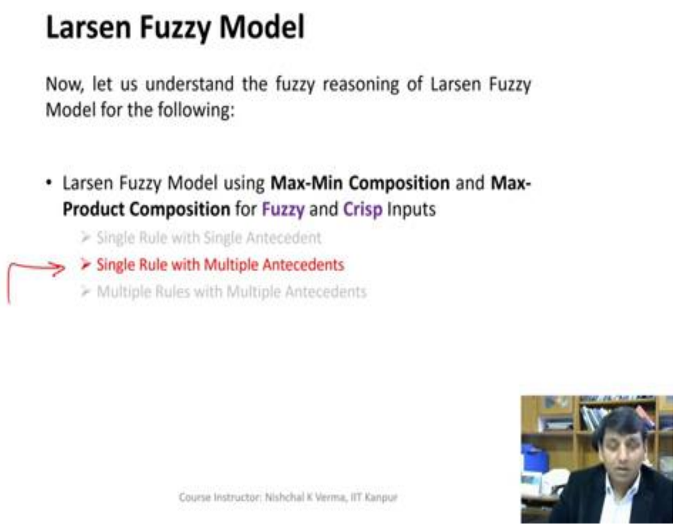
**pFuzzy Sets, Logic and Systems and Applications**
**Prof. Nishchal K. Verma**
**Department of Electrical Engineering**
**Indian Institute of Technology, Kanpur**

**Lecture - 57**
**Larsen Fuzzy Model**
**(For Single Rule with Multiple Antecedents)**

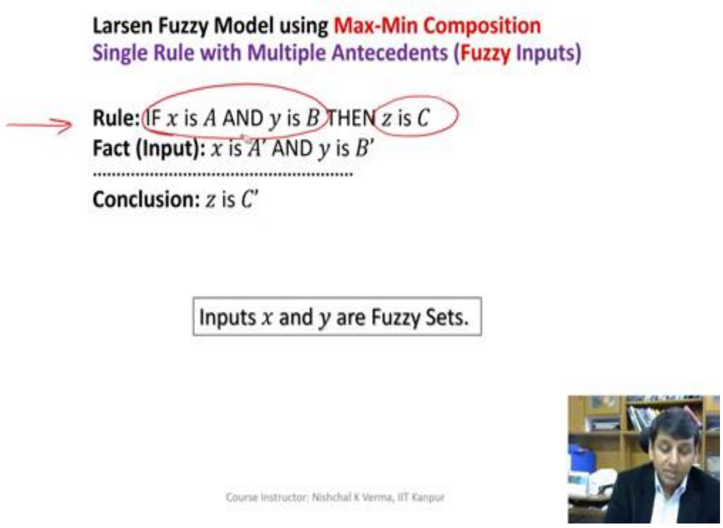
Hi, welcome to the lecture number 57 of Fuzzy Sets, Logic and Systems and Applications, in this lecture I will continue our discussion on Larsen Fuzzy Model. Here in this lecture I will discuss the single rule with multiple antecedents for Larsen fuzzy model.

(Refer Slide Time: 00:29)




Now, coming to the second case here of the Larsen fuzzy model and in this second fuzzy second case, second case of Larsen fuzzy model. We have the model which is characterized which is defined by single rule only one rule with multiple antecedents. So this is very important to note. So, we already know that how will the rule of this kind of model will look like.

(Refer Slide Time: 01:04)

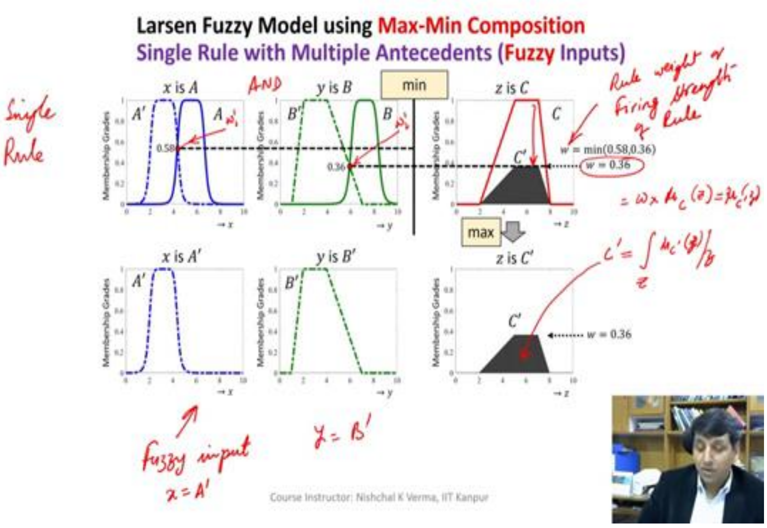



So, we see that we have the rule here and this rule, since the model is defined by only one rule. So, you see only one rule we have a single rule, but this model is defined by single rule with multiple antecedents. So, we can have multiple antecedents for simplicity we have here only two antecedents but it can be even more antecedents.

So, we have here two antecedents and both the antecedents here $x$ is $A$ and $y$ is $B$ are connected with any with a connective that is AND here, but it can be OR also, O-R. So, either we use AND connective or we use OR connective or we use or we can use any other connective also in between the two antecedents, any two antecedents.

So, we see that this kind of rule will look like this, this kind of fuzzy rule will look like this. So, if $x$ is $A$ and $y$ is $B$ then $z$ is $C$. So, this is our premise part and this is our consequent part. So, we have in premise part to antecedents as I have mentioned. Now, if we have a Larsen fuzzy model which is characterized by this rule this kind of rule; now if any unknown input comes to this model. So, this input can be either fuzzy or crisp.

(Refer Slide Time: 02:50)




So, let's now discuss this one by one. So, here in this case we have the fuzzy rule the single rule is our single rule and we have multiple antecedents. So, we have first antecedent and then we have the second antecedent, this must be noted here that we have in this kind of multiple antecedent we have only two antecedent. We can have so many other antecedents also.

So, I mean we can have few more antecedents are more antecedents. So, here for simplicity I have taken only two antecedents. So, first antecedent here is $x$ is $A$ and the second antecedent is $y$ is $B$ and both are connected with the connective AND. So, we can we could have as I mentioned we could have any other connective like OR, OR.

So, let us now apply the fuzzy input first. So, here we have the this as the fuzzy inputs. So, we are now supplying Larsen fuzzy model this fuzzy input. So, I can write this fuzzy inputs. So, if we have supply in this fuzzy input as $A'$, $x$ is $A'$ which is fuzzy and $y$ is $B'$ which is also fuzzy.

So, when we supply this input and so, $x$ is going to the $x$ part means $x$ antecedent first antecedent and then $y$ is $y$ the fuzzy input $B'$ is going to the second antecedents. So, when we superimpose here these two fuzzy sets like $B'$ and $A$ we see that we are getting the point of intersection. So, we are getting only one point of intersection as I mentioned earlier that we could get many points of intersections. So, in those cases normally we get two. So, in all the cases if we are getting multiple points of intersection then what we do we take the maximum of this.

So, here since we are getting only one point of intersection we call this as w 1 and here we are getting the another intersection. So, let us call this as $w_2$ of rule 1, of rule 1. So, I can write here w 1 and here I can write; so, $w_1^1$ and $w_2^1$. So, then what we are getting here is that we take the min of these two weights, as I have already mentioned that the we call either weights or we call the firing strength of the rule.

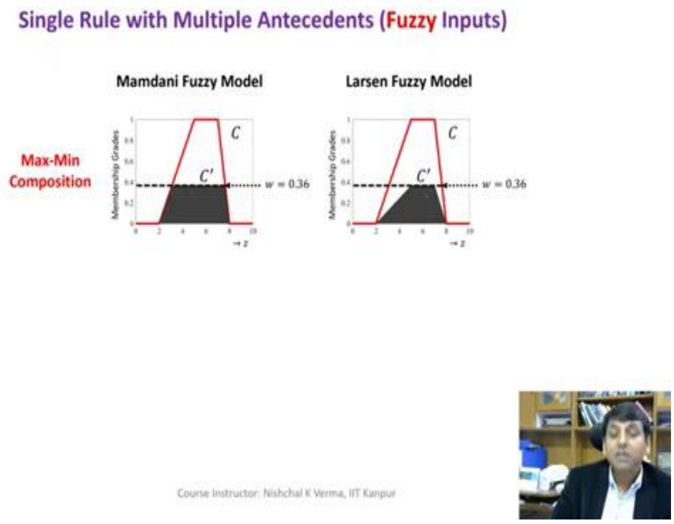
So, what is the firing strength of the rule is $w$ here this is the final firing strength of the rule is called the rule weight. The rule weight or firing strength of rule all right. So, now, since we have 2 weights here to membership values here. So, these are nothing, but the membership values or grades. So, we take the min because we are using max-min composition here. So, when we take max min composition we use the min of these 2. So, we see that when we take min we are getting this value.

So, min is min of the 2 will be 0.36. Now, we will use this to a scale down the height of the fuzzy set $C$. So, output fuzzy set is $C$. So, we will bring this down to here to this value 0.36. So, let me repeat here that the height of the fuzzy set we will bring down to the 0.36 and we know how did we get this value this value that is 0.36. So, we bring the height here of the $C$ fuzzy set to 0.36 and early mentioned that we can get it very easily.

So, what we do here we just take this w as 0.36 and then we multiply it with simply $\mu_B$ as the membership function of the fuzzy set $\mu_B$ which is characterizing the output fuzzy set $C$. So, this is the, this $\mu_C$ not $\mu_B$. So, w into $\mu_C(z)$ and this is going to give me $\mu_{C'}$ there is going to give us $\mu$, I can write here $\mu_{C'}(y)$. So, this is how we get $\mu_{C'}(y)$ and then from here we can write the $C$. So, what is $C'$ here? C dash will be nothing, but the fuzzy set defined in terms of $\mu_{C'}(z)$ now is this is not $y$ this is $z$.

So, this is z and then here we have $z$ a $z$ this is $z$ this is also $z$. So, this is how we get this fuzzy set as the output. This is $y$ is equal to $B'$. So, this is very simple and this we have seen that we have a Larsen fuzzy model which is defined which is having only one rule, but with multiple antecedents. So, when we supply the fuzzy value as inputs. So, this is how we are getting the fuzzy output and this fuzzy output is the $C'$ here in this case.

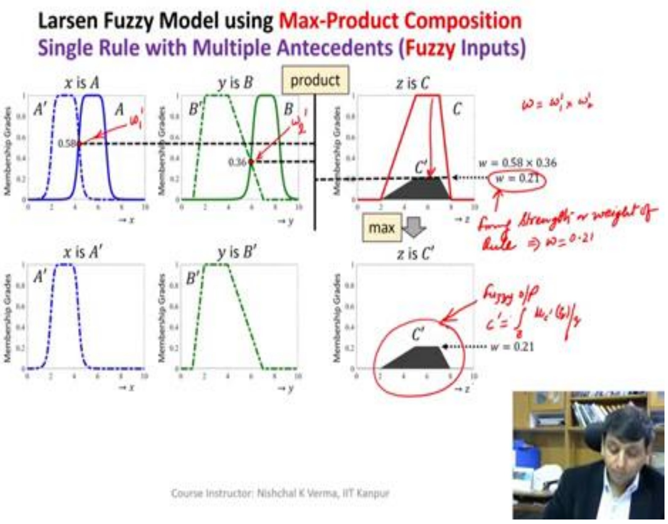
(Refer Slide Time: 10:24)




So, here we have compared here means we are comparing the output values for the same input when we use Mamdani fuzzy model and Larsen fuzzy model we get here in this case our $C'$ like this and here and when we use Larsen fuzzy model we get this output.

So, we see that when we use Mamdani fuzzy model the w that we have gotten the weight of the rule this firing strength of the rule that we have gotten is used to simply chop off the fuzzy set output fuzzy set. Means the to truncate the fuzzy set, but here in Larsen fuzzy model the same value is used to reduce the to scale down the output fuzzy set. So, this is how we can see we can compare the outputs of the Mamdani fuzzy model and Larsen fuzzy model for the same input and here the input is the fuzzy input.

(Refer Slide Time: 11:48)

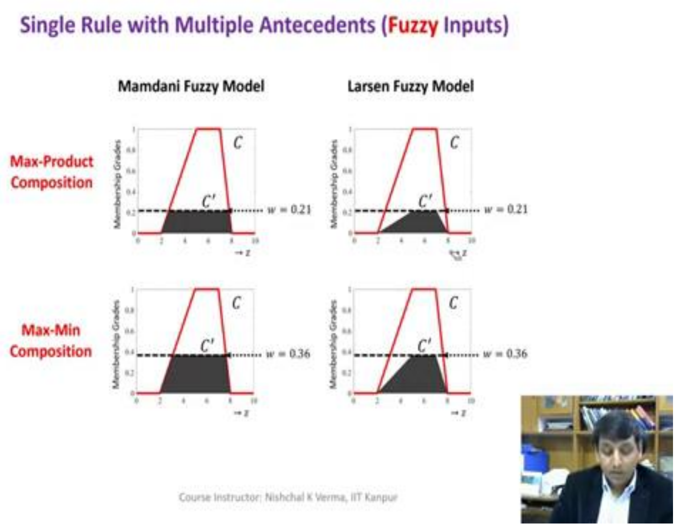



And, here in this case let us use the other composition other composition that is max-product composition instead of max min composition. So, since we have multiple antecedent here we have in this case in a particularly in the rule that we have here is having two antecedent which is the case of multiple antecedent. So, here the max product composition will give us the different result because there in the first case first composition we have taken the max-min.

So, we took the firing strength the minimum of the firing strength minimum of the firing minimum of the weights and which was in the previous case 0.36, but here the inputs remain the same for the same input we are getting here these values as $w_1^1$ and here we have $w_2^1$. So, here since we are using max-product composition. So, we have to take the product of these two. So, simply w will be nothing, but $w_1$, I can write it $w_1$ $w_1$ and. So, here is also $w_1$. So, there is 1 and then we have the $w$. So, w so here this is $w_1$ there is for the first rule a single rule that is why.

So, whether we write it or not write it does not make any difference. So, let us not use 1 ok. So, when we multiply we get multiplied the two $w_1^1$ and $w_2^1$ we get the rule of the firing a strength of the rule.

(Refer Slide Time: 14:25)




So, this is firing a strength or the weight of rule which is $w$ is equal to which is $w$ is equal to 0.21 here we have got 0.21 because they have multiplied the two points of intersections these values. So, this is less than the previous case where we use max-min composition. So, please note this and with this now when we scale down. So, the output. So, let us now a scale down.

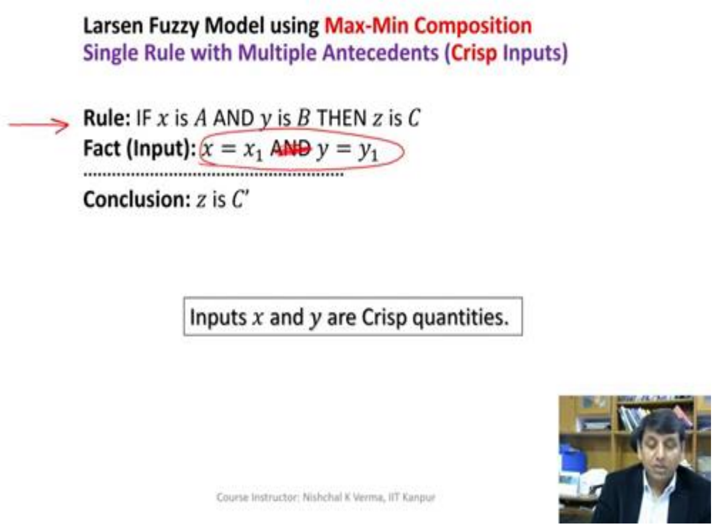
So, when we scale down the output here to let us a scale down this to 0.21. So, this is scale down here. So, what we are getting here is this output because we have a single rule. So, max does not make any difference see max of this will remain the same. So, here we are getting this as the output the fuzzy output corresponding to the fuzzy inputs. So, this is this is the fuzzy output and this is nothing but the $C'$ and what is $C'$, $C'$ is nothing but the fuzzy set defined by here $\mu_{C'}(z)/z$. So, this is how it is defined.

Now, let us compare this output which the Mamdani fuzzy model let us compare the Larsen fuzzy model output with the Mamdani fuzzy model and we can clearly see the difference here the rule strength the firing strength of the rule of the weight here is the same in both the cases and that the same rule strength the firing rule strength we when we truncate the output fuzzy set we get this as the output. Whereas, here in Larsen fuzzy model we scale down the height we scale down the fuzzy set. So, we see the difference in the output for the same fuzzy input for the same fuzzy input all right.

So, now let us compare the Larsen fuzzy model output again in the case of max-min composition. So, we have two compositions that we have used here. So, here since we have the antecedents multiple case and here in our case we have two antecedents. So, since we have two antecedent. So, max-product and max-min will make the difference. So, here we see the Larsen fuzzy model with max product composition the output and here we see the max min composition but for the same fuzzy inputs.

So, we see the difference as to how we get the output for the same fuzzy inputs and we see here again the difference in between the Mamdani fuzzy model output and the Larsen fuzzy model output.

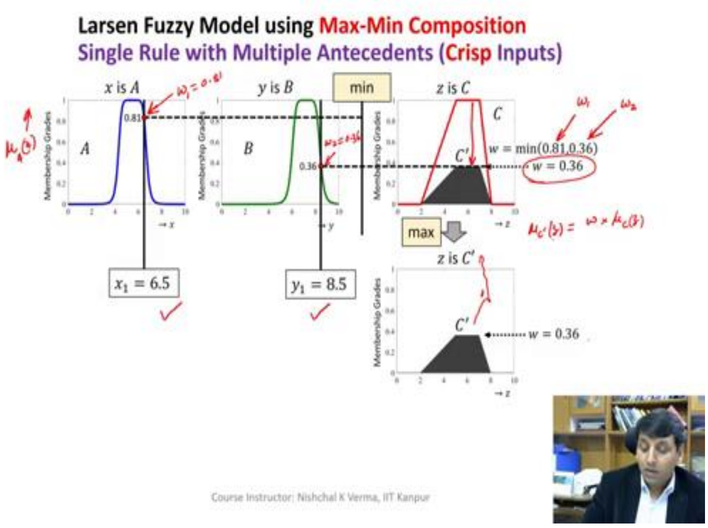
(Refer Slide Time: 18:44)




Now, let us go to the crisp inputs here. So, in the second case itself let us use a crisp input instead of fuzzy inputs and see what we are getting. So, the rule remains the same because we are dealing with the Larsen fuzzy model with single rule and multiple antecedents. So, this remains the same here and now when we supply the input to this model here $x$ is equal to $x_1$ and $y$ is equal to $y_1$.

So, both the inputs we are supplying here and both the inputs are crisp inputs both the inputs are the crisp inputs.

(Refer Slide Time: 19:51)



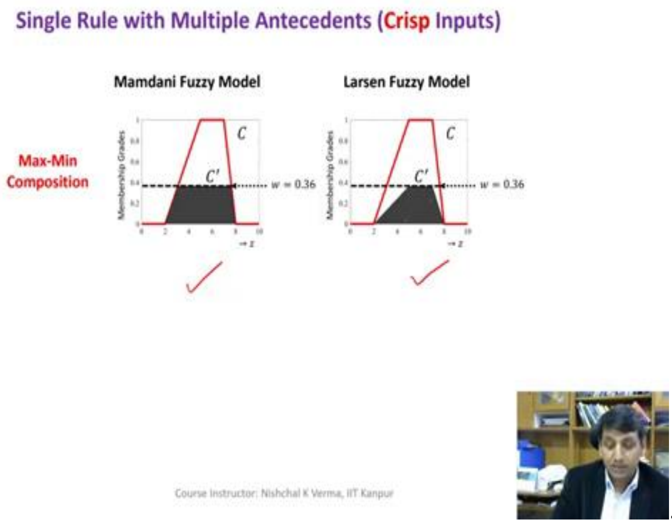

So, let us see what we are getting as the output when we use the Larsen fuzzy model using first using max-min composition. So, let us take the two crisp values, that means the our $x_1$ is let us say 6.5 here and $y_1$ here is the 8.5 as the crisp values. So, you can choose any value which should be applicable, which should be falling within the fuzzy region and should be further should be in the universe of discourse within the universe of discourse.

So, here in this case we are taking for simplicity $x_1$ is equal to 6.5 and $y_1$ is equal to 8.5. So, we see when we take the $x_1$ is equal to 6.5 and when we draw a line parallel to the $y$ axis that means, the parallel to the ordinate that means, the $\mu_A(x)$ here. So, we get the point of intersection here as mu as $w_1^1$, let us say $w_1$ and since we have only one rule. So, we did not write this. So, let us say this is $w_1$ is equal to this and $w_2$ is equal to, $w_2$ is equal to 0.36.

Now, since we are using the max min composition. So, let us take the minimum of these two when we take minimum of the two yes. So, this was nothing, but the $w_1$ this was nothing, but the $w_2$. So, when we take the min of these two we are getting 0.36 here, now again as we have done in the previous slides since we are dealing with Larsen fuzzy model. So, we have to a scale down the height of the original fuzzy set of the output. So, $C$ that is $C$ here the fuzzy set. So, the height of the fuzzy set we will a scale down to we will bring down to here to this value $w$.

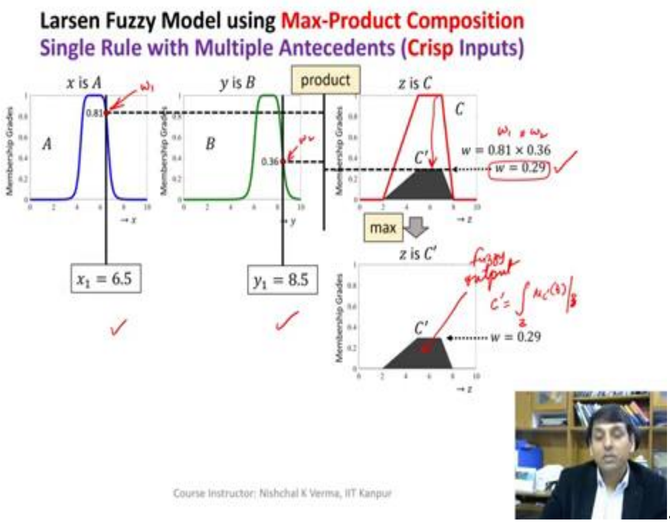
So, this $w$ is equal to 0.36. So, we will use this value here and we will bring the height here accordingly the fuzzy set will be scaled. So, the formula that we use here is we use we get the new fuzzy set like this the membership function of the new fuzzy set let us say $\mu_{C'}$ let us say $\mu_{C'}(z)$ and this is nothing, but the $w$ into $\mu_C(z)$ ok. So, this way we will get the new the output membership function and here this fuzzy is the output fuzzy set here will be $C'$ and this we can represent by simply $\mu_{C'}$.

(Refer Slide Time: 23:28)




And since we have only one rule. So, max is really not applicable here is whatever outcome that we are getting here will be will remain the same. So, now, let us compare this Larsen fuzzy model output for the input that we have supplied with the Mamdani fuzzy model. So, let us now compare we see here that the for max min composition we get Mamdani fuzzy model here truncated output truncated fuzzy set whereas, in Larsen fuzzy model we get the scale down fuzzy set.

(Refer Slide Time: 24:37)

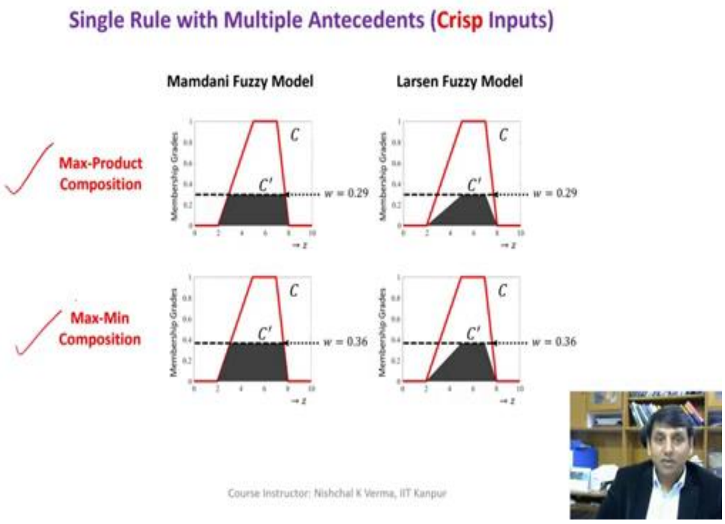



Now, let us move ahead and for the crisp input let us use the max product composition. So, we see that here in max product composition here let us quickly use this. So, since we have already found the points of intersection 0.81. So, we can write here as $w_1$ and we can write here as the $w_2$.

So, here in max product composition is simply instead of taking the min we will take product of these two. So, we take the product here $w_1$ into $w_2$ and we get the rule strength the firing strength of the rule or the weight final weight of the rule is equal to $w$ is equal to 0.29 you can see here. Now, this value will be used to scale down the original fuzzy set which is the output fuzzy set to this value to $C'$ we call this as the $C'$.

So here so this way we scale down the output fuzzy set and we have already seen that as to how we write the output final output. So, here since we have the single rule. So, max is really not applicable we have only one the whatever output comes will remain. So, fuzzy output is $C'$ and $C'$ is what the $C' = \int_z \mu_{C'}(z)/z$ and this is how this will be represent that this output will be represent. Now, let us compare this output with the output of Mamdani fuzzy model.

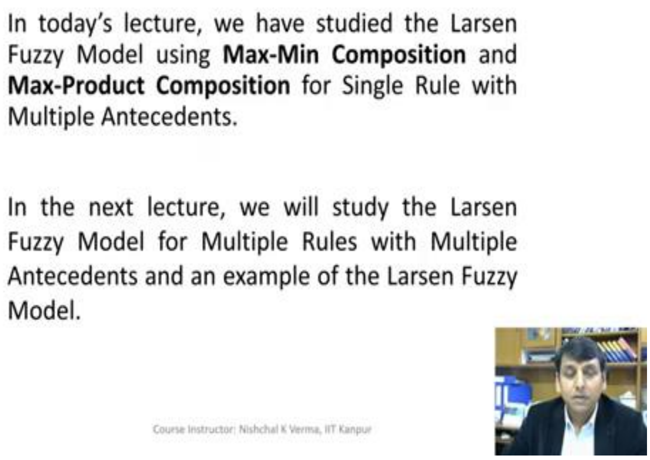
(Refer Slide Time: 27:13)




So, we can clearly see that when we have used the max product composition. So, how we get the output in case of Mamdani fuzzy model and in case of Larsen fuzzy model for the same crisp inputs. Now, let us compare all the outputs like when we have supplied the crisp inputs and when we have used the max product composition and max min composition, how these two models Mamdani fuzzy model and Larsen fuzzy model are giving the fuzzy output.

And as I as I have already mentioned that this fuzzy output is not the final output basically, the further the defuzzify this fuzzy output using the suitable defuzzyfication method to get the crisp value. So, we can here see that the you can compare the outputs of the Mamdani fuzzy model and Larsen fuzzy model for the crisp inputs for the same crisp inputs and in case of the max-product composition and the max-min composition.

(Refer Slide Time: 28:30)




In today's lecture, we have discussed the Larsen fuzzy model using max min composition and max product composition for single rule with multiple antecedents. So, with this I would like to stop here and in the next lecture we will discuss the Larsen fuzzy model for multiple rules with multiple antecedents and we will discuss the example also on the Larsen fuzzy model.

Thank you.