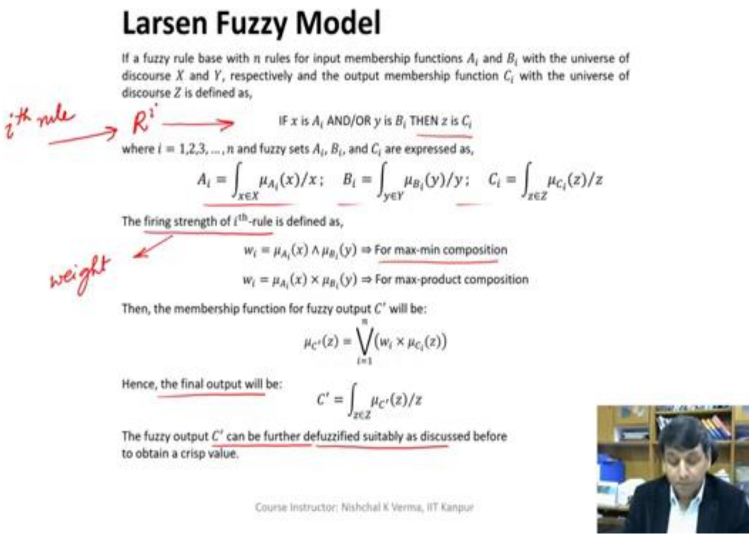
Fuzzy Sets, Logic and Systems and Applications
Prof. Nishchal K. Verma
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 56
Larsen Fuzzy Model

(Refer Slide Time: 00:14)



Hi, Welcome to lecture number 56 of Fuzzy Sets Logic and Systems and Applications. Today in this lecture I will discuss Larsen Fuzzy Model. We have already covered the Mamdani fuzzy model and now in today's lecture, I will discuss with you the Larsen fuzzy model which is very similar to the Mamdani fuzzy model, in the sense that the output here is the fuzzy.

So, if we have fuzzy rule base with n number of rules and our input membership functions could be A_i, B_i and so on with the universe of discourses X and Y and so on respectively. So, the output membership function of the resultant fuzzy set let us say here in this case C_i, if we are taking only two antecedents where the first antecedent is x is A_i and the second antecedent here is y is B_i and both are connected by the connective either A-N-D AND, or, O R. So, both the antecedents are connected here by the connective A-N-D or O-R.

So, here we have taken a typical fuzzy rule the typical fuzzy rule here is that as you see if x is A_i and then we have the connective AND oblique or so, either of these could be and then the other antecedent here is y is B_i. So, these two are the antecedents and then we have then part; that means, the consequent part starts now. So, then z is C_i, so here we have the consequent part and this is the premise part or antecedent part.

So, we have a rule a fuzzy rule where we have two antecedents, here this can have multiple antecedents or even the single antecedents also. So, we can have multiple cases which we have discussed in the coming slides and here in this particular rule we have two antecedents, first antecedent and the second antecedents and then we have the connective here. So, these two antecedents here will be connected with any suitable connective either AND or by OR.

So, as I already mentioned that this premise part can have either only one antecedent or it can have multiple antecedents. So here for simplicity, we are taking only a typical rule with two antecedents and then we have in the consequent part a fuzzy output. So, in Mamdani fuzzy model also we saw the rule set and these rule sets were basically having the antecedent part fuzzy and the consequent part fuzzy.

So, here also we have for the Larsen fuzzy model the set of fuzzy rules that will be applicable will be the antecedent part fuzzy and the consequent part fuzzy. So, both Mamdani and Larsen will have the same kind of fuzzy rules applicable. So, here A_i in A_i and B_i we have written, see here and this A and B are nothing but the fuzzy reasons or fuzzy value corresponding to the input x and B_i is the fuzzy value which is nothing but fuzzy set corresponding to y. i here signifies the this fuzzy set for i^{th} rule.

So, if this is the i^{th} rule I can write here R_i, or in other words I can write here R_i. So, I represents the i^{th} rule I can write here the i^{th} rule. Now if we have let us say n number of rules, so if my A_i is like this, B_i is like this, we know this just the A_i is a fuzzy set for i^{th} rule, B_i is the fuzzy set for antecedent of the i^{th} rule, similarly C_i is the fuzzy set for the output.

Now, the firing strengths of the i^{th} rule here is defined as we have already done in Mamdani. So, firing strength is also known as the weight. So, I have use weight word in the Mamdani fuzzy model. So, weight and firing strength both are the same. So, here weight we write by w_i, w subscript i. So, if I have let us say first rule so, for the first rule I will have the w_1 as we have written here written there in case of Mamdani.

So, w_i we can find by simply taking the min of these two antecedents corresponding membership values and these we could find by the points of intersection. So, we can have max min and then we can have max products. So, here if we have let us say two antecedents. So, if we have two antecedents are more in antecedents right. So, for two or more antecedents in max min, we will have the we will take here for max min we will take min and in case of max product, we will multiply simply. We will take the product of the antecedent values the weight values to get the final weight.

So and when we have done this then comes the membership function for the fuzzy output C'. So, let us say we have here in the as in this case we have the output fuzzy set C. So, then if we have C it and accordingly after the truncation with the help of the weight values are firing strength of the rule, we will be getting the truncated fuzzy value which if it is denote by C'.

So, then membership value of the membership function of the this C' fuzzy set can be like this here. And if we have let us say n number of rules so, we take the maximum of all the fuzzy outputs. So, for that we have used the maximum. In other words the final output we can say that we add them we take the maximum or we say that we take the union of these.

So, the fuzzy output C' can be further whatever for the output that we will be getting will be fuzzy. So, C' basically is fuzzy and then this C' can be further defuzzified suitably as discussed before to obtain a crisp value.

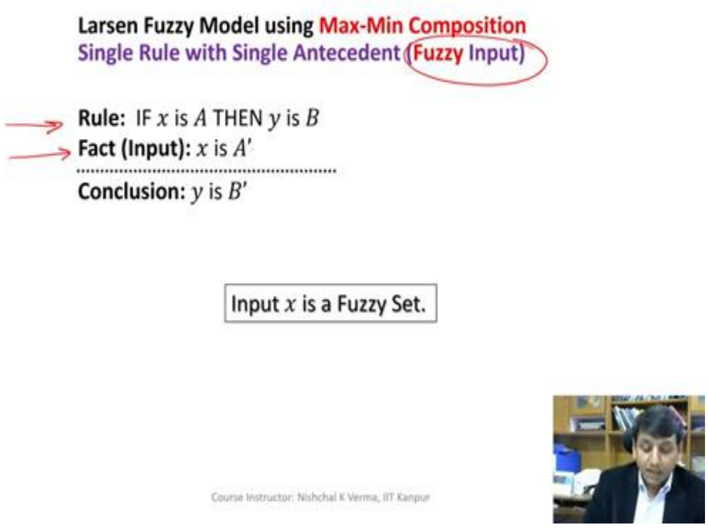
(Refer Slide Time: 08:53)

So, let us now take some of the cases which we have already discussed for the Mamdani fuzzy model as well. So, here in Larsen fuzzy model, we have the cases the first case is single rule with single antecedent. You can see here single rule with single antecedent then we can have the single rule with multiple antecedents and then we have the third case multiple rules with multiple antecedents.

So, all these three cases for Larsen fuzzy model I will discuss in this lecture and then again all these three cases we will discuss for the composition that is max-min and the max-product composition. So, both the composition will be discussed here in this lecture.

So, we have two compositions; first composition is max-min composition so, both the compositions, the first is the max-min composition and the second one is the max-product composition. So, both the compositions will be discussed here for all the cases and then again we will discuss the all these cases will be discussed with respect to the fuzzy input and the crisp input both. So, let us now first take up this case, there is a first case where we have single rule with single antecedents.

(Refer Slide Time: 10:46)

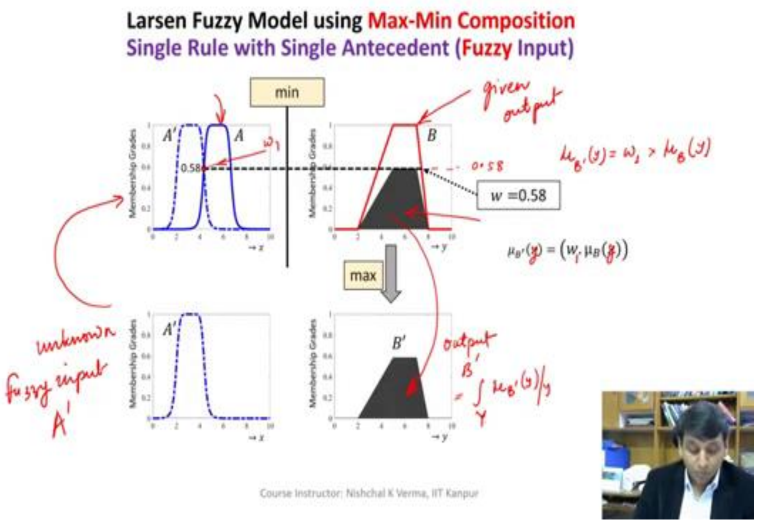


So, single rule with single antecedent, we have the rule of this type. So, here we have the rule please see here. So, here we have the single antecedent and the single rule. So, we have a Larsen fuzzy model which has only one rule here as mentioned here and then in the rule we have the antecedent only one so, means the single antecedents. So, these we can see here in the premise part, we have only one antecedent that is x is A.

So, since we have only one antecedent, we do not need any connective here. So, we have only one antecedent that is x is A; x is A. x here is nothing but, the input variable input generic variable and A here is nothing but the fuzzy region in which this x can fall. Now, if we have only one rule here as the Larsen fuzzy model or in the Larsen fuzzy model so, then if we have any unknown input comes as x is A'.

So, if any unknown input that comes as the input to this model and this input please understand is the fuzzy input means a fuzzy input is fed to the fuzzy model and for this unknown input what will be the corresponding output. So, let us understand how we can get it. So, if we have the input here.

(Refer Slide Time: 12:56)



We have, they are same as the premise part remains the same as the Mamdani fuzzy model. So, we as we have already seen in Mamdani model we have the premise part antecedent.

So, the antecedent have we have as x is A so, this is already known. Now this input is coming, this is the unknown input, this is unknown input that is coming and this unknown input is a fuzzy input. So, I can write here the unknown fuzzy input. So, when this input is coming this input is superimposed here. So, we can clearly see that A' is superimposed here over A. So, when both are shown together so, A' and A both are superimposed we see a point of intersection here as w_1, I can write here w_1. So, this we call as the firing strengths for single antecedent type of rule fuzzy rule.

So, w_1, we get the point of intersection here we have already discuss this in the Mamdani fuzzy rule. So, there should not be any confusion over here and this w_1 comes out to be so, what is w_1 here is the point of intersection. We might get two points of intersections since we are taking fuzzy value fuzzy set so fuzzy set is characterized by a fuzzy membership function. So, we might get multiple points of intersections normally two at the max.

So, in that case what we will do? We will take only the max of all the intersection points. So, here we are getting only one point of intersection that is w_1. Here this is 0.58 and we have designated this as w_1, this is called the weight of the rule number 1. So, here since we have only one rule so, we do not worry about the number.

So, we have only one rule. So, we can call either this by w or w_1 so, we have this as the w_1. This is as I mentioned this is the weight of the rule this is this is also called as the firing a strength of the rule. So, boths of these terms the weights and the firing strengths remain the same they are used interchangeably. So, now, what next? So, next is that we take this point five eight value here and with this value we find the output.

So, how do we do that? So, here since B is given this is given membership function, this is the given output fuzzy set and here we have the corresponding fuzzy set that is μ_B. So, how do we get the corresponding output fuzzy set corresponding to the A'? So, here this value the highest value is scaled down to 0.58, here we have 1, the highest value here is the highest the height of the fuzzy set B is 1.

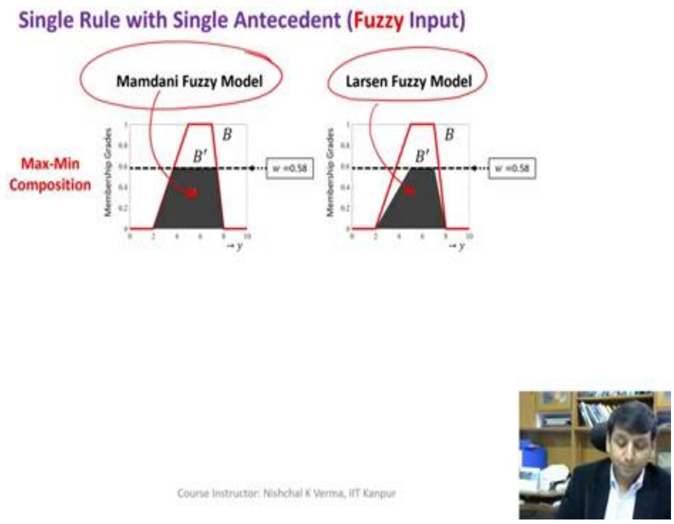
Now, the height of the resulting fuzzy set has to be the maximum height of this resulting membership function or the fuzzy set has to be 0.58. So, this has to be 0.58. How do we get that? And correspondingly what we need to do here is all the other points, all the other edges have to be is scaled down. So, how do we get this? So, for this as I have already mentioned that we take this weight here we have w 1 and then we multiply this with the $\mu_B(y)$ and when we multiply this with mu y, we are going to get the membership function of the resulting output fuzzy set that it B'. So, I can write here $\mu_{B'}(y)$.

So, this $\mu_{B'}(y)$, I can write it like this $\mu_{B'}(y)$ in this case, this $\mu_{B'}(y)$. So here, so with this we have gotten the output fuzzy set here which this output fuzzy set is this fuzzy set which we have got just multiplying w_1 the weight of the fuzzy rule with the $\mu_B(y)$.

So, since we have only one antecedent here, since we have only one antecedent and only one rule. So, since we have only one rule which means there is no other rule so, we cannot take the max. So, here the max of this will remain the same. So, we are just taking this here and we say that corresponding to A' as the A' as the unknown fuzzy input to Larsen fuzzy model the output is B' the output is B'.

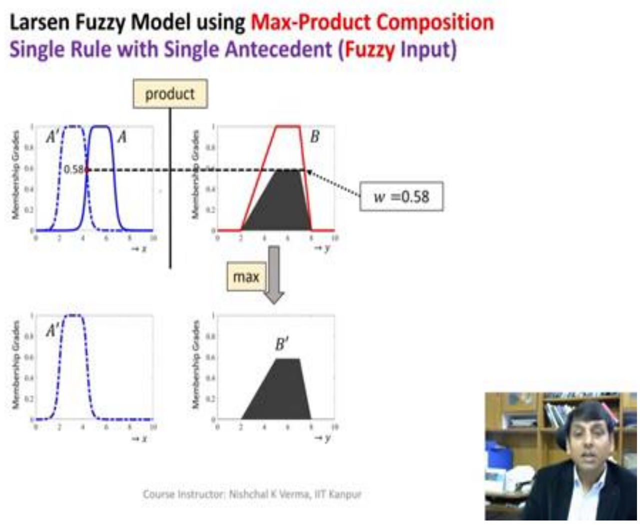
And this B' can be denoted by this B' can be denoted by simply I can write here this is the output and this B' and this is nothing but the summation μ_B or y μ_B. So, $\mu_{B'}(y)$ and then y here we have the universe of discourse Y. So, this is how we can get the output.

(Refer Slide Time: 20:37)



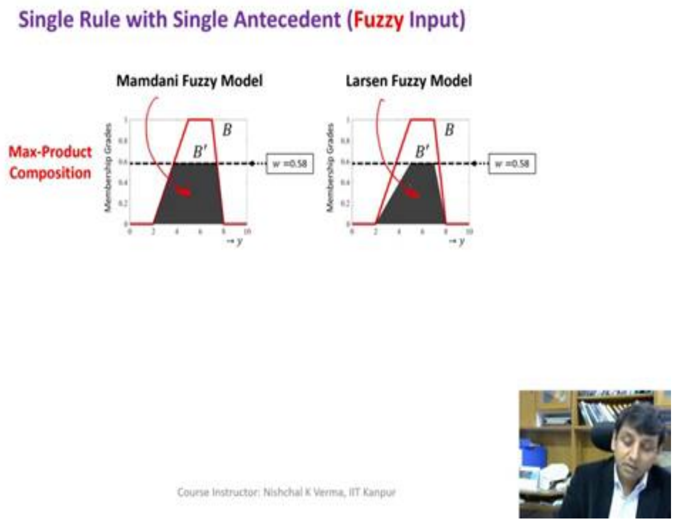
Now if we compare this output here with Mamdani fuzzy model, we see that this is the output which is which we get by using the Larsen fuzzy model, we can compare this fuzzy output this output with the Mamdani fuzzy model. So, we see that in Mamdani fuzzy model this w was just used for truncating the fuzzy set B, but here in Larsen fuzzy model we get the trunk we get the scaled down output. So, this is how we can compare the Mamdani fuzzy model and Larsen fuzzy model for single rule with single antecedent fuzzy input A'.

(Refer Slide Time: 21:38)

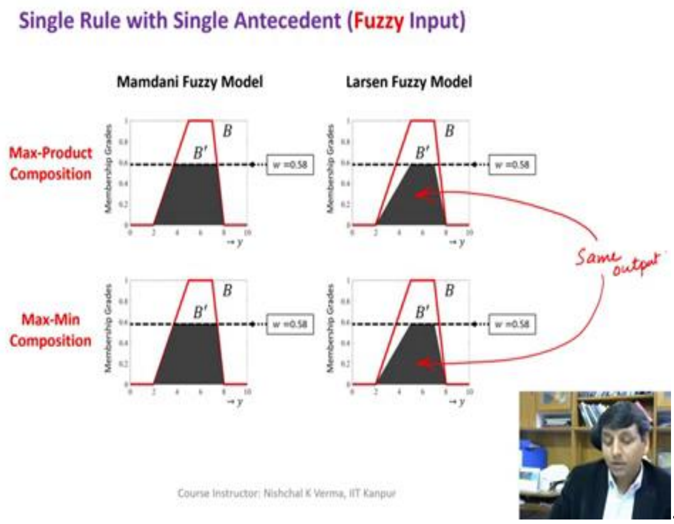


Now let us take the fuzzy the max product composition here. So, max-product composition is going to give us the same result because we have only one rule we have the single rule. So, max so, whether we take min product that there is we have here this single antecedent. So, this product are min are not going to play any role in single antecedent case. So, since we have here the single antecedent product and min both are going to give us the same output. So, we can see here both in both the cases we are getting the same output here.

(Refer Slide Time: 22:23)



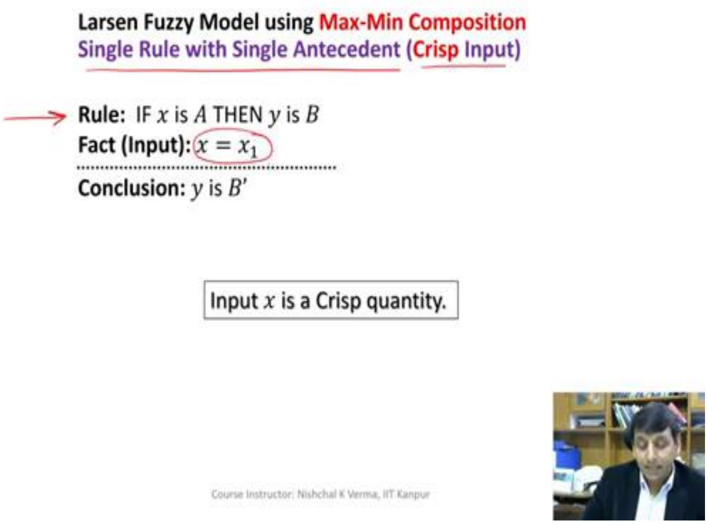
(Refer Slide Time: 22:31)



.

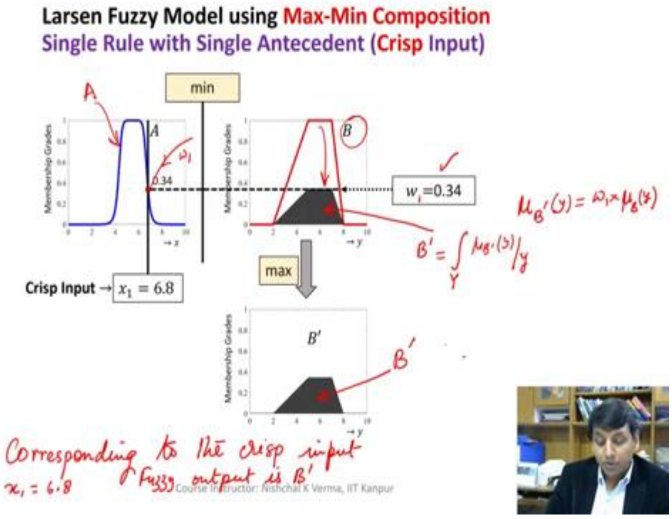
If we compare the max-min composition and max product composition here, we see that we are getting both same. So, I can write here the same output.

(Refer Slide Time: 22:59)



Now, let us move ahead and take the second case in this the so, the this this in this case what we do here is instead of the fuzzy input what happens when we use the crisp input, so the rule here in this is actually in the first case. So, first case where we have the single rule single antecedent, but here we have the instead of fuzzy input. We are taking crisp input. So, here if we are taking the crisp input; that means, x is equal to x_1, let us see what is the output that we are going to get.

(Refer Slide Time: 23:51)



So, we have here the A fuzzy region which is represented by a fuzzy set A and similarly here we have the input that is x_1, we have just taken x_1 is equal to 6.8, we can take any input here in the fuzzy region. So, if x_1 is equal to let us say point let us say 6.8, then here we get the point of intersection here as so we try to find the point of intersection with x is equal to 1. So, what we get here is the point of intersection for x is x_1 is equal to 6.8 on this membership function for the fuzzy set A.

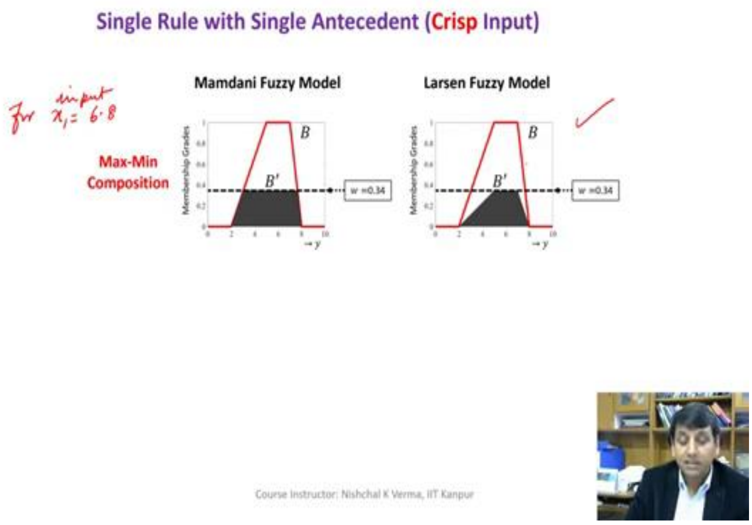
So I can write here I can represent this by the w_1. So, this is also called as the weight or the firing strength of the rule. So, with this firing strength, with this weight again we scale down the B members B fuzzy set, the membership function of this fuzzy set we scale down here to this height 0.34, 0.34 height. So, how do we do that? Here again we use the same we have the w.

So, here in this case we have w_1 or w whatever since we are here we have only single rule we can write only w or w_1. So, what is the outcome here? What is the membership let us say if this fuzzy set is B', the outcome fuzzy set is the output fuzzy set is B' so what is the corresponding membership function? What is the membership function for this B fuzzy set B' fuzzy set is basically, I can write it like this μ_B let us say this is my B fuzzy set.

So, the membership function of B fuzzy set, let us say this is y. So, this is going to be w_1 into $\mu_B(y)$. So, this is how we can get the resulting output the scale down membership function. And this is B' so, we can. So, this will be nothing but $B' = \int_Y \mu_{B'}(y)/y$ and then. So, this is how the output will be represented. And since we have only one rule that means, the single rule so the max is not applicable here. So, whatever is here the output here will be transferred here. So, this is the output B'.

So, corresponding to, corresponding to the crisp input here crisp input x_1 is equal to x, x_1 is equal to 6.8, the fuzzy output the fuzzy output is B'. And as I have already mentioned that the output here is fuzzy so we can use any suitable defuzzification technique, defuzzification methodology to convert this fuzzy value into the crisp value as we have discussed this in Mamdani fuzzy model in our last lecture.

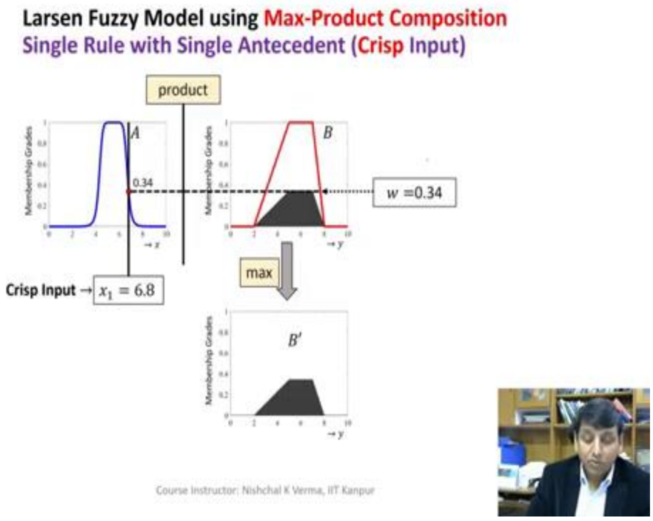
(Refer Slide Time: 28:22)



Now, how will it look like? So, here we have the corresponding to the crisp input x_1 is equal to 6.8 here, we are getting this output so, we have presented here and then the fuzzy Mamdani fuzzy model for the same input how will it look like. So, we see that if we would have used Mamdani fuzzy model the output will be like this. So, here we have compared the output for x_1 is equal to 6.8 as the input, I can write here as the input.

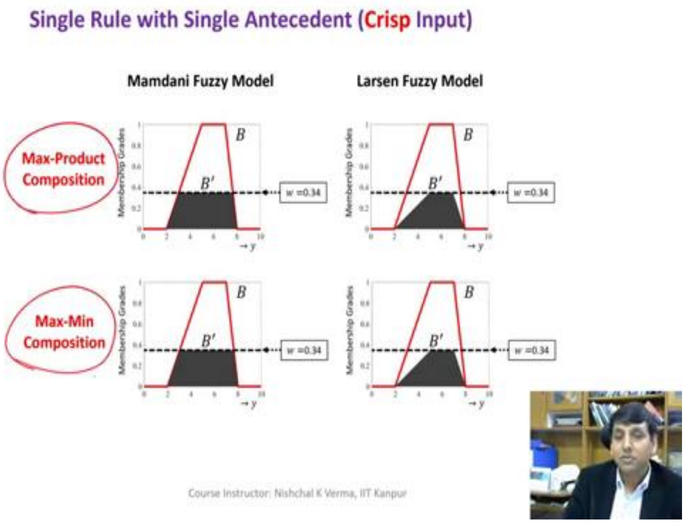
So, we can see we can compare the input so, in the Mamdani model we see we use the weights are the firing strength of the rule to truncate the fuzzy set B. But here in Larsen fuzzy model the we use the weight or the firing strength of the rule to scale down the fuzzy set B, the output fuzzy set B. So, we have scaled down the fuzzy set and the corresponding membership function, you can see here and similarly the and similarly it can be shown by B'. So, we can clearly see the difference here.

(Refer Slide Time: 30:05)



Now, with the same crisp input, now if we use max-product composition here so, since we have only one antecedent, so this product or min is not going to make any difference. They are only going to make difference when we have more than 1 antecedents. So, the output is going to remain the same here whether we use the max-product composition or max-min composition.

(Refer Slide Time: 30:41)



So, here we when we compare this again with the Mamdani and Larsen we see the difference. And now when we see the difference in all the four cases like we have the all the two cases like for max-product composition and max-min composition, we see the output in case of Mamdani fuzzy model and the Larsen fuzzy model with the respect to input the crisp input that we have fed to the system fed to the model fuzzy model, Larsen fuzzy model so, here we have fed x_1 is equal to 6.8.

So, what we are interested to note is that that for single antecedent whether the we use max-product composition or we use the max-min composition, the result is going to remain the same the output is going to remain the same. So, as I have already mentioned that as output we are getting a fuzzy value and this fuzzy value can be defuzzified by using suitable techniques of defuzzification and we can get as a result the crisp value.

(Refer Slide Time: 32:12)

So, in today's lecture, we have discussed the Larsen fuzzy model using max-min composition and max-product composition for single rule with single antecedent and with this, I would like to stop here. And in the next lecture, we will discuss the Larsen fuzzy model for single rule with multiple antecedents.

Thank you very much.