Fuzzy Sets, Logic and Systems and Applications

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Lecture - 58
Larsen Fuzzy Model (For Multiple Rules with Multiple Antecedents

Welcome to the lecture number 58 of Fuzzy Sets, Logic and Systems and Applications. And here, we will continue our discussion on the Larsen Fuzzy Model for Multiple Rules with Multiple Antecedents and this also we will discuss with max-min composition and max-product composition for fuzzy and crisp inputs both.

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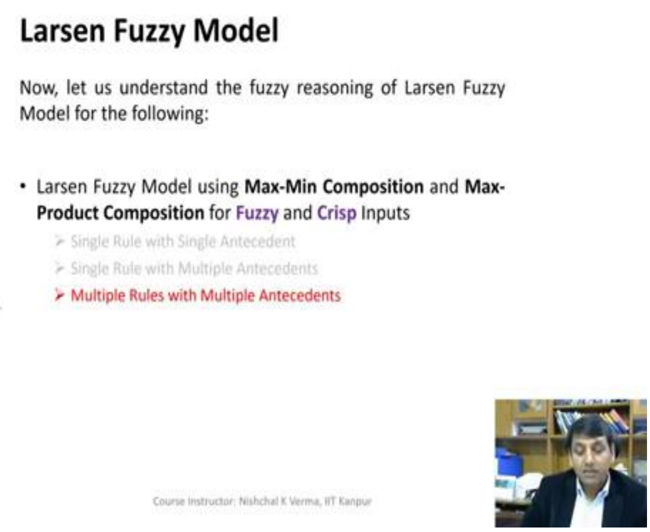

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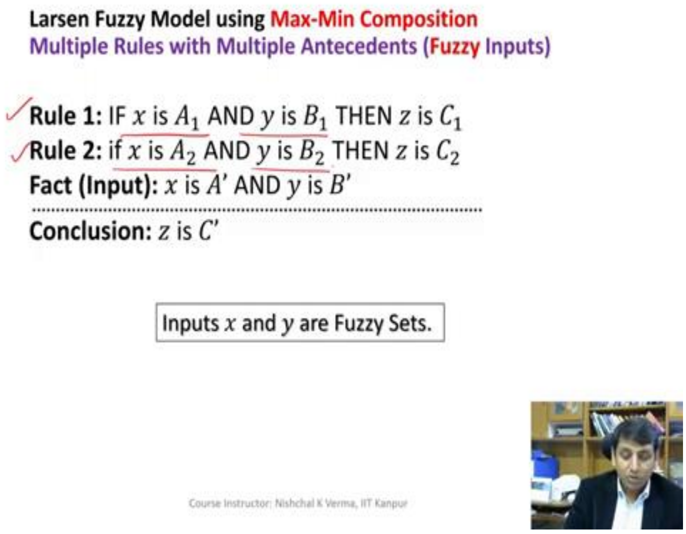

So, since this class is having the multiple rules and multiple antecedents. So, we are taking 2 rules here for simplicity, but we can have multiple rules like n number of rules we can have and similarly, we can have multiple antecedents. So, here also we are taking two antecedents only.

So, 2 antecedents and 2 rules we are taking for simplicity. So, rule number 1 and rule number 2, but if we understand this then we can apply this for multiple antecedents and multiple rules, means we can apply to any number of rules and any number of antecedents.

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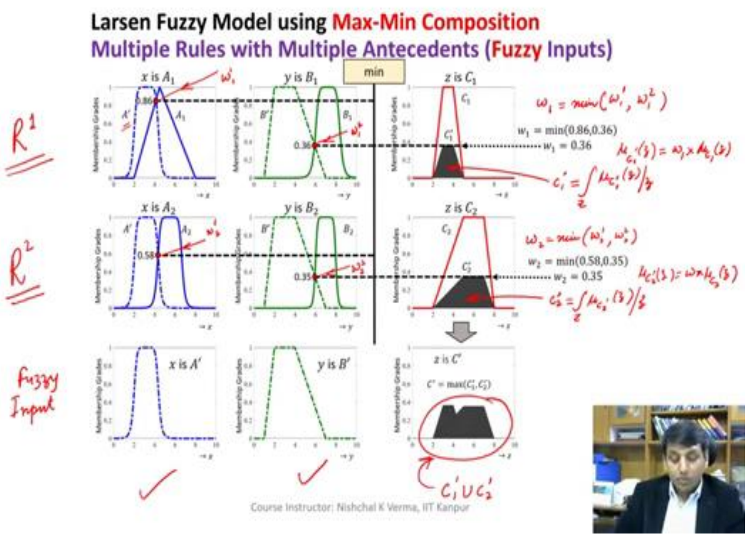

So, here we are taking only 2 rules and let us go to the first case, where we are taking the fuzzy input. So, we have this as the rule number 1 and this as the rule number 2. These two are the rules already given for the fuzzy model Larsen fuzzy model and this dotted one is the applied input.

So, we may not at the moment consider when we are discussing the rule. Now, this is the input, this is the fuzzy input this is a fuzzy input that we supply to the Larsen fuzzy model. And we have this fuzzy input for x this fuzzy input for y means the fuzzy input the fuzzy value A' that means the A' is nothing, but a fuzzy set here and B' is also a fuzzy set.

So, these two fuzzy values are applied fuzzy values are given as the input to the model. So, when we do that, we see that for the first rule let us understand that when we apply this so we superimpose A_1 and A'. So, we superimpose A_1 and A', we get the point of intersection as 0.86 and similarly here for y input we superimpose B' and B_1, B_1 was already there and B' is the given fuzzy input. So, when we superimpose these two or we superimpose B_1 on superimpose B' on B_1 and we see that there is an intersection here point of intersection and this point of intersection is 0.36.

So, we have two points of intersection first is for the x first antecedent and the second one is for the second antecedent. I can call this as w_1 and I can call this as the w_2. Now, since here we are taking max-min composition. So, we will take the min of these two weights w_1 and w_2.

So, when we take the min and since this is for the first rule, where there are multiple rules here means two rules. So, we write the symbol of weight like this. So, upper subscripts here is for rules, so first rule similarly here the upper subscript will be the w^1. So, w^1basically I can ok.

So, let us have this have it like this the symbol is a bit, we can take here in this particular case you can take this as the w_1^1 and w_1^2. So, the lower subscript is for the rule and so the w_1 here is the w_1 is equal to the minimum of the 2.

So, w_1 is equal to the minimum of w_1^1 and w_1^2. So, this way we have the min of the two as w_1 w_1 is 0.36 and this we used to a scale down the output fuzzy set C_1. So, we can click, we can very easily get this value the C_1. So, there is the fuzzy output here this is the fuzzy output this is $C_1{}'$ and this is nothing but $\mu_{C'}(z)/z$ and what is $\mu_{C'}(z)$ is $\mu_{C'}(z)$ is nothing but μ but w into this is $\mu_{C'}(1)$.

So, $\mu_{C_1'}(z)$ is $\mu_{C_1'} = w_1 \times \mu_{C_1}(z)$. So, this is how we get this value of C_1' . Similarly, we find the points of intersection here for the second rule, we call this as the w_2^1 and then we call this as the w_2^2. So, $w_2 = \min(w_2^1, w_2^2)$.

So, this way we get w_2 here and similarly here we get the C_2'. So, this is the fuzzy set and this is nothing, but see, if this is $w_{C_2'}(z)/z$ and $\mu_{C_2'}(z) = w \times \mu_{C_2}(z)$. So, this is how we get the output fuzzy set scaled. Now, since here we have multiple rules so we have to when we apply max-min composition.

So, now the max is relevant here max of max-min composition is irrelevant. So, we have multiple rules here we have two rules. So, both the outputs are now included both the outputs are now accounted and this accounting is done by taking the union of the two. So, we take the union of the two and that is how we are getting this as the output.

So, this is the union of C_1' and C_2'. So, we can write it like this C' and then C_2' and this is our final output in this case. So, now, this output is the fuzzy output and as I have already mentioned that when we are interested in crisp equivalent of this crisp value of this, then we use suitable defuzzification methods and we get the crisp value the corresponding fuzzy set.

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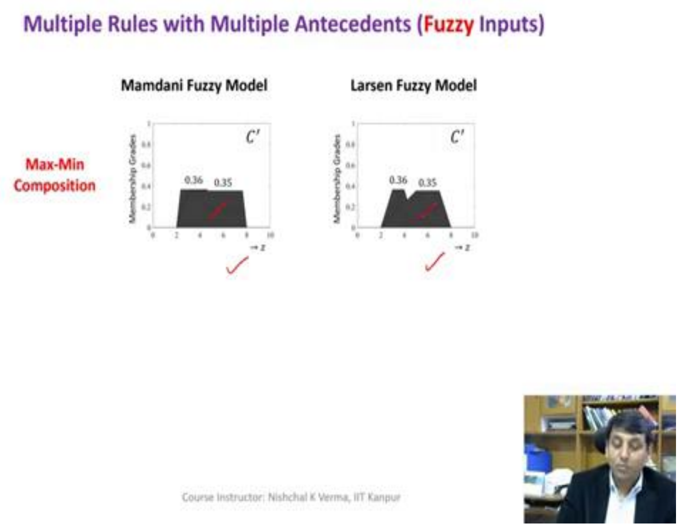

Now, let us compare this output of Larsen fuzzy model with the Mamdani fuzzy model.

So, this means that when we have for the same inputs for the same fuzzy inputs, if you would have used Mamdani model what will would have got against here. So, for the same max-min composition and for the same fuzzy input, we are getting the different outputs. So, Mamdani model is giving us here, this output this fuzzy output whereas, the Larsen fuzzy model is giving this output

So, please look at the outputs and these two are different outputs. Now, let us go ahead and use the other composition. Let us go ahead and use the max-product composition. So, when we use max-product composition.

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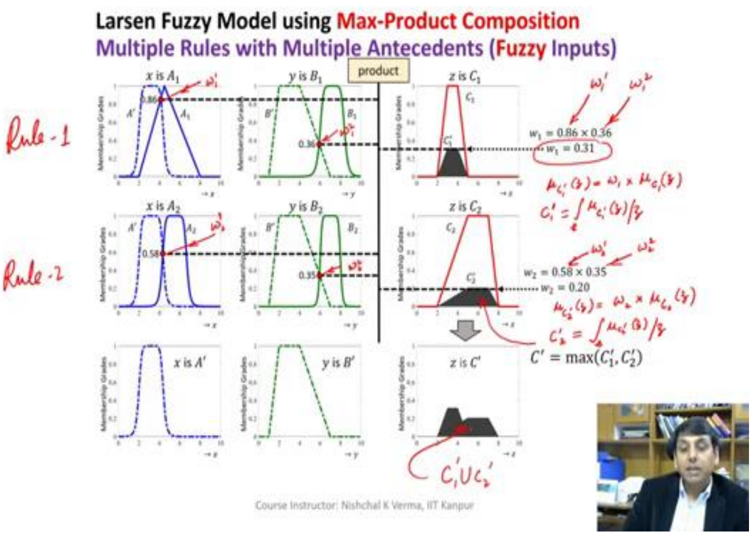

So, since we have already this as the w_1^1 and this we have w_1^2. So, our $w_1 = \min(w_1^1, w_1^2)$. So, we are getting here. No this is the product this is not minimum. So, this is the product. What is this? This is this we have w_1^1 and this is we have w_1^2. So, when we multiply this the value w_1 that is the firing rule strengths is coming out to be 0.31 and we use this value to the scale down the height of the C_1 the fuzzy set C_1 to 0.31.

So, this is how it is done. Now, the new membership function of C_1' the new membership function the scale down membership function $\mu_{C_1'}(z) = w_1 \times \mu_{C_1}(z)$ and then you can write the C_1 here we can write the C_1' the output is scaled down fuzzy set here $\mu_{C_1'}$.

Alright so now similarly, when we apply the second rule when we apply the input the fuzzy inputs to the second rule. This was the rule number 1 the first rule and then we have the second rule. Now, when we apply this w_2 this is the second rule. So, we write w_2^1 and then we write here w_2^2. So, this is w_2^1 this is w_2^2. Similarly, here also we have the value that we are getting as the firing strength of the rule as 0.2.

So, this value will be used to a scale down the C_2 to C_2'. So, the membership function here of this scaled down fuzzy set will be $\mu_{C'}(z) = w_2 \times \mu_{C_2}(z)$ and the $C_2' = \int_z \mu_{C_2'}(z)/z$. So, this is how we get the C_2' as the output the fuzzy output. Now, since we are applying here a max-product composition. So, we take the union of the two outputs.

So, we take the C_1' take C_1' and C_2'. So, we take the union of these 2 and this is what we are getting here as the output. So, we see that we are finally, getting the fuzzy output and this fuzzy output can be defuzzified further to get the crisp output and this is the output that we obtained using Larsen fuzzy model using the third case the third case of the Larsen fuzzy model that is multiple rules and multiple antecedents and this output is with respect to max product composition with fuzzy inputs.

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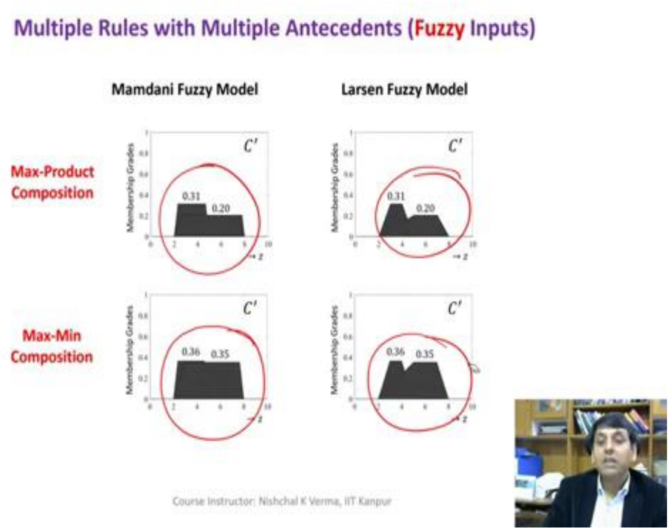

So now, let us compare this case with the this output with the Mamdani fuzzy model. So, had it been a Mamdani fuzzy model you would have gotten this output for the same input for the same composition, that means the max-product composition. So, see here that, we are getting different fuzzy outputs for the same inputs and for the fuzzy for the same compositions.

So, we see clearly that we have different fuzzy outputs and since we have the different fuzzy outputs, obviously, we are going to get the different crisp values as well. Now, let us compare this with the max-min composition and we see that all four are different when we use max-min composition here also the Larsen and Mamdani both are producing different fuzzy outputs.

So, we can see that as to how when we use the same even if the same max-min composition different models are producing corresponding to the same fuzzy inputs different fuzzy outputs and so the crisp out outputs also will be different.

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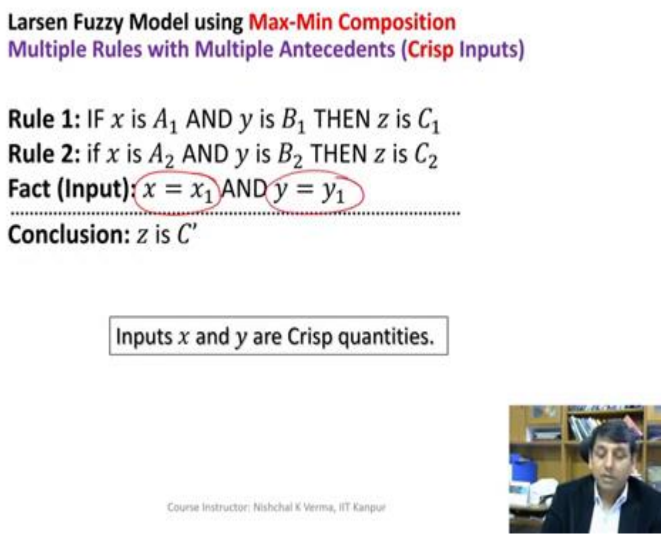

Now instead of fuzzy inputs, let us use crisp inputs and see what happens. So, we have the again for this case also we have two rules and the input here is different that means, x is equal to x_1 instead of the fuzzy set and here we have y is equal to y_1 that means the crisp input instead of the fuzzy input. So, let us see what we are going to get when we apply this crisp input.

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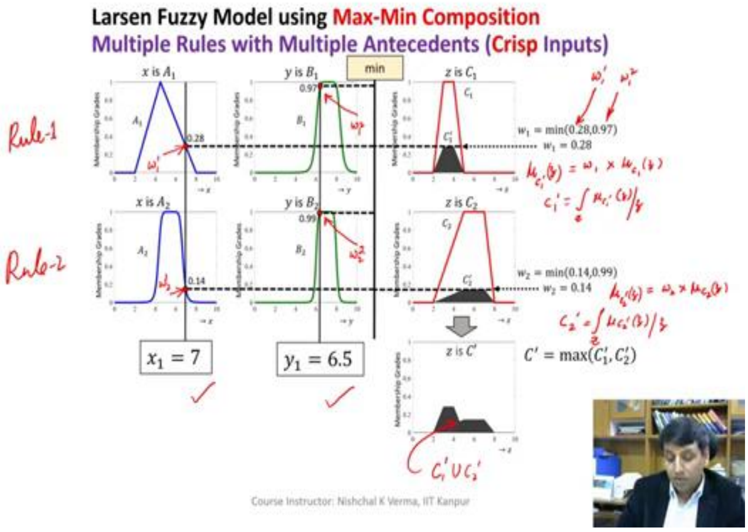

So, here we have let's us assume that x_1 is 7 and y_1 is 6.5. So, when we take this x_1 for the first antecedent, so first antecedent is x is A_1. So, when we take x_1 is equal to 7. So, corresponding to x_1 is equal to 7 we see that this is cutting this is intersecting the A_1 fuzzy set at 0.28 membership value. So this we call as w_1^1, for the first antecedent and first rule this is rule number 1 this is rule number 2.

So, similarly here y_1 is cutting y_1 is also intersecting here at 0.97. So, we call this as w_1^2 the second antecedent and first rule. So, if we use max-min composition, then we have to take the min of these two. So, w_1 is equal to or we can simply write here like this that we have this as the w_1^1 and this as the w_1^2. So, the value the minimum value is coming out to be 0.28.

So, since we are taking max-min composition. So, the minimum here is 0.28. Now, we use this value to scale down C_1 to C_1' means the new fuzzy set is the scale down fuzzy set is C_1' and as I have already discussed as to how we are going to get the membership function of the scaled down C_1' fuzzy set. So, here C_1' will be like this, w_1 multiplied by $\mu_{C_1}(z)$ here also we will have $\mu_{C_1'}(z)$ and then here we will have w_1 into $\mu_{C_1}(z)$.

So, with this we will be getting $\mu_{C_1'}(z)$, the membership function of the scaled down fuzzy set and this is scaled down fuzzy set is C_1' and C_1' is this $\mu_{C_1'}(z)/z$. So, similarly, when we apply the same input same crisp inputs to the rule number 2. So, here we get my w_2^1, 0.14 and w_2^2 as 0.99. So, since we are taking the max-min composition.

So, minimum of the two will be 0.14 and similarly here also the membership function of the scale down fuzzy set will be $\mu_{C_2'}$. So, I can write here $\mu_{C_2'}$ is going to be w_2 multiplied by $\mu_{C_2}(z)$ since this is defined in z. So, we can write here $\mu_{C_2'}(z)$ so like that and then we have this C_2' as $\mu_{C_2'}(z)/z$. So, this is how we get the expression for C_2'. Now, since we are using here the max-min composition.

So, the outputs corresponding to rule number 1 and rule number 2 are unionized is a union of the two outputs are taken. So, $C_1' \cup C_2'$ and this is what is the output that we get when we take union we combine these two. So, either we call this as the union or we taking max. So, this is nothing, but the union of the two outputs corresponding to the rule number 1 and rule number 2.

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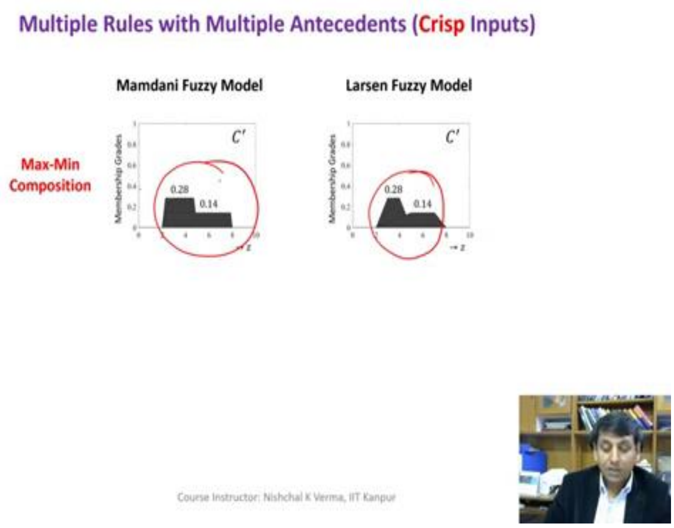

Now, let us. So, since this is fuzzy output. Now, let us compare this fuzzy output of the Larsen fuzzy model with the Mamdani fuzzy model with the same max-min composition and with the same crisp input. So, we see that the outputs again here will differ. So, Larsen fuzzy model produces this fuzzy output whereas the Mamdani produces the different fuzzy output compared to Larsen fuzzy model. So, similarly when we defuzzify this the crisp outputs also will remain the different.

So, let us now for the same input and in the same class that means, the multiple rules with multiple antecedents let us use the max product composition and let us see what we are getting.

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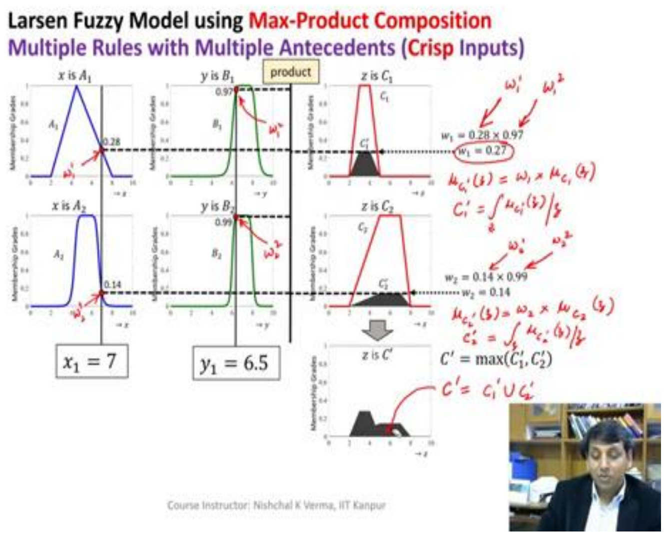

So, here we are again for the same input we are getting w_1^1 as 0.28 as the point of intersection here we are getting w_1^1. So, w_1^2 here as the point of intersection here. So, this part remains the same, the only thing is this multiplication here because we are using max-product composition.

So, we multiply these two the w_1^1 and then w_1^2. This is w_1^2. So, final value that is w_1 is coming out to be 0.27 which is the firing strength of the rule 1 and again it is needless to mention as to how we get the C_1' here. So, what is done here is that the height of C_1 is brought down to or the is brought down to C_1' and the accordingly the whole fuzzy set is a scaled down and this membership function of this fuzzy set C_1' that means $\mu_{C_1'}(z) = w_1 \times \mu_{C_1}(z)$.

So, when we have this then, we can simply write the expression for fuzzy set C_1' and C_1' is $\mu_{C_1'}(z)/z$. So, this is how we write C_1' fuzzy set. Similarly, when we apply this crisp input x_1 is equal to 7 y_1 is equal to 6.5 to the second rule the output is C_2' and here this intersection is w_2^1 and this intersection is w_2^2.

So, this is w_2^1 and this is our w_2^1 w_2^2; w_2^2. Similarly, the membership function of the C_2' so the membership function of $\mu_{C_2'} = w_2 \times \mu_{C_2}(z)$. So, this $C_2' = \int_z \mu_{C_2'}(z)/z$. So, this is how we can get the membership value and the fuzzy set. So, both the outcomes are now maximized the outcomes that union of the two membership two fuzzy sets are taken.

So, here we take the max of C_1' and C_2', we and this is same as this is same as the $C_1' = C_1 \cup C_2$ here and this is the outcome. So, this is the fuzzy outcome and we use the defuzzification methods suitable defuzzification methods to get the crisp output. Now, let us compare this output of the Larsen fuzzy model here and let us compare this with Mamdani.

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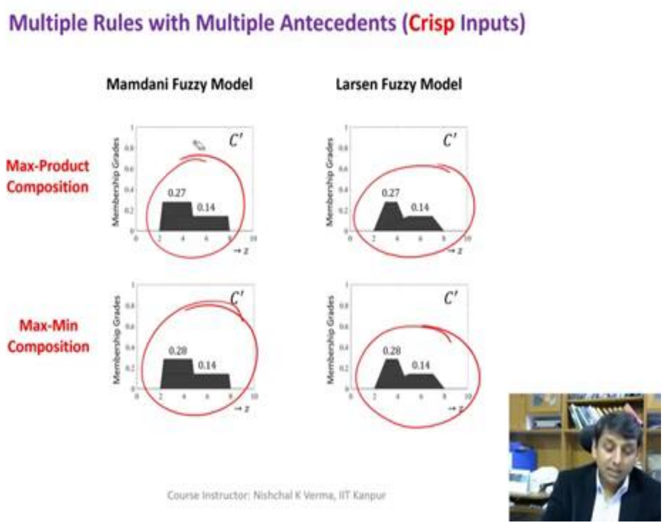

So, had it been the same input same crisp inputs same set of fuzzy rules and same composition that that means, max-product compositions. So, C here the Larsen fuzzy model is giving this output and Mamdani fuzzy model is giving this output which is different from the Larsen fuzzy model for the same crisp inputs and same max-product composition and now let us compare the outputs of the max product composition with max-min composition as well.

So, we see that here we see that here all the four outcomes are different means, the fuzzy values are changing the output of the Larsen fuzzy model in all the cases in both the compositions are different for the same input and same composition and same input and similarly here the Mamdani fuzzy model also we have different outputs. And similarly, since we have the fuzzy values are different. So, the corresponding crisp values are also going to be different.

So, we see that, we have the outputs fuzzy outputs in as the result of Larsen fuzzy model with crisp model crisp input and max-min composition and then the same for max-min composition and we see that the results are the fuzzy outputs are different. The different the outputs are different and then accordingly, we can say that the crisp values are also going to be different.

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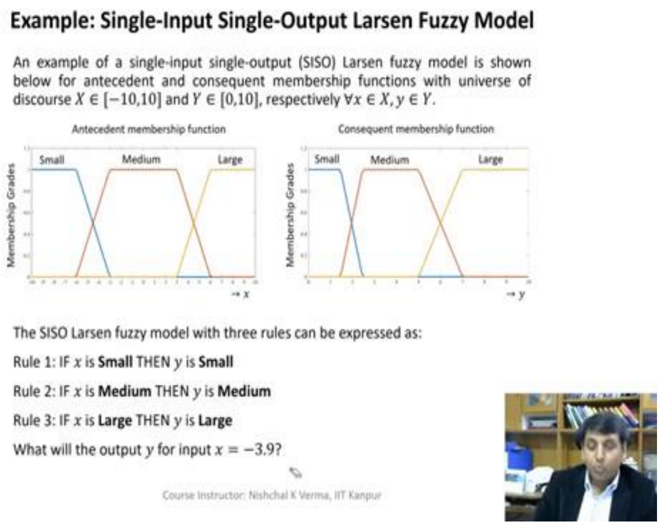

Now, let us take a simple example a very simple example here of the case the single input and single output Larsen fuzzy model.

So, here we have the single input it means we have the single antecedent, but here we have multiple rules. We have single antecedent, but multiple rules and in this case we have the single input single output that means the SISO Larsen fuzzy model and which is shown here for antecedent and consequent membership functions with universe of discourse. x belonging into belonging from -10 to 10 and y from 0 to 10 respectively for every $x \in X$ for every $y \in Y$.

So, we see here that the x is having three fuzzy regions small, medium, large and all these fuzzy regions are defined by or represented by the corresponding fuzzy sets for small for medium for large. Similarly in consequent part that means, the output is also divided into

y is also divided into three fuzzy regions and every region is represented by a fuzzy value fuzzy set. So, small, medium, large.

So, we have the input and we have the output and here we have the three rules of the model which is given. So, rule 1 says if x is small then y is going to be in the small means, if any input x which is falling in the small region then the output has to be fall has to fall in the small region only. Similarly what rule 2 is saying is if x is medium means x is going to be medium.

Then y is also going to be in the medium means what does this mean exactly is if any input x is falling in the medium region then y will also be falling in the medium region. Similarly for rule 3 if x is large means if the x values if the input x is falling into large region large fuzzy set region, then the corresponding y will also fall in the large region. So, these are very simple case to make you understand. So, if we have these three rules present for this fuzzy model for this SISO fuzzy model for this SISO Larsen fuzzy model, then let us find the output corresponding to the input x is equal to -3.9.

So, let us apply this input. So, please understand that the output input here is the input that we are giving to the model is a crisp input. So, we have x is equal to -3.9 and this is a crisp input. So, this input when we supply to the Larsen fuzzy model, let us see what is the output that we are going to get corresponding to this input.

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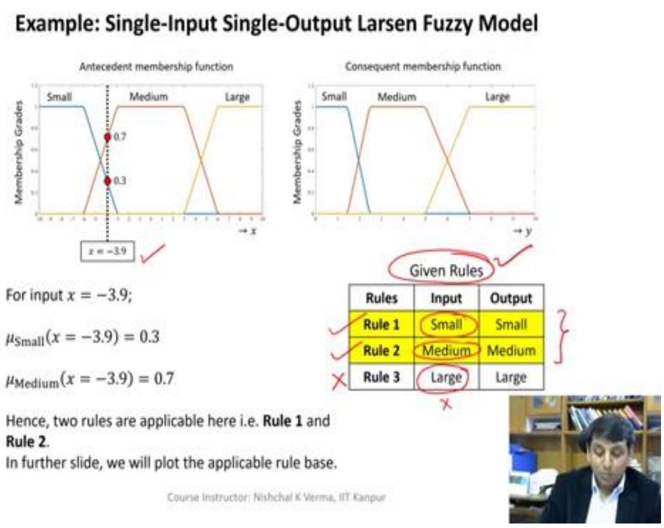

So, when we apply this input, we see that corresponding to x is equal to -3.9, we get here the two points of intersection. So, we see that a small region is cut at one place a small region a small fuzzy set is cut at is intersected at one place by x is equal to -3.9 line and similarly the medium is also intersected at 0.7 and then when it comes to large is not at all affected.

So, the input that we are supplying here which is x is equal to -3.9 is applicable is falling in to two regions a small and medium it is not falling in the large region. So, large is large fuzzy region is irrelevant for this input. So, let us now further understand that, corresponding to this input we are getting 0.3 as the point of intersection in a small region and 0.7 in medium region. Now, let us check this in the given rules.

So, let us now look at the rules and see whether the rules are applicable for this input or not. So, we have we have the input x is equal to -3.9 which is falling in the small region. So, if it is falling in the small region, we have the given rule we see here and in given rule if we if our input is falling in the small region. So, we have the output also in the small region.

So, this means that the input since our input x is equal to -3.9 is falling in the small region. So, this means rule 1 is applicable, no matter what is the output. So, we will first see the input. So, input is falling in this small region, then we see the input is also falling in the medium region. So, rule 1 and rule 2 are applicable and rule three is not applicable because the x is equal to -3.9 is not falling in the large region.

So, this is this rule is not applicable so only two rules are applicable. Now, let us proceed with these two rules.

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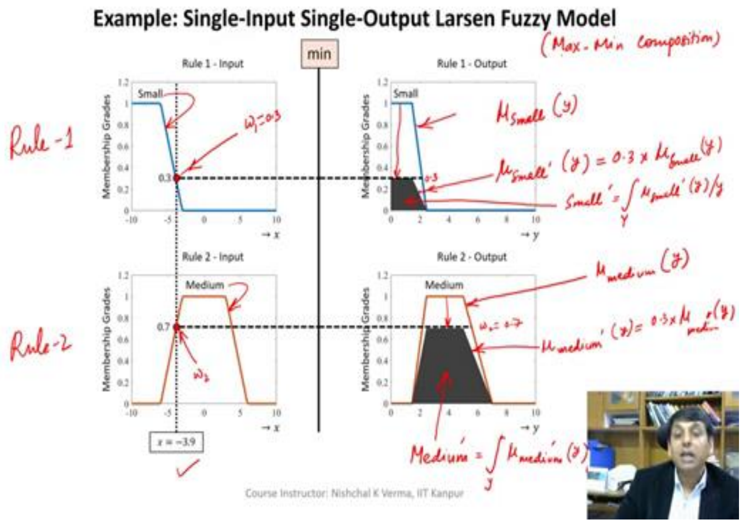

So, we have the small fuzzy set here the fuzzy set for a small here and then we have the fuzzy set for medium here this is medium, this is medium fuzzy set and this is small fuzzy set and these two are given. So, since we have two rules here, rule number 1 let us now apply the input.

So, for rule number 1 and then we have the rule number 2. So, when we apply the input here we find here for the first rule we are getting the point of intersection as the weight which is 0.3; 0.3. Now, if we apply Larsen fuzzy model. So, the height of this output fuzzy set that is small here in this case we have to bring it down to the 0.3 value here.

So, and the membership function, the new membership function let us say the membership function of the small here the small here let us say this is y and the membership function of they small dash let us say will be the w; that means, the $0.3 \times \mu_{Small}(y)$.

So, similarly here, when we apply a rule number 2 we get here the point of intersection as w_2 and we scale down the medium to medium height of the medium to w_2; that means, the w_2 is equal to 0.7 and this is the outcome that we get a scale down fuzzy set and the corresponding membership function. So, let us call this as the dash medium dash

So, this is y so μ_{Medium}. So, this will be basically, $0.7 \times \mu_{Medium'}(y)$. And here we have the membership function mu medium let us say y and this is dash. So, this is how we are getting the scale down fuzzy set and this is the membership function is the membership function. We can get the fuzzy set a $Small' = \int_y \mu_{Small'}(y)/y$ similarly here the fuzzy set and so this will be the fuzzy set basically, this is the fuzzy set and here this fuzzy set there the mu medium this is fuzzy set that you write it here. This fuzzy set is μ, let us say $Medium'$ and this $Medium' = \int_y \mu_{Medium'}(y)/y$. So, that is how we get the scale down membership function and the corresponding fuzzy set for medium dash like scale down medium dash scale down fuzzy set.

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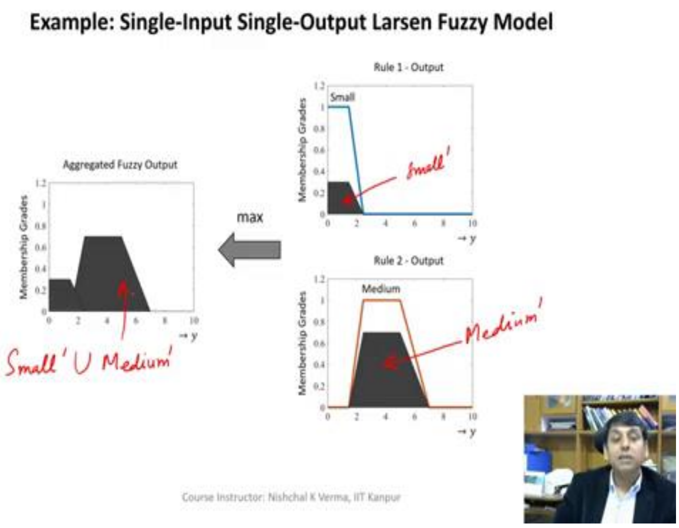

So, the same is shown here. Now, since we are using max-min composition. So, this is for max-min composition here, this is for max-min composition. So, we use max-min composition. So, minimum we have already taken here the minimum was not applicable because, we have only one antecedent. So, max-min composition or max-product composition both will give the same result because, we have only one antecedent here.

So, that is not really going to make any difference. So, I can simply remove this because this is applicable to both min and max-min and max-product composition. But finally, in both the cases we have to take the max of the two output. So, both the output and what is this output this is the small dash here this is a $Small'$ and this is $Medium'$ as the fuzzy output as the fuzzy output. So, both these outputs are now taken as the union.

So, I can write here as a small dash union medium dash. So, this is what is the output. So, this is finally, again the fuzzy output.

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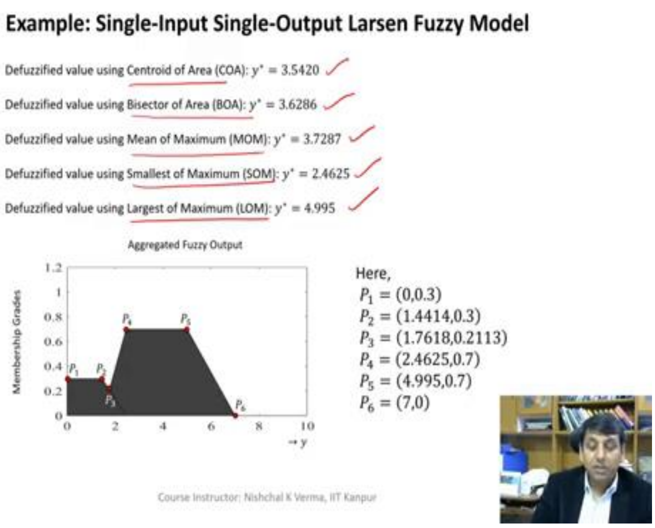

Now, we can use any suitable methods of defuzzification to get the crisp value. So, here this is the output and fuzzy output and when we use the centroid of area the defuzzification gives the y^* as the crisp value. So, y^* is 3.5420, the formula I have already discussed all these defuzzification I have already discussed.

So, if we use all these, we will get the center of area if you use center of area the same fuzzy value the same fuzzy quantity is giving us different crisp values. So, when we use center of area method of defuzzification, we get 3.5420, when we use the bisector of area we get 3.6286 as the crisp value, when we use max of mean of maximum we get 3.7287 as the crisp value, when we use the smallest of maximum then we get 2.4625 as the crisp value.

And then when we use the largest of maximum here we get a 4.995 as the crisp value for the same fuzzy output. And here it is shown the fuzzy output that we have got is shown and their vertices also all these vertices $P_1, P_2, P_3, P_4, P_5, P_6$ are shown here, is you are interested you can use these vertices to compute and compare the and match the results that we have obtained.

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So, we have discussed in today's lecture. So, many things and I hope you enjoyed the lecture in detail I have discussed the Larsen fuzzy model using max-min composition and max-product composition for both the inputs fuzzy and crisp. And all these three cases we have discussed, when we have the Larsen fuzzy model with single rule with single, single antecedent and then this was the first case.

The second case was that we discussed was single rule with multiple antecedents and the third case that we discussed for Larsen fuzzy model was the multiple rules with multiple antecedents and this covered almost all the cases that we could apply. And with this I will stop here and in the next lecture we will discuss the Tsukamoto fuzzy model.

Thank you very much.