Fuzzy Sets, Logic and Systems and Applications
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Lecture – 55
Mamdani Fuzzy Model

Hi, Welcome to the lecture number 55 of Fuzzy Sets, Logic and Systems and Applications. In this lecture, we will discuss another example which is based on the Mamdani Fuzzy Model.

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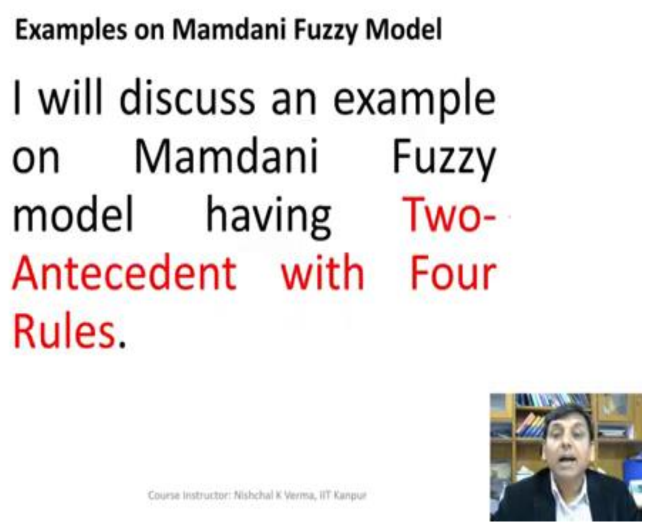

So, here in this Mamdani fuzzy model, we will have two antecedents with four fuzzy rules.

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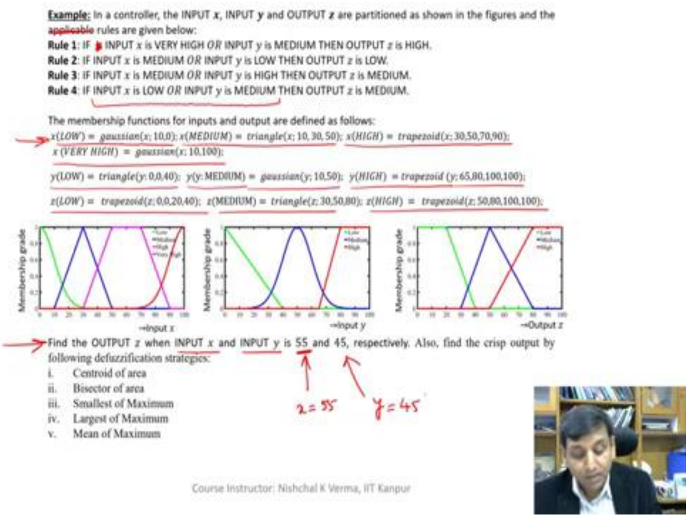

So, let us quickly go through this example and in this example we have a fuzzy model which is also called a controller here. So, in this controller, the input is x and another input is y. So, we have two inputs input x and then another input is y. And corresponding to these two inputs we have the output which is z the z is here.

So, this input x input y input z are partitioned as shown in the figure, figures here and the applicable rules are given. So, I will not say applicable rules rather I will say the rules that are there for this model, the available rules are available here. So, we have four rules that are available. So, we have rule number 1 and then we have rule number 2, rule number 3, rule number 4.

So, rule number 1 says that if input x is very high or input y is medium, then output z is high. Similarly, rule 2 says that if input x is medium or input y is low, then output z is low. Similarly, rule 3 says if input x is medium or input y is high, then output is medium. Rule 4 says, if output x is low or input y is medium, then output is medium. So, let us first understand the rule here, the rule we have four rules, four fuzzy rules in this Mamdani type of fuzzy model.

So, the in this fuzzy rule basically we have two antecedent. So, if we look at the premise part here of each rule we have two inputs x and y. So, that is why we can say that we have two antecedents see here. So, we have two antecedent x and y. x is low and y is medium for example in rule number 4.

So, we have two antecedents and now how many rules do we have here is? We have four rules four fuzzy rules. So, we can say this is a class of multiple antecedents and multiple rules. And we see that the inputs are divided input fuzzy regions are basically input is divided into multiple fuzzy regions. So, input x you see that here please look at this input x is divided into multiple fuzzy regions in each region is represented by a fuzzy set low, medium, high, very high.

Similarly, the input y also is divided into multiple fuzzy regions. So, that is here in this case we this is divided into 3 fuzzy regions, the low, medium, high and the output here is divided into again 3 regions which is low, medium and high. One more thing I would like to mention here that in this fuzzy rule in this fuzzy Mamdani fuzzy model where these four rules multiple fuzzy rules are there and these rules have the connectives and the connective here is OR instead of AND.

So, when we have OR connective it means we take the for all for the both the inputs when we find the when we compute we use for OR we take the union. So, we will see when we will be discussing this ahead. So, let us understand here that the membership functions for input and output for inputs and output are defined as follows. So, x is this $x(LOW)$ we have gaussian here gaussian function which is x and then we have the mean and variance. So, the mean here is 0 and variance here is the standard deviation here is 10.

Similarly, $x(MEDIUM)$ is triangular membership function and you can see here triangular membership function, then $x(HIGH)$ is trapezoidal membership function and then $x(VERY\ HIGH)$ is gaussian membership function.

Similarly, we have y fuzzy set, the membership function for $y(LOW)$ is triangle triangular membership function and the vertices are 0, 0, 40, Similarly, for y we have $y(MEDIUM)$, we have gaussian membership function with 50 mean and 10 as the standard deviation. Similarly, $y(HIGH)$ is trapezoidal membership function. And then we have output $z(LOW)$, $z(MEDIUM)$ and THEN $z(HIGH)$, the trapezoidal membership functions respectively.

So, what is the problem here? What is the, what we need to do here is we need to find here the OUTPUT z for INPUT x and y. So, when the INPUT is 55, INPUT x is equal to 55, x is equal to 55 and OUTPUT and INPUT y is equal to also 45 here. So, we have two inputs, one input is 55 another input y is the 45. So, when these two inputs are there, corresponding to these two inputs x and y we need to find the z.

So, let us now first find that as to how many rules are applicable, how many fuzzy regions are relevant for these two inputs as we have seen in the previous example which was discussed in the last lecture, the previous lecture.

So, let us find first the output and then of course, yes because this is a Mamdani fuzzy model the output is going to be the fuzzy value finally, after getting the aggregated value. So, and then this aggregated value will be converted into the crisp through it is a defuzzification method. So, these defuzzification methods are mentioned here that the you have to find the crisp value by using centroid of area, bisector of area, smallest of maximum method, largest of maximum method, mean of maximum method. So, these we have to use.

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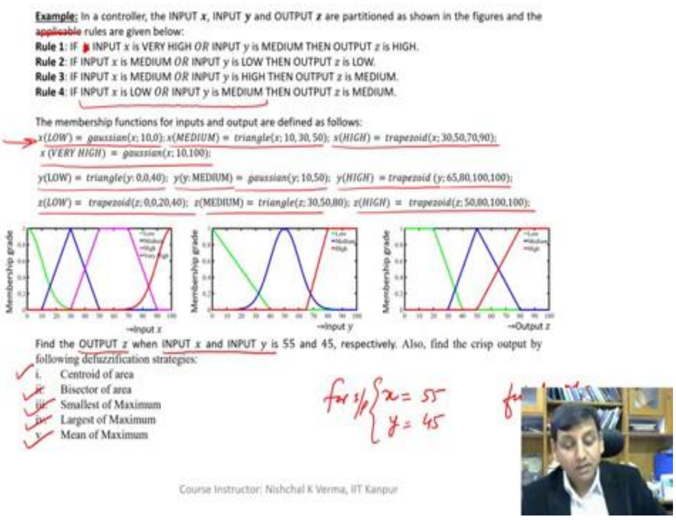

So, let us first find the for x is equal to 55 and for y is equal to 45. So, x is equal to I am writing here for x is equal to 55 and let me write here no the for INPUT x is equal to 55 y is equal to 45. So, let us find the OUTPUT. So, this is what is the question in this example.

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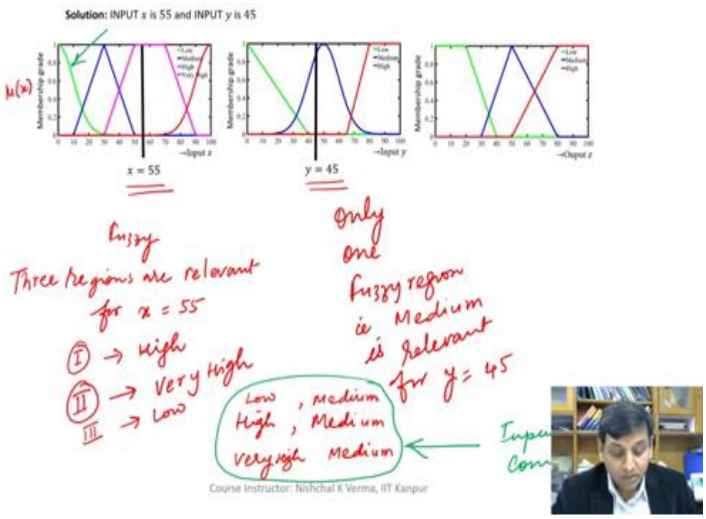

So, the target here is that you have to find the OUTPUT. So, let us first find that as to how many fuzzy rules are applicable. So, for finding the applicable fuzzy rules, we have to first look at the relevant fuzzy regions which are relevant for the particular INPUT. So, when we see that for x is equal to 55, you can see here for x is equal to 55, this x is equal to 55 is falling under the region of high only. So, this x is equal to 55 is falling in the high region, no other region is visible to be relevant.

So, maybe because we have very high which is a gaussian. So, maybe it may be relevant here, because this is not very visible, but very negligible value we will be getting, but let us see. So, since very high is a gaussian so, of course, this is not going to be 0 for x is equal to 55 similarly, for y is equal to 45. So, y is equal to 45, we see that only medium is applicable because low is having 0 value for corresponding to y is equal to 45 and similarly high is also having 0 value. So, I can write here fuzzy regions are applicable are relevant for x is equal to 55.

So, I should write it like this I^{st} fuzzy region is high region and the II^{nd} region is the very high region. Here only one fuzzy region is relevant, only one fuzzy region that is medium is relevant for y is equal to 45. So, here we have three fuzzy regions. So, II^{nd} fuzzy region here, III^{rd} fuzzy region here is that the low is also there.

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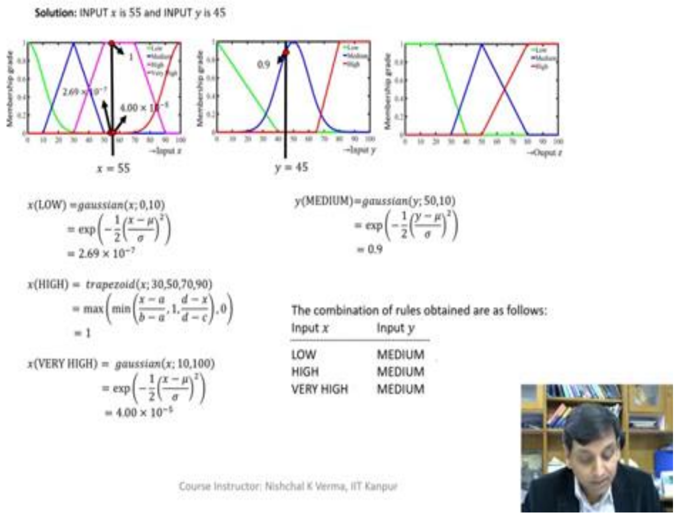

And before this let us first find the values also like the points of intersection, the intersection values. So, here the values are there you can just look at these values. So, we can come to this later also and these are the membership functions which are characterizing the corresponding fuzzy sets.

So, the combination of rules that are obtained here is. So, all these combinations are there. These are the input combinations. So, since we have these input combinations here $LOW, MEDIUM, HIGH, MEDIUM,$ here $VERY\ HIGH, MEDIUM$.

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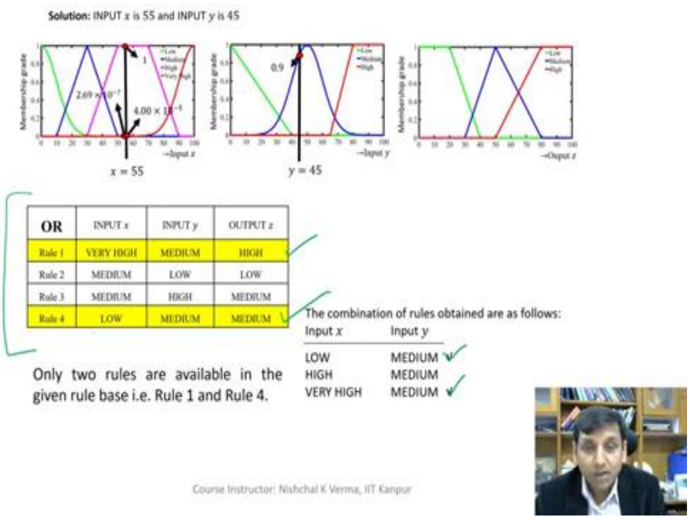

Now, let us check whether these combinations are existing in the rules that are provided or not. So, we see that we have been given only four rules and in this four rules we have two rules which are taking our combinations. $LOW - MEDIUM$, that means, the highlighted ones and then the $LOW - MEDIUM$ is here and then we have $VERY - HIGH\ MEDIUM$. So, we see that these two are there which are existing in the set of rules that are given to us, when we say the set of rules that are given to us means the model is having these rules.

So, then when we look at the second combination that $HIGH - MEDIUM$ which is not there in the set of rules. So, we just we will just discard it. So, we'll only take this rule number 1 and rule number 4 and we will proceed further.

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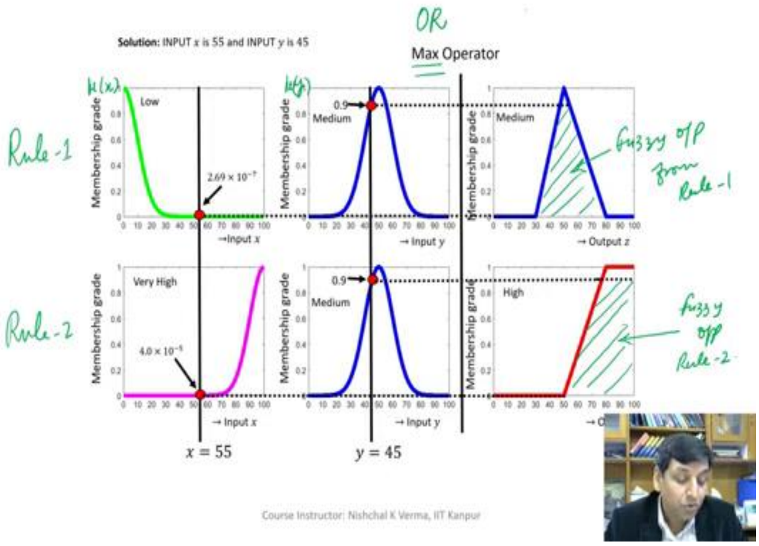

So, when we do that. So, for the 1^{st} rule, 1st rule where we have if x is LOW, y is $MEDIUM$ and then what we have here is the z is $HIGH, x$ is LOW and y is $MEDIUM$ then z is $MEDIUM$. So, the $MEDIUM$ is here. Similarly the rule number similarly the rule number 2 is here. The rule number 2 is that we have $VERY\ High$ if x is $VERY\ High, y$ is $MEDIUM$ then z is $HIGH$. So, this way when we as we have already done in the previous example.

So, when we see the intersection points. So, the intersection points as these are computed here from the membership functions $\mu(x)$ these are the $\mu(x)$ and this is $\mu(y)$. So, we see that we have two membership functions membership grades. So, here the first intersection point is 2.69×10^{-7} which is very low, but we have we need to write this here because we are computing and then we have another value which is 0.9 and since we are taking here the max-min composition.

So, what we are doing here is that since we have the connective OR please understand we have OR combination we have the OR connective. So, for OR connective we will use max instead of the min. So, that is why we are taking the max of the 2.69×10^{-7} and 0.9. So, this is going to be the, if we use 0.9 for truncation. So, we use this value as the fuzzy output.

So, this is the fuzzy output from rule number, from rule 1 for the input x is equal to 55 and y is equal to 45. Now on the same lines if we move for rule number 2, we get point of intersection here for very high 4.0×10^{-5} and y is equal to for 45 for y is equal to 45, we get 0.9. Here also we get 0.9 as the max of these two values and this when we use this value for truncation we are getting this is this as the fuzzy output here. And this fuzzy output is coming out of coming out of the fuzzy rule, fuzzy rule 2. So, now since we have the output from both the rules since only two rules are applicable here.

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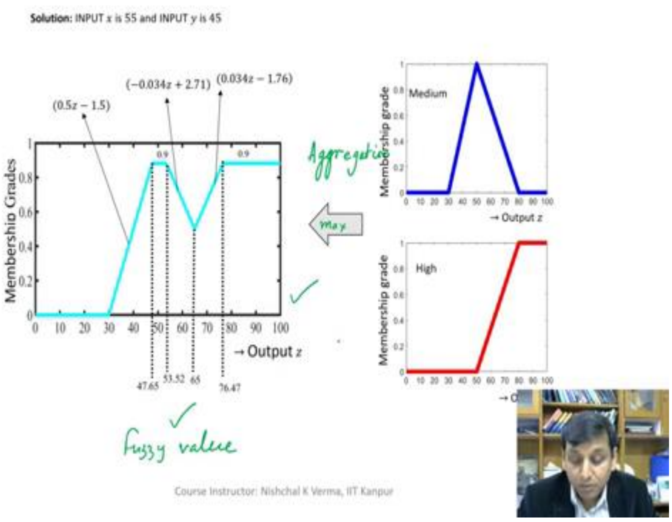

So, these two rules basically, now further when we aggregate these rules will be after get up after aggregating this I am writing here the aggregation, aggregation of the outputs and what we do here is we take the MAX of the two. So, we take the MAX, I am just writing the MAX ok, means we take the maximum of this. We take the union of this. So, I am writing the MAX. So, when we take MAX so, what we are getting here is, this structure and this is nothing but, a fuzzy value the output here is the fuzzy value.

Now, it is quite interesting here, this structure this fuzzy value is the irregular structure irregular shape. So now we have to convert this into crisp and this irregular structure, irregular shape can be converted into the crisp, that means, that means, it is a fuzzy value which needs to be converted into crisp. So, we have the methods for defuzzification that are available.

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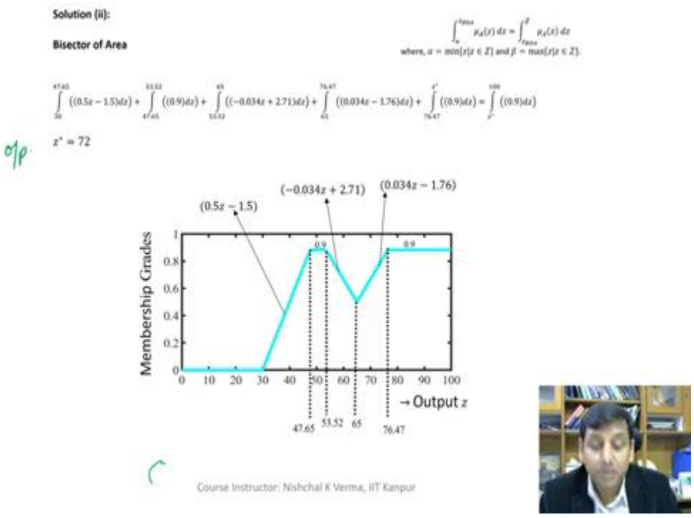

The first of the methods is here, first method is the centroid of area. So, when we use centroid of area. I am not going to discuss the again because we have discussed in detail in the previous example in detail. So, we get corresponding to x is equal to 55, y is equal to 45, we are getting the output as z^* which is 69.84 all right. And then when we use bisector area we are getting z the output here as z^* is equal to 72.

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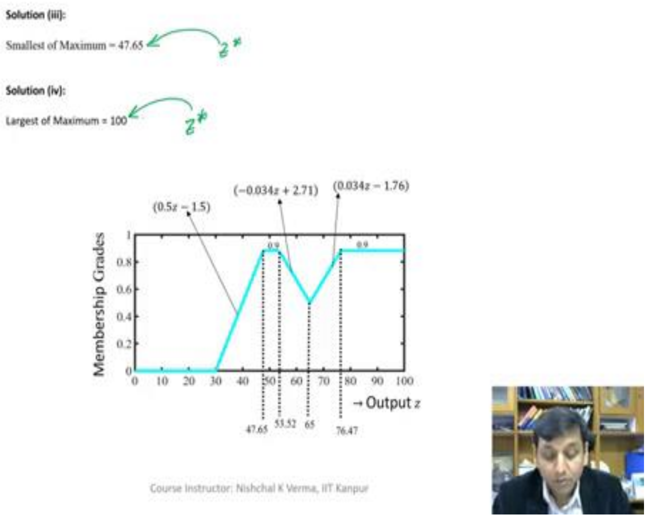

Similarly, when we use the smallest of maximum method here, we get the z^* here, z^* is our z^* which is the output corresponding to x is equal to 55 and y is equal to 45 that is 49.65. Similarly, when we use largest of maximum our z^* is here 100. So, this you can see from the figure here the structure irregular shape you can very easily get these values.

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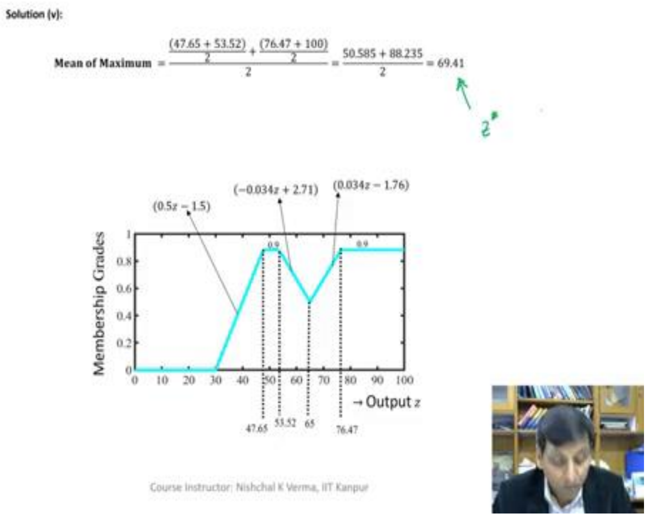

And similarly, when we take the mean of maximum the z^* that is the output corresponding to x is equal to 55 and y is equal to 45, we are going to get 69.41.

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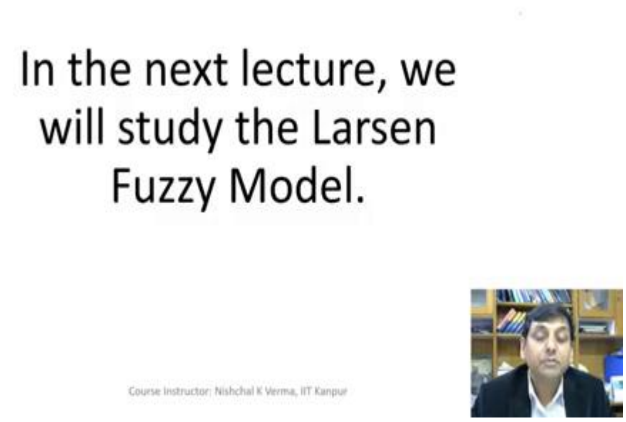

So, this way we see that the output that we are getting are the values which we have discussed from various methods of defuzzification and that is how we get we obtain these values, the test values. So, with this I would like to stop here and in the next lecture we will discuss the Larsen fuzzy model.

Thank you.