Fuzzy Sets, Logic and Systems and Applications
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Lecture – 54
Mamdani Fuzzy Model

Hi, welcome to the lecture number 54 of Fuzzy Sets, Logic and Systems and Applications. In this lecture we will cover the remaining part of the Mamdani Fuzzy Model.

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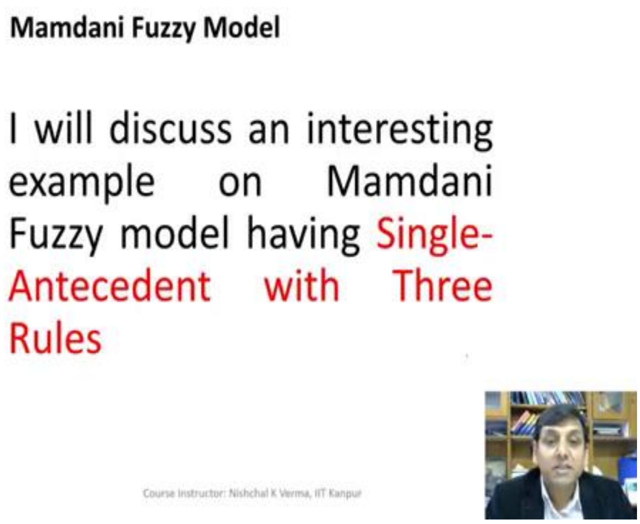

So, in this lecture, I will discuss an interesting example on Mamdani fuzzy model having single antecedent with three rules.

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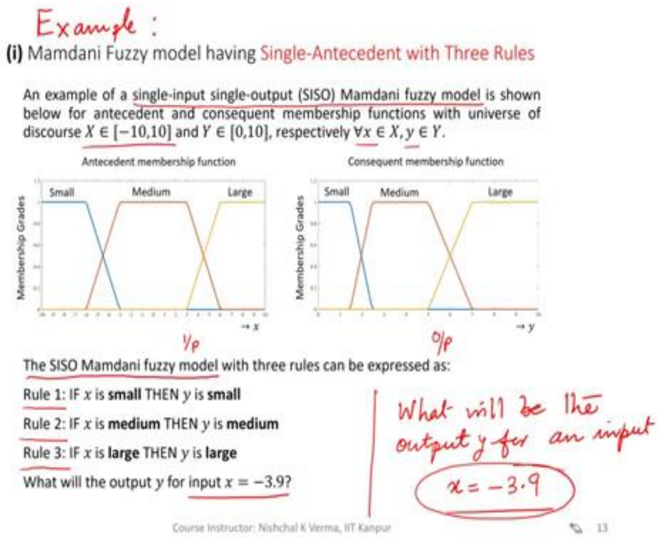

So, let us have a look here at the example here. So, this is the example and this example basically has. So, this is the example and this example basically has single antecedent with three rules. So, what is this example let me just read. An example of single input you can see here single input single output that means, this is a SISO Mamdani fuzzy model.

And which is shown below for the antecedent and consequent membership functions with universe of discourse X here, which is from -10 to 10. And the universe of discourse for output that is Y, which is from 0 to 10 respectively. And again for every x for every x here and for every y belonging into the X and Y respectively.

So, here what we have is that we have the fuzzy reason for the input x. So, this is the fuzzy reasons. And this fuzzy reasons have basically three these fuzzy reasons are characterized by three fuzzy sets, first fuzzy set is the fuzzy set for small, the second fuzzy set here is the fuzzy set for medium, the third fuzzy set here is the fuzzy set for large. So, this means that the total input region. So, total input x that is the generic variable.

So, this x has been divided into three fuzzy reasons; small, medium, large. And this small medium large is within the universe of discourse -10 to 10 you can see here is -10; that means, x is equal to -10 to x is equal to plus 10. So, this means this x is within the universe of discourse -10 to 10. Similarly the output y so, x is our input and y is the output. So, y is again having the universe of discourse right from 0 to 10.

So, you can see here the y starts from 0 here and this ends at 10. So, the y of the output again is divided into three fuzzy reasons, the first fuzzy reason is characterized by the fuzzy set small. And then the second fuzzy reason is characterized by a fuzzy set for medium. And the third one is the fuzzy set for the fuzzy region is characterized by the fuzzy set for large. So, we can see here. So, we have the left side, left hand side here we have the fuzzy regions and the corresponding fuzzy sets for the input. And here the right side we have the fuzzy sets for the outputs for the fuzzy regions.

So, as it is mentioned that this fuzzy model has single input and single output. So, we can say this SISO fuzzy model. And this fuzzy model is a Mamdani type fuzzy model because you see all the rules are with a special type of rules where the premised part is fuzzy and the consequent part is also fuzzy. So, you can see that the premised part of each rule is fuzzy and the consequent part is also fuzzy.

So, and we know that Mamdani fuzzy model, Mamdani fuzzy model deals with such kind of rules. So, we have a set of three rules; the rule number 1, rule number 2, rule number 3. First rule says that, if x is small then y is small. Then the second rule says if x is medium then y is medium; third rule says if x is large then y is large. So, if a model is known if Mamdani model is known, this means that all these three rules are known the parameters of all these three rules are also known.

So, when we say any model is already known, it means all the parameters here of the rules. Like small for input small for output fuzzy reason medium for input medium for output large for input large for output all these are known; means all these fuzzy reasons are known. Means the fuzzy sets for characterizing these reasons are also known. Now, this model this Mamdani fuzzy model is known.

Now, the question is that since this model is known if we have an unknown input and we are giving this unknown input to the model what will be the output corresponding to this input? So, we all know that the Mamdani fuzzy model can take the input in the form of a fuzzy set are in the form of a crisp value. So, here in this problem in this example basically, we have the input which is given as the x is equal to -3.9. So, when we supply this input -3.9 what will be the output? So, precisely the question is that what will be the output for and input x -3.9?

So, this is what is the question. So, let us now go ahead and see how we can manage to solve this, how we can manage to get the answer for this. So, the question is that, we have an input and the input the value of the input is -3.9 and comparable to that are corresponding to this input what will be the y. So, I can write y here. So, let us now use the Mamdani fuzzy model and find the output corresponding to x is equal to -3.9. Here in this example we have three rules and each rule has single antecedent.

So, we can see premise part has single antecedent, means single inputs. And we also see that all these premise part of the rules are fuzzy like small, medium, large. And the type of input that we can supply here in fuzzy can be can be fuzzy can be crisp, but here in this case the type of input that we are supplying is crisp.

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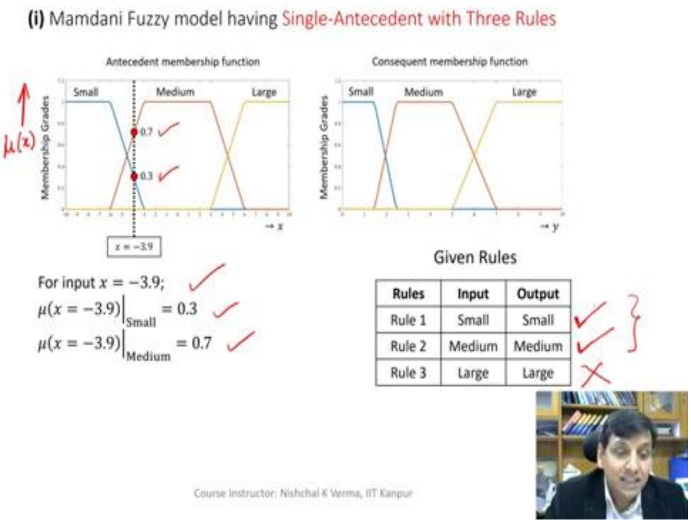

So, let us now go ahead and see what is going to be the output corresponding x is equal to -3.9 here. So, what we first do is, we first try to get the points try to get the input.

So, our x is equal to -3.9 is here see here. And here we try to see if we draw a line a straight line at parallel to the $\mu(x)$ axis this is the membership grade axis this is $\mu(x)$. So, parallel to the y axis are $\mu(x)$ axis I would say. So, when we draw a line we see that the x is equal to -3.9 is giving us the two intersection points, two points of intersection, one point of intersection is at 0.3 another point of intersection is at 0.7. And then x is equal to -3.9 is cutting or intersecting the fuzzy set is small at 0.3 and it is intersecting medium fuzzy set at 0.7.

So, we have two intersect points of intersection. So, two fuzzy sets are being intersected by the line which is parallel to $\mu(x)$ and this is at x is equal to -3.9 here. So, once we have now come to know that, we have the point of intersection and we also see that when we draw this line parallel to $\mu(x)$ axis. Then we see what are the fuzzy reasons, which are applicable which are being which are belonging to this particular input. Or in other words we can say that what are the fuzzy reasons which are relevant to this input x is equal to -3.9.

So, for -3.9 we have two fuzzy reasons; one is a small and the another one is medium. Large is not relevant because the x is equal to -3.9 is not intersecting the fuzzy set large. Or in other words we can say x is equal to -3.9 is not intersecting fuzzy set or fuzzy region large. So, we have two points of intersection. So, once we finish this then we now come to the rules. So, the given rules are here for the fuzzy set. So, rule number 1 so, we have two fuzzy sets which are relevant.

So, we first seen what are the rules which what are the fuzzy reasons which are relevant for this input. So, for this input we have a small and medium and these two are relevant. So, applicable rule is I will just tick. So, we since we have the input which is falling in the fuzzy reason is small.

So, this rule will be applicable, means rule 1 will be applicable. And then the medium rule number 2, since our input is also falling under medium fuzzy reason. So, this second rule the rule number 2 will also be applicable. So, these two rules will be applicable. Third rule will not be applicable because our input x is equal to -3.9 is not falling under the large fuzzy set.

So, this way we see that we have two rules that are applicable. Now, when we know now that there are two rules which are applicable. Now, let us find the output value corresponding to each rule as we have done, when we have discussed the Mamdani fuzzy model for single antecedent multiple rules.

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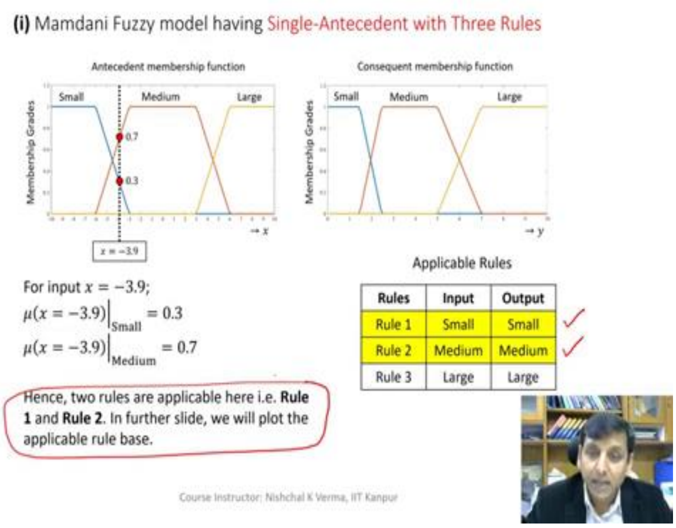

So, we can see that this two rules which are applicable here, both the rules are highlighted here only two rules are applicable. And now let us find the outputs of the corresponding rules for the input value x is equal to -3.9.

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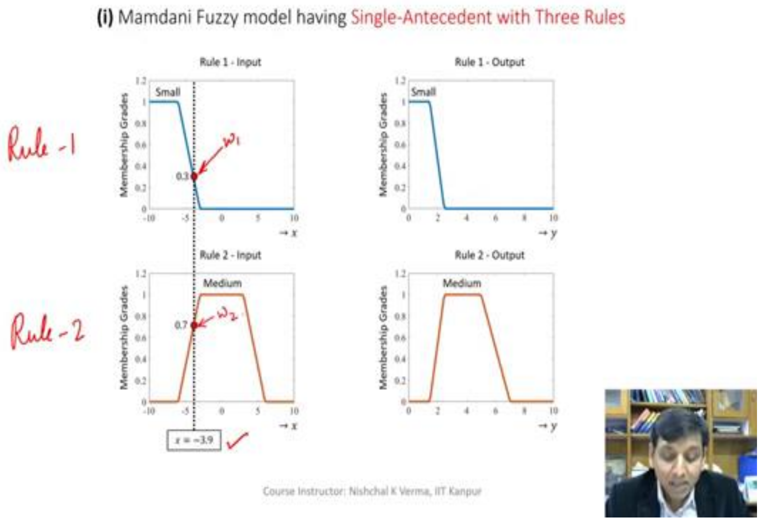

So, here we have the first rule where this is rule number 1. So, first rule says that, my input is falling in the small, then the output is also falling in small. Second rule says the input is falling in medium, then the output is also falling in medium. So now, let us find the point of intersection here for these two rules. So, the rule number 1 and rule number 2. So, since we have the x that is the input that is equal to -3.9. So, corresponding to this value x is equal to -3.9, we see here that we are getting two points of intersection.

So, let us call this as the w_1 and this as w_2 point of intersection. So, we have single antecedent, but multiple rules, so multiple rules means here we have two rules. So, now, as we have done while we discussed the Mamdani fuzzy model having single antecedent multiple rules. So, in that case, what we did was that for max-min comp max-min composition we could get the minimum of this like.

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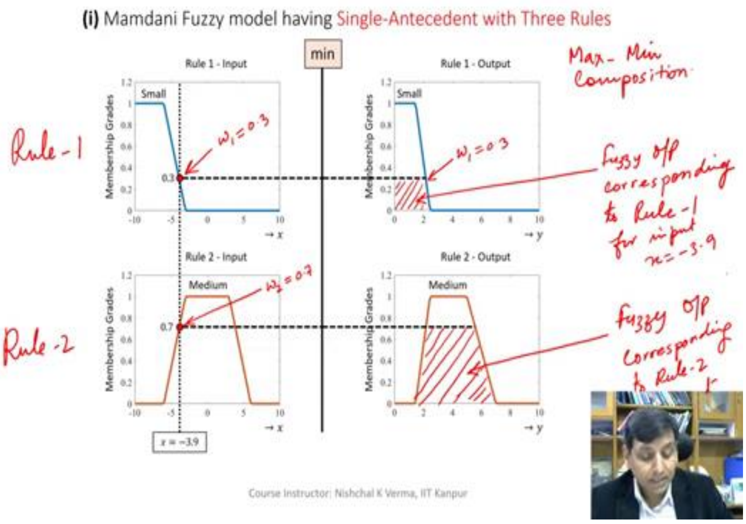

So, since we have single antecedents. So, minimum will be 0.3 only here, so w_1 which is 0.3. So, since we have only one value that is w_1 and the minimum will also be w_1. So, here this is going to be the w_1, which is 0.3. Now, we have to truncate the small fuzzy set, the fuzzy set for a small like this here and we have to only keep this value that the truncate here.

And similarly this was for the rule number 1, this was for rule number 2. So, corresponding to rule number 1, means when we apply the input to rule 1, the output of the rule is this, you can see here the output of the rule here the fuzzy output I can write here fuzzy output corresponding to rule number 1.

Similarly, now here we have the point of intersection which is nothing but the weight. So, w_2 let's say and this is 0.7. Now, corresponding to w_2 in other rule the second rule, when we truncate the fuzzy set for medium and we choke chop it off and then the remaining part of medium fuzzy set will be the output of.

So, output fuzzy output corresponding to corresponding to here the it is for input x is equal to -3.9. Similarly, here also the fuzzy output corresponding to rule number 2 for input x is equal to -3.9. So, now, since we have two fuzzy outputs, we had two rules that are fired that are applicable in this for this input x is equal to -3.9 and these two inputs now have to be aggregated.

So, these two input because we are applying they are taking the max-min composition, max min composition. So now we have to take the maximum of the two inputs means we have to aggregate.

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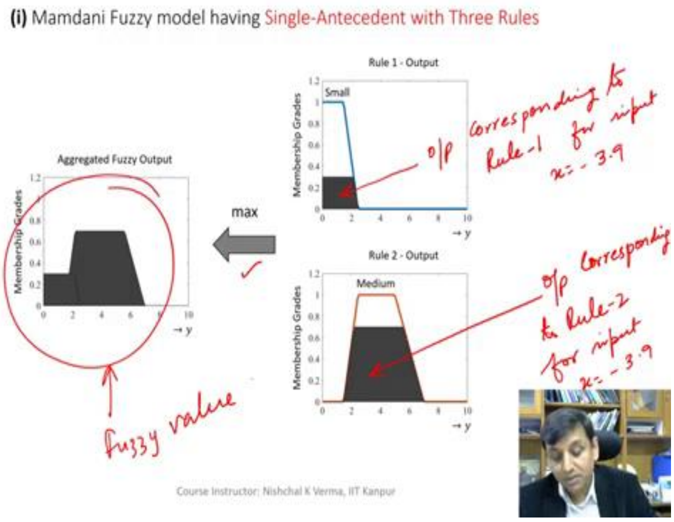

And then, when we aggregate when we take max you see here and basically max means we are taking the union of these two. And we are getting here the after taking the union we are getting and in an irregular shape basically and which is nothing, but a fuzzy value.

So, this is a fuzzy value. Now, our next job is to get the crisp value, we have to get the crisp value after converting this fuzzy value. So, this is fuzzy value now we need to convert it.

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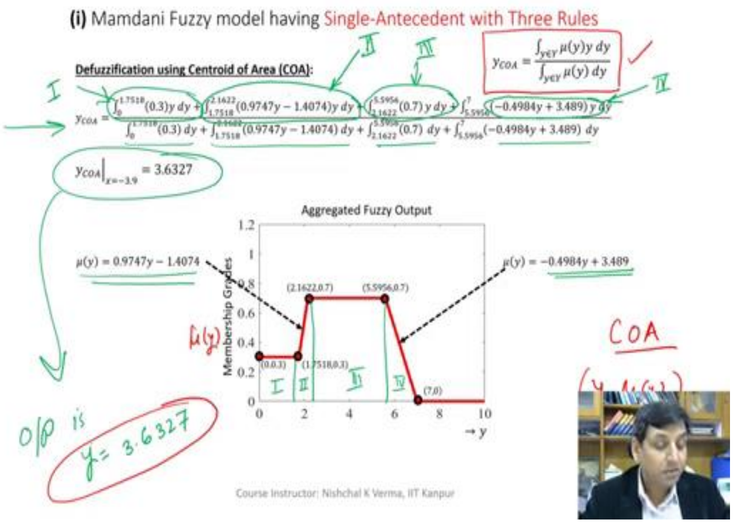

So, let us defuzzify here using the center of area. Let us now go ahead and use the center of area formula, which I discussed in the previous lecture. And here the center of area is very simple, here we all know that the center of area we can find by integrating here like this and then we divide it by the as you mentioned here.

So, $y_{COA} = \frac{\int_{y \in Y} \mu(y) y \, dy}{\int_{y \in Y} \mu(y) \, dy}$. So, since this is an irregular shape, so what we do is we first find all these vertex vertices all these points here, the first point here is the start point that is basically the 0 and then $0, 0.3$. And then similarly we have this vertex here as $1.7515, 0.3$ similarly here, so I just use different colors so, that. So, here we have this I am using green color for all showing all the vertex here and then this vertex and then this vertex.

So, all these vertex are known because we know the shape. And if we draw this on the graph paper or even otherwise also we can know the vertex. And then when we know this then we can use the formula for centroid of area or center of area here and we apply this formula. So, we can get y_{COA} you can see here y coordinate of the center of area. So, what we do here is that, we integrate the $\mu(y)$, the $\mu(y)$ from here to here you see for the first segment $\mu(y)y$. So, $\mu(y)$ is all throughout 0.3 and then we multiply this by y and then we take dy.

So, we have one integral and this limit is from 0 to 1.7518. So, the limit is here this limits 0 to 1.7518. So, when this is done then so, this part is over. Now, now let us go to this part this part. So, I will show it like this. So, here we have first part then second part area and then third part and then fourth part here. So, we have basically four parts. So, first part is here this is first part. Now, this is the second part that we have here since its second part.

So, second part is basically here, we have the equation of line here which is you can see here the $\mu(y)$. So, $\mu(y)$ is point 0.9747 $y - 1.4074$. So, this is the expression for the $\mu(y)$ here for this segment the second segment. So, this we write as it is here you can see. And, when we write this then we multiply this the whole thing by y and we put the limit here as you can see here the limit. The limit of the second segment starts from 1.7518 to 2.1622.

So, this is the limit this one and this one, so from this to this alright. So, now, once this is done now we move to the third segment, third segment is this is the third segment. Third segment $\mu(y)$ we see that here $\mu(y)$ this is the this is for the second segment. So, let me first name it. So, we have the first segment this is the integral for first segment. And then we have the second segment and then here we have let me write this also and then is and then here we have the third segment.

So, third segment here $\mu(y)$ is constant and this value is nothing but here 0.7. So, $\mu(y)$ is 0.7 for this structure and we multiply this with y. So, 0.7 into y and then we take dy, we write dy because we had to integrate it with respect to y. And the limit will be right from the start of the third segment to end of the third segment. So, this is going to be 2.1622 to 5.5956 you can see here. So, this way we write the expression for the third segment.

Now, fourth segment is here this is fourth segment. So, for fourth segment now we get the equation of $\mu(y)$ here $\mu(y)$, $\mu(y)$ which is here you see $\mu(y)$. And this $\mu(y)$ is nothing but -0.4984 y + 3.489. So, we use this value here as it is and then we multiply this by y. So, we multiply with y and then we write dy because we have with respect to dy we have to integrate.

So, here we see that, this is the expression for the fourth segment. And once this is over then comes the denominator part where we have the expression for the integrate integrations here. And then when we solve this with all these limits what we get here is this.

So, y_{COA} means the y coordinate of the center of area or centroid of area here for x is equal to minus 3.9, we are going to get 3.6327. Output is by this method y is equal to 3.6327.

Now, as I have already mentioned that we have other methods also to defuzzify the fuzzy output.

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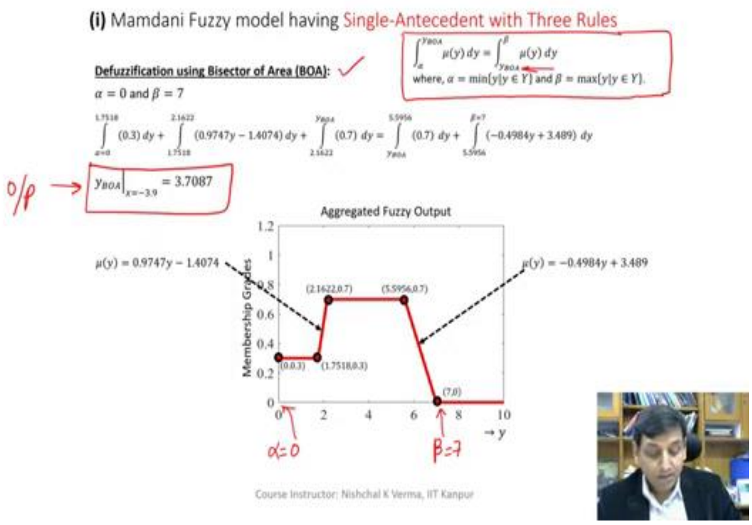

So, let us now see what we are getting when we use the bisector of area. So, bisector of area method is basically the defuzzified value here is the crisp value and we defuzzify the fuzzy value. So, here this bisector of area method, it actually divides the whole fuzzy value into two parts and the line which divides which bisects basically is the representation of the y value the output.

So, the formula is here the expression here is here for finding the bisector area. So, let us assume that we have the y_{BOA} you see here $\int_{y_{BOA}}^{\mathrm{B}} \mu(y)dy$. So, where and then we have here alpha and then the $\int_{\alpha}^{y_{BOA}} \mu(y)dy$. And you see that this left hand side is equal to the right hand side, where $\alpha = min(y|y \in Y)$ and $\beta = max(y|y \in Y)$.

So, when we write the expression here. So, our α is here you see the α is 0 and then because this is the minimum value minimum y; means the lower limit and the β here is 7 this value. So, when we apply this formula and we write the expression here corresponding to our problem, so with all the limits and all. So, when we solve this, we are getting the output through the bisector of area method. So, y we call this as y_{BOA}, for x is equal to the input -3.7, so then we get this as the crisp value. So, this is the output that we get output.

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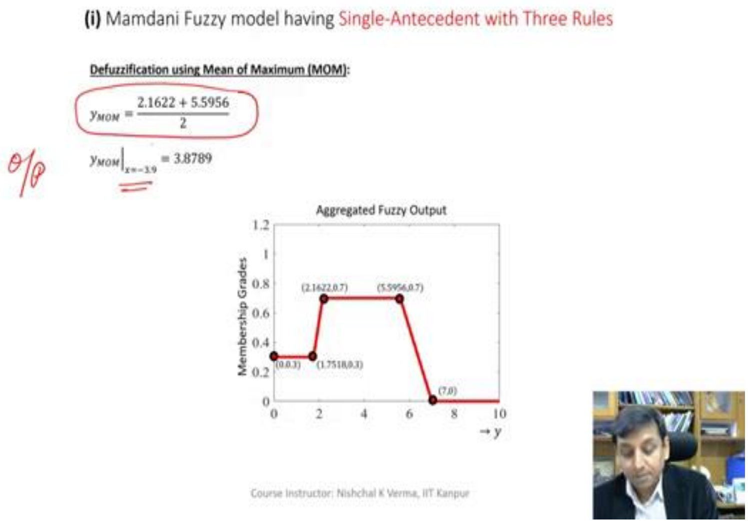

Similarly, when we use the defuzzification method, mean of maximum we have already discussed this in previous lecture. So, we have this shape here see. And we find the maximum first, so maximum is this. And then we take mean of this we know these limits here these two limits, then we take the mean of this. So, we see that you see here 2.16 here and then 5.5956 and then when we take mean we divide we add these two together and then we take the average of it.

So, when we do this y_{MOM} is giving us the output corresponding to x is equal to minus 3.9 as 3.8789. So, this is the output.

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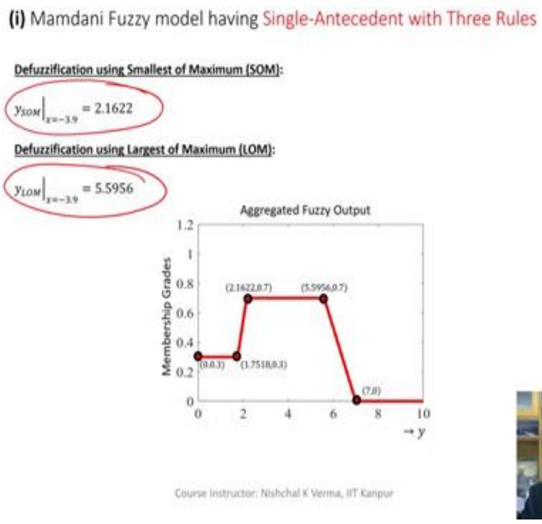

Similarly, now when we use another defuzzification is key, let us say which is smallest of maximum. So, smallest of maximum is going to be this here the maximum we know and then the smallest is the 2.1622. So, just by looking at the fuzzy value as the output we can get the crisp value through this defuzzification method.

So, here we have got the corresponding to the input x is equal to -3.9, we get 2.1622. Similarly now, we when we use the largest of maximum we get this value as the defuzzy defuzzified value that is the crisp value corresponding to x is equal to -3.9. So, that is how we are getting these values as the crisp value corresponding to the fuzzy value through the various defuzzification schemes.

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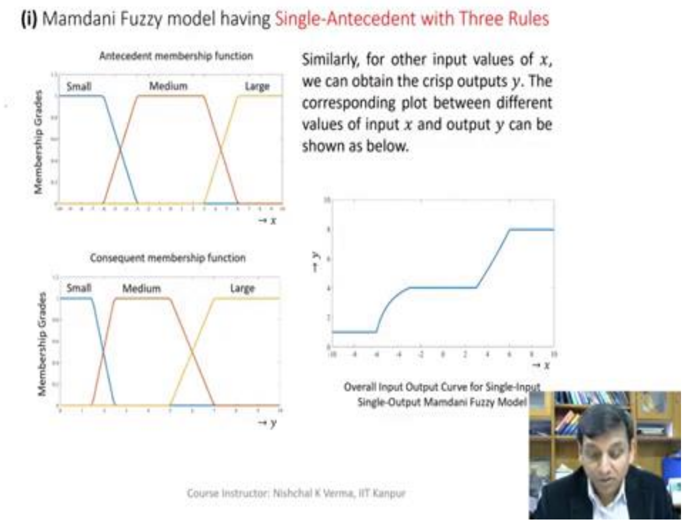

So, here basically in this example that we just discussed, we have input fuzzy regions; you see here we have three input fuzzy regions; the first one was the small and then second is the medium the third is the large. And similarly, the output also is divided into three fuzzy regions. So, when we here in this problem, we took only one value of the input that it x is equal to -3.9. So, let us say when we take various values of x as input and then what would be the corresponding output?

So, here when we take the x value right from -10 to 10 what we see here is the output will be like this. So, we can say that for other inputs input values of x we can obtain the crisp outputs, the corresponding plot between the different values of input and output can be shown as below. So, this exercise I am leaving for you and you can try for various values of x and see that your output will be falling on this input output characteristic curve.

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So, at this point I would like to stop. And in the next lecture, I will discuss an example on Mamdani fuzzy model having two antecedents with four rules.

Thank you.