second third fourth we see the only l w is varying.

So, this is this has done intentionally to make you understand that if we vary if we change l w only what is happening to the pi shaped membership function. And here in all the three cases only lw is changing rest three parameters are remaining the same. So, here if we see as I mentioned that lw is controlling the left feet. So, if we see here the left feet is changing by changing by varying the values of lw; rest of the rest other values all three values are the constant the lp is constant rp is sorry the lp lp rp rw remains the same.

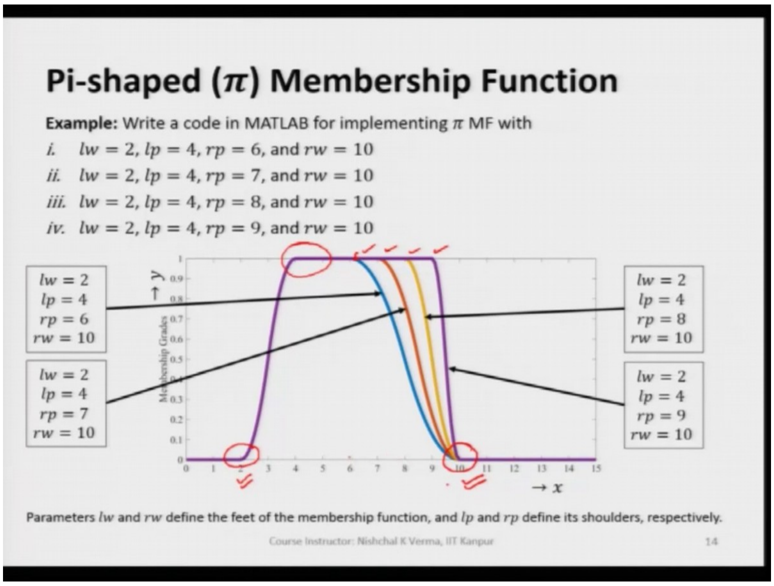
So, that is why only feet is changing and that is and only left feet is changing rest other things remain the same for this pi membership function. So, if you see here in all the three cases right feet remains the same right feet is this. So, right feet remains the same and then shoulder here the left shoulder remains the same, right shoulder remains the same. So, we clearly see that only by changing the lw only left feet is changing. So, this has to be noted here.

(Refer Slide Time: 24:18)



And then similarly if we take another example, where we see that left feet remains the same in all the four cases only lp is varied. So, we are changing the values of lp only and we clearly see that left since the left feet values are same in all the four cases so, left feet remains the same. And what is interesting here to see that by changing lp the values of lp in all four cases, you see the left shoulder is changing this is clearly visible here that the shape of the left shoulder is changing.

(Refer Slide Time: 25:12)



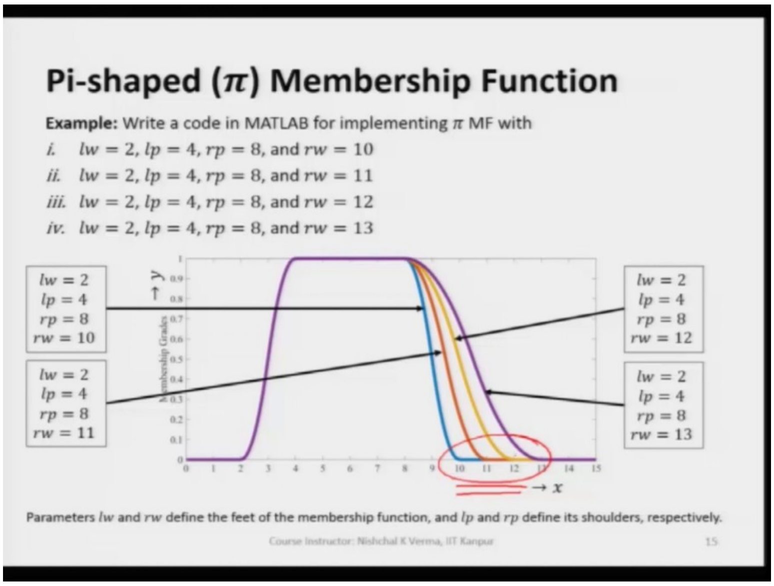
Similarly, if we take another example where the out of all the four parameters of pi shaped membership function, if we change only rp that is right shoulder and the rest other parameters remain the same. If you see here, lw is not changing in all the cases it remains at 2. So, that is why it is not changing it remains the same in all the four cases. r w is 10 and this is also remains at the at 10. So, this is also not changing.

So, it means that left feet is not changing right feet is also not changing. And here if we look at the shoulders like left shoulder see left lp here in all the four cases remain the same and it remains at 4. So, this is this also remains the same in all the four cases. Whereas, since we are changing rp; so, we see here the rp is changing means the shoulder width is changing in all the four cases.

So, first case we have rp is equal to 6. So, at 6 you see here the shoulder width is shoulder is coming at 6 and then again if you are changing rp which is here is at 7 and then it is changing at 8. In the third case and then again it is changing to 9 at in the fourth case.

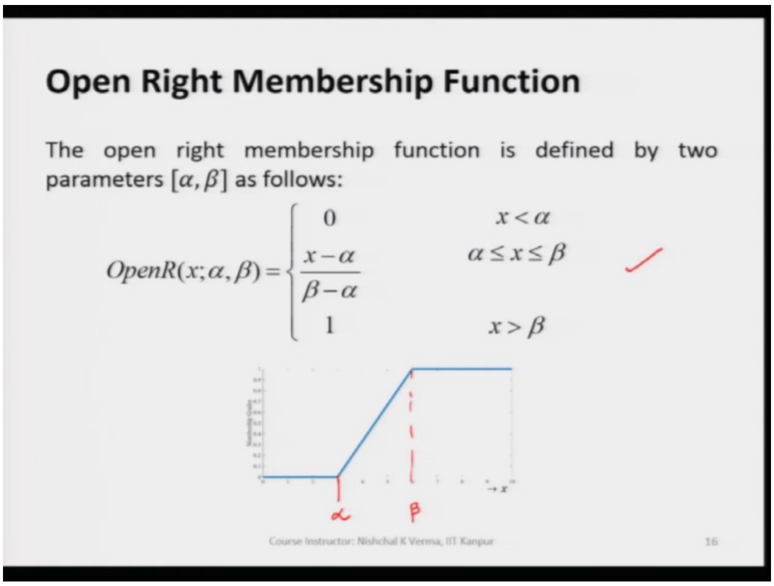
So, this way we understand as to how all these four parameters are changing the left feet, right feet, left shoulder, right shoulder of a pi shaped membership function.

(Refer Slide Time: 27:18)



So, yes so here we are changing the right feet. And if we change the right feet here we see clearly that the feet is changing So, in all the four cases in this example only rw is changing, rest three parameters lw, lp, rp remains the same. So, here also we clearly see that the feet the right feet is changing as per rw values.

(Refer Slide Time: 27:57)

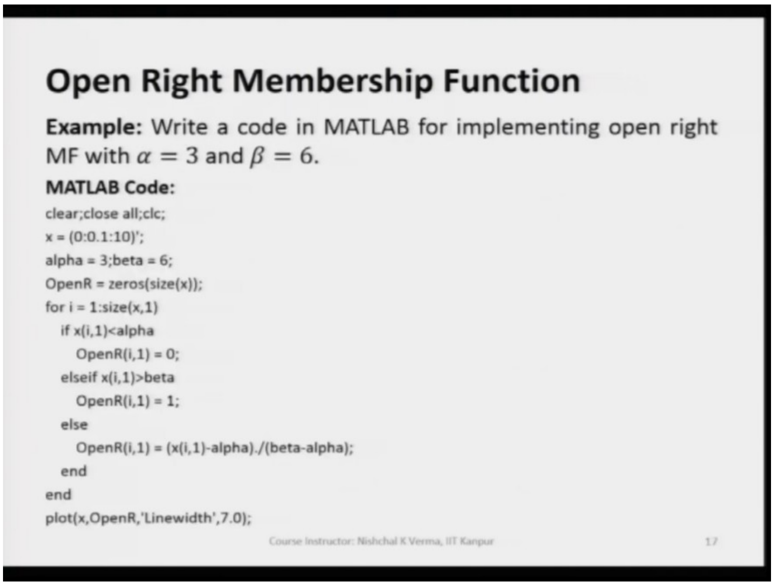


So, then let us discuss the open right membership function. So, in the open right membership function, we see that this kind of membership function is defined by

$$OpenR(x;\alpha,\beta)=\begin{cases} 0 & x<\alpha \\ \frac{x-\alpha}{\beta-\alpha} & \alpha\le x\le\beta \\ 1 & x>\beta \end{cases}$$

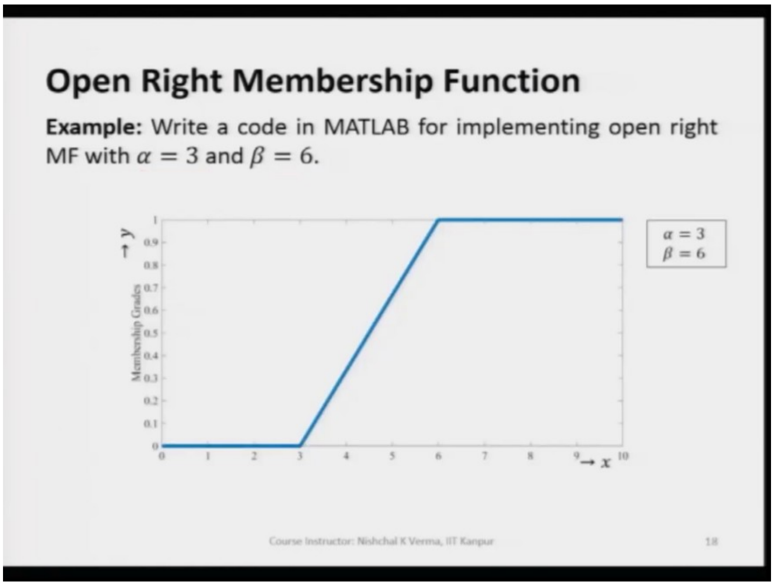
So, when we try to plot the membership function or when we try to draw a membership function based on this function, we will be going to get this kind of shape. So, this point on the x axis we will have as alpha and this point here we will have as beta. And we can clearly see that this function is open in the right side. So, this is important to be noted.

(Refer Slide Time: 29:44)

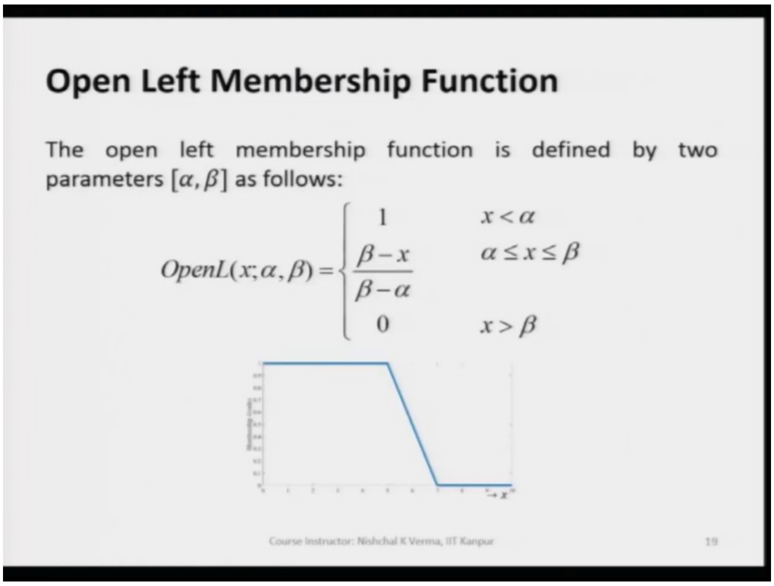


And similarly for this also we have a MATLAB code for your help. So, you can clearly copy this MATLAB code or you can rewrite this MATLAB code in the MATLAB. And you can run this MATLAB code with $\alpha=3$ and $\beta=6$ you will be going you will be getting this kind of shape.

(Refer Slide Time: 30:10)



(Refer Slide Time: 30:12)

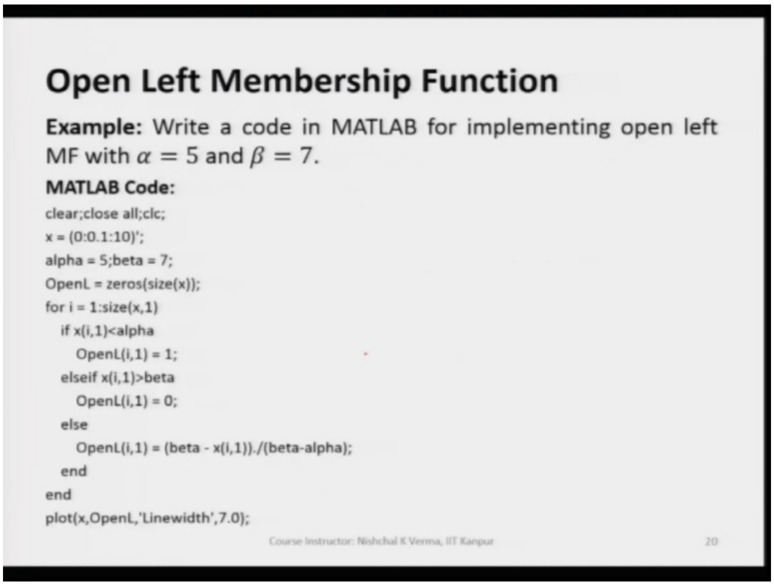


Similarly, since we saw here the open right membership function here where open right membership function because the right side of this membership function remains open. So, when we say right side of this membership function remains open means limit x tending to infinite function the open right function will always be equal to 1. So, when this function finally, is this function is going to attained the highest value which is 1.

So, likewise we have the open left membership function. And this membership function can be defined by

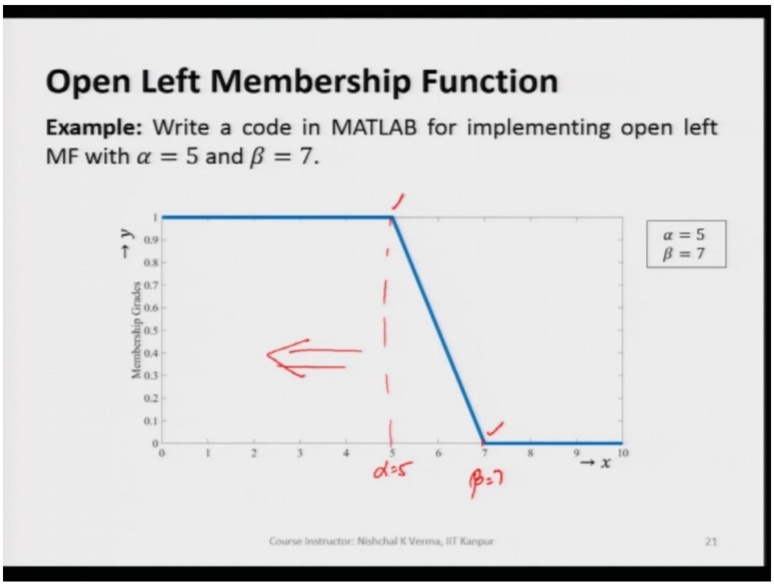
$$OpenL(x;\alpha,\beta)=\begin{cases} 1 & x<\alpha \\ \dfrac{\beta-x}{\beta-\alpha} & \alpha\le x\le\beta \\ 0 & x>\beta \end{cases}$$

(Refer Slide Time: 31:43)



So, similarly here also we have a MATLAB code that will help you to plot open left membership function by substituting different values of alpha and beta. So, in this example we have chosen the value of α as 5 and value of βas 7.

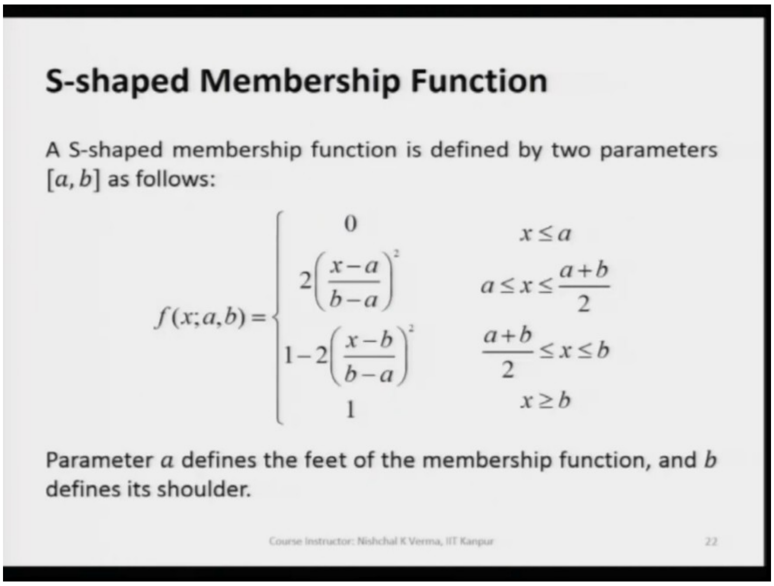
(Refer Slide Time: 32:12)



So, by substituting this these values of α and β, we see here open left membership function. And we can clearly see that left side of this membership function is open in the sense that we have the values the final values or we have the left side if we if we take this value as alpha, this value as beta, this is $\alpha=5$ and the $\beta=7$.

So, left of 5 left of $\alpha=5$ the membership function, or the membership values or I would say the membership function will have membership values equal to 1. Now, we come to another membership function which is called S shaped membership function. So, S shaped membership function is defined by two parameters and these parameters are a and b.

(Refer Slide Time: 33:09)

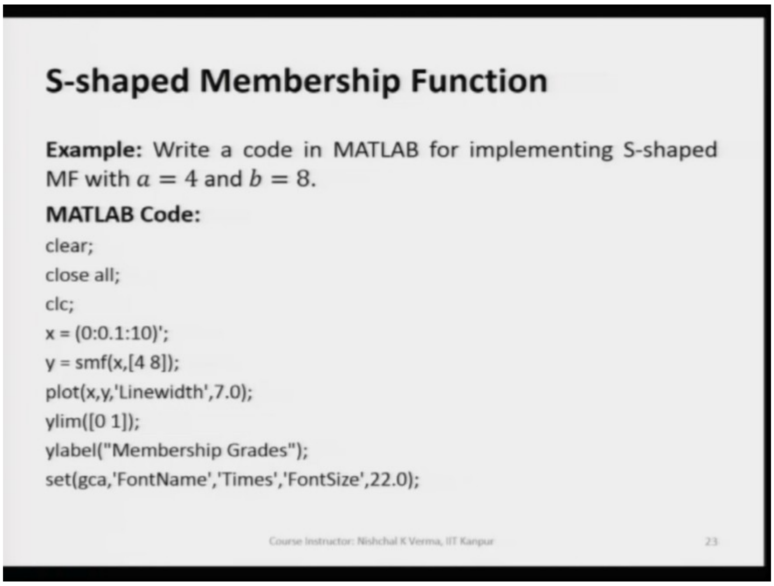


So, this is defined by

$$f(x;a,b)=\begin{cases} 0\, x\le a \\ 2\left(\frac{x-a}{b-a}\right)^2 a\le x\le \frac{a+b}{2} \\ 1-2\left(\frac{x-b}{b-a}\right)^2 \frac{a+b}{2}\le x\le b \\ 1\, x\le b \end{cases}$$

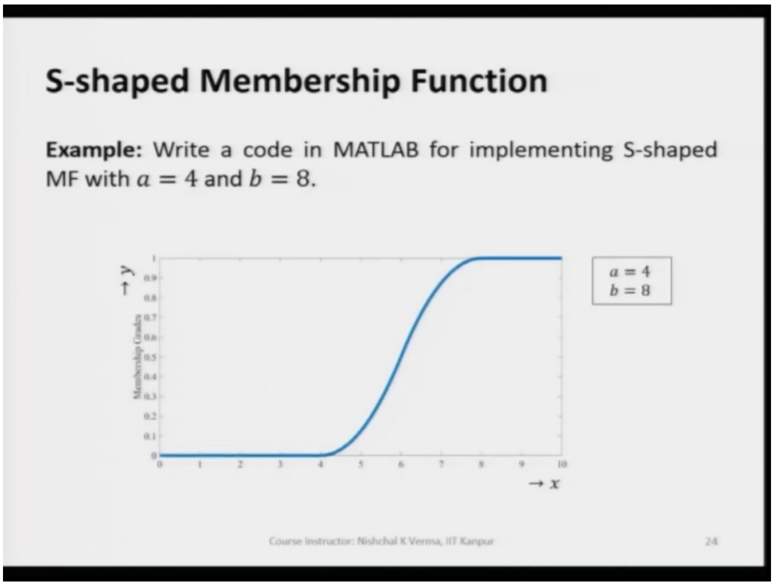
And let me also tell you that this parameter a defines the feet of the membership function and b defines its shoulder.

(Refer Slide Time: 34:58)



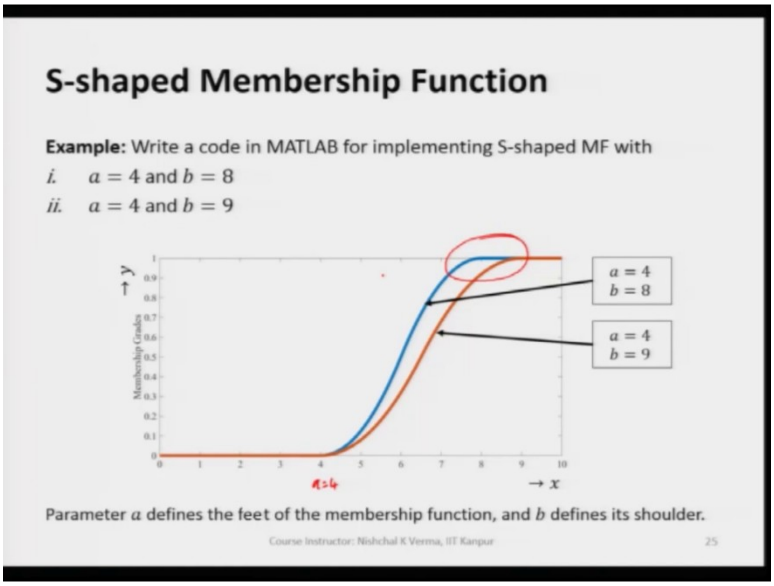
So, like other membership functions here also for S shaped membership function we have a MATLAB code ready for you. And if you would like to take this and execute in the MATLAB with $a=4$, $b=8$.

(Refer Slide Time: 35:23)



We clearly see that this kind of membership function the S shaped membership function we are going to get. So, as I already mentioned that a, b these two parameters are characterizing s shaped membership function a controls the feet of the membership function and b controls the shoulder of the membership function.

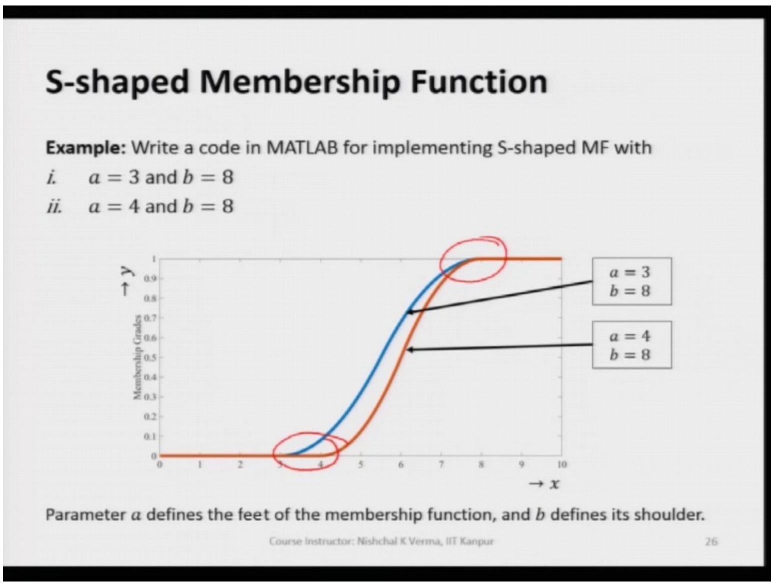
(Refer Slide Time: 35:47)



So, here we have few more cases. So, in this example we have two cases; the first case we have a=4, b=8 and a and the second case we have a=4 and b=9. So, we can clearly see that in this example the feet of the curve is not changing feet of the curve is remain remaining at the 4.

So, here a=4 we have taken and you see the shoulder is changing here the shoulder is one the first shoulder is at b=8 and the second the shoulder for the second plot is at 9. So, by changing the suitable values of a and b we can keep changing the feet and the shoulder.

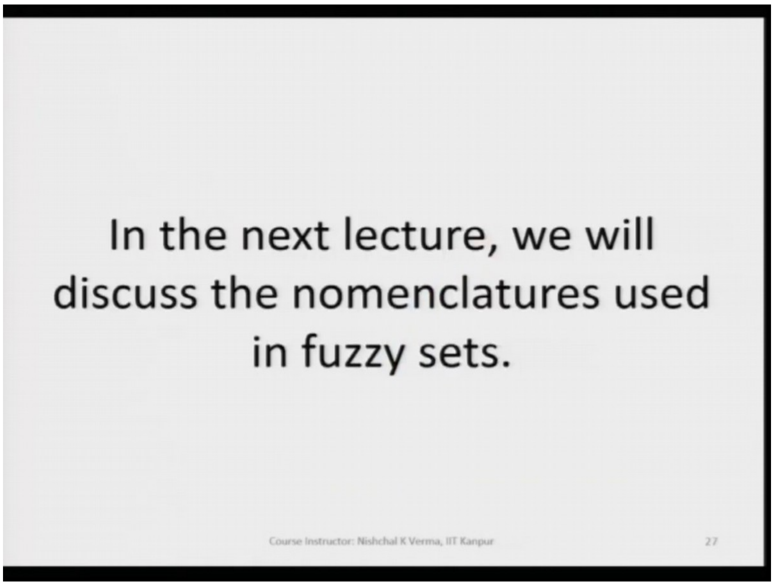
(Refer Slide Time: 36:56)



So, here as I mentioned that if we change the values of the feet we suitably we can change the places of the feet. And here in these two cases we have b constant. So, b is not changing so, that is why the shoulder remains at the same point. So, shoulder remains in both the cases same. So, this way the S shaped membership function can be plotted, And let me also tell you here it’s very important to mention that s shaped membership function fuzzy membership function, can also be used as the right open or left open type of membership functions.

So, this membership function since the left right side of this membership function remains at finally, at one all the time, so this qualifies to become the right open type of fuzzy membership function.

(Refer Slide Time: 38:15)



So, with this I would like to stop here. And in the next lecture we will discuss the nomenclatures used in the fuzzy sets.

Thank you very much.