Fuzzy Sets, Logic and Systems and Applications
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Lecture - 53
Mamdani Fuzzy Model

Hi, welcome to the lecture number 53 of Fuzzy Sets, Logic and Systems and Applications. In this lecture we will continue our discussion with the remaining part of the Mamdani Fuzzy Model.

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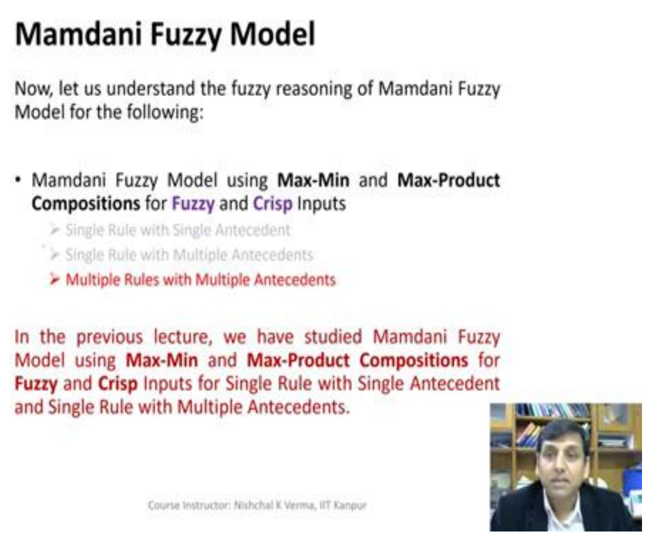

Here, let us understand the fuzzy reasoning of Mamdani fuzzy model for the following. Mamdani fuzzy model using max-min and max-product compositions for fuzzy and crisp inputs for multiple rules with multiple antecedents. In the previous lecture what we have done was, the Mamdani fuzzy model using max-min here max-min and max-product compositions for fuzzy as well as the crisp inputs for single rule with single antecedents and single Rule with multiple antecedents.

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Now, let us discuss the third scenario where we have a Mamdani fuzzy model with multiple rules more than one rule. And, even in the rules itself we have multiple antecedents. So, let us now discuss this scenario.

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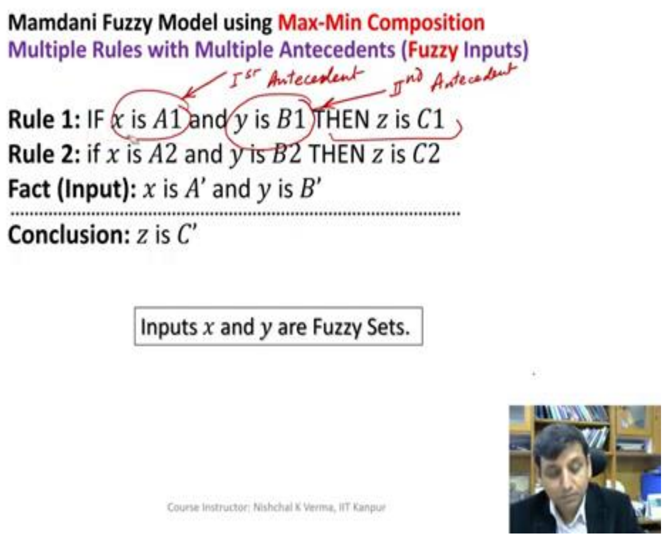

So, here we are taking for simply simplicity we are taking 2 rules only under this multiple rule case with multiple antecedents here also we are taking only 2 antecedents. So, let me just make it very clear that we have we have this as the I^{st} antecedent, I^{st} antecedent.

And then we have the II^{nd} antecedent and we have 2 rules. So, 2 rules 2 antecedents and we all already know that this is the premise part and this is the consequent part this premise part, this is consequent part. And here also we see that premise part is a fuzzy and the consequent part is also fuzzy.

So, both premise and consequent both parts are fuzzy, when we say fuzzy it means we have this premise and consequent both part are designated by the fuzzy values like here in this case A_1, B_1, C_1, A_2, B_2, C_2 are fuzzy values.

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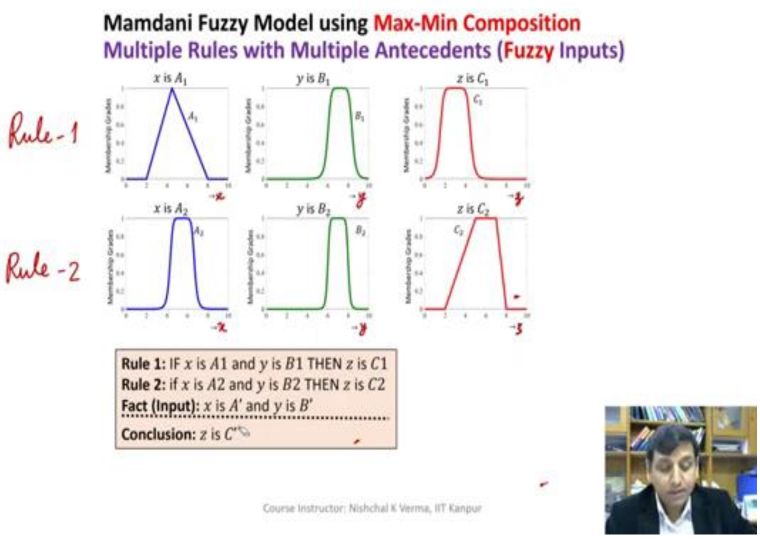

Now let us on the same lines as we have discussed the two previous scenarios, let us discuss this also. So, here we have rule number 1 where we have x is A_1, y is B_1 this x this y this is z, x, y this is rule number 2. So, we have two rules and then two antecedents. So first antecedent is this, second antecedent is this. And these two antecedents are connected again by AND, you can see here and so both the rules are containing AND as the connectors.

Now, these two rules are given in the fuzzy model Mamdani fuzzy models. Or in the other words we can say this this Mamdani fuzzy model is described by or characterized by the 2 fuzzy rules and these fuzzy rules are with 2 antecedents.

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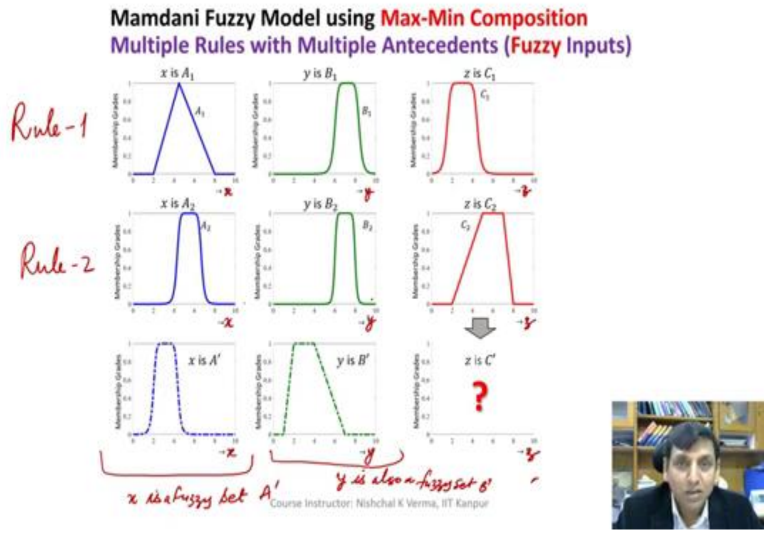

And which is clear case of the multiple rules with multiple antecedents. So, let us first take the fuzzy input. So, Rule 1 and this is Rule 2. So, here let us take the same input here x, y and x is fuzzy y is also fuzzy, when we say fuzzy again fuzzy means of fuzzy value and fuzzy values always characterized by or represented by a fuzzy set. So, we are taking a fuzzy set alright. So, now, let us apply this input to the fuzzy model and let us see what we are getting as the output to the corresponding input.

So, this is fuzzy input, first fuzzy input. So, here x is the fuzzy input and what is this fuzzy set here? Fuzzy set A', here y is again is a is also a fuzzy set which is B' this is y alright. So, now, let us apply this. So, this is very simple and as we have done in second scenario, so let us take the Rule number 1 first and take these two x, y fuzzy values superimpose.

And then finds the points of intersection and these points of corresponding to the points of intersections, the membership values will become the w's like w_1 and w_2 like this here.

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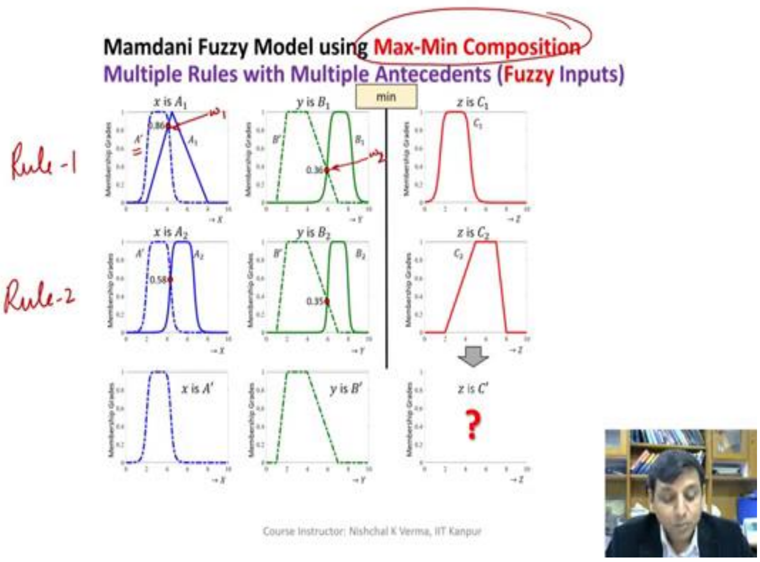

So, when we superimpose this is our Rule number 1 and this is Rule 2. So, when we superimpose A', so you see here A is superimposed on A_1, B' is superimposed on B_1.

So, A' is superimposed on A_1 and we are getting here, we are getting here as the w_1, here also we are getting the point of intersection as w_1. Now, here in this case since we are using the max-min composition. So, now, let us take w_1 this w_2. So, let us take the minimum of the w_1 and w_2 and when we take minimum of the w_1 and w_2 we are getting w as 0.36.

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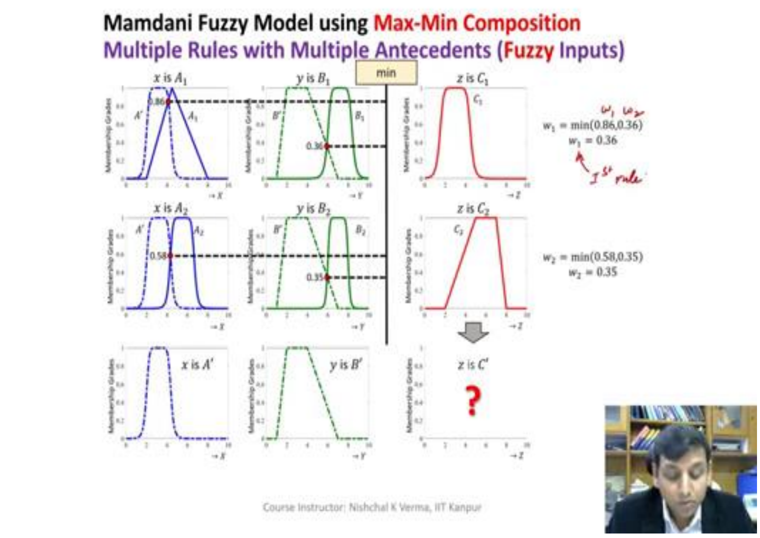

So, this is w_1 this is w_2, so w is. So, let us take this as the w dash, this is also w dash and when we take this as the w_1 and w_2, w_1', w_2' let us call this as w_1 ; 1 is for the Rule here this for the first rule, first rule it is for the first rule.

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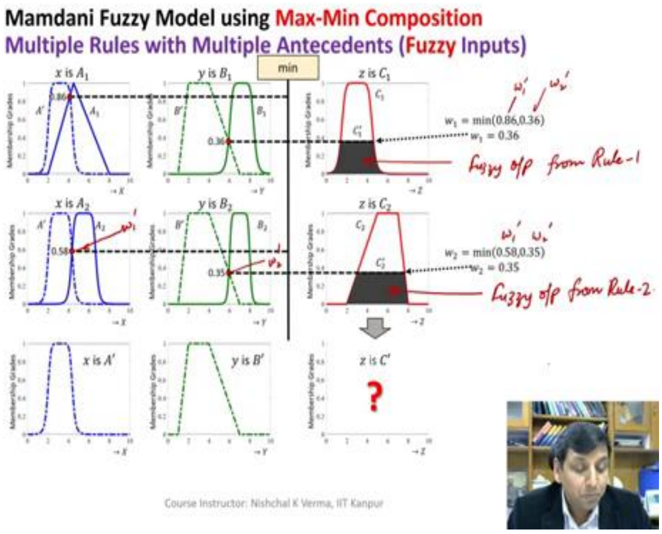

So, when we get w_1 here as the minimum of the w_1' this was the w_1' this is w_2' alright.

So, w_1 is coming out to be 0.36 and again we use this w_1 to truncate C_1 and this is the output is the fuzzy output corresponding to the fuzzy inputs if fuzzy output from Rule 1, from Rule 1. Similarly, we do the same exercise using min composition here also we are getting w_1', w_2'. When we take w_2' here in this case.

So, when we take min because we are using we are taking max min composition, we take min. So, w_2 which is the outcome which is corresponding to the second rule. So, w_2 with this w_2 we truncate C_2 and this becomes C_2'. So, this is this we can write as the fuzzy output from Rule 2.

Now, here we are using max-min composition. So, min is utilized here now whatever outcomes that are coming from each Rule we are taking the maximum of it, maximum means we are taking the union of it. So, let us take the let us aggregate it with union.

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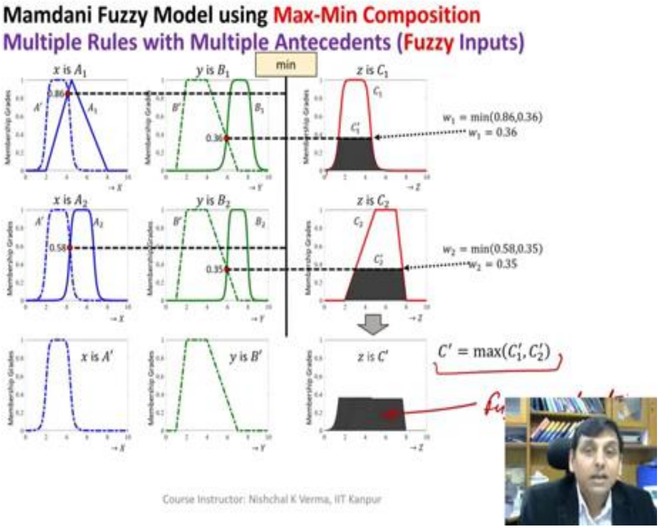

So, when we take the union, so we are getting this kind of thing you see here when we plot it we are going to get the union see here. So, we aggregate the output from Rule 1 and then we get the output from Rule 2 both outputs are aggregated, we are taking the union and the union is giving us see here the max is giving us the fuzzy output corresponding to the fuzzy input. Now, this fuzzy output can suitably be defuzzified by using suitable defuzzifiers. Now, in this scenario the third scenario let us see when we use the max product composition.

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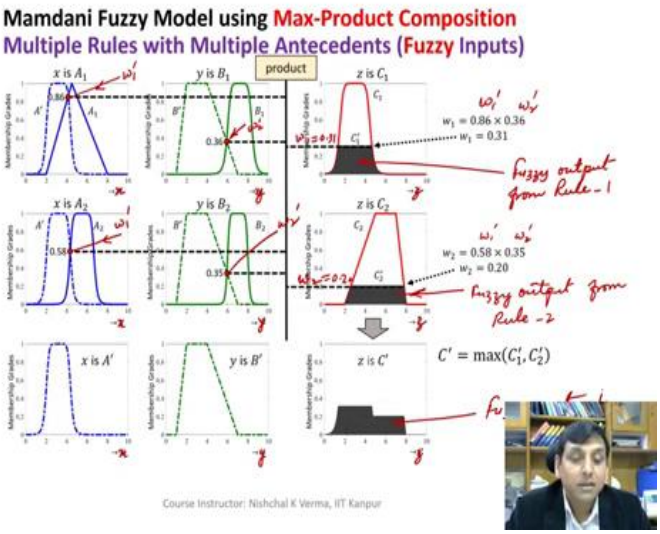

So, this again very simple here when we superimpose the fuzzy inputs here. So, A_1 is superimposed to A' is super imposed A_1 this is w_1', this is w_2' similarly this is w_1, w_2'. So, since here we are using the max product composition, so we take max we take the product of these two.

So, product is going to give us 0.31 and this is the output of the Rule output from the Rule fuzzy output that we are getting, the fuzzy output from Rule 1. Similarly, here also we are getting some output on the same lines here also we have the w_1', w_2' when we use max-product max-product composition.

So, we take product in this case product of w_1', w_2'. So, when we do this we are getting here w_1', w_2' and yes these are x, y, z; x, y, z; x, y, z everywhere.

So, this way we are getting the again now coming back to the third scenario, when we use max-product composition. So, in max product composition we takes here again the product of the w_1', w_2' and this truncated portion becomes the and we truncate with the value w_2. So, this is w_2 yes w_2 which is nothing but 0.20 and here w_1 is 0.31.

So, this way we are getting two outputs, the first output here is corresponding to Rule 1 and the second fuzzy output is corresponding to Rule 2. So, here I can write fuzzy output from Rule 2. So, every rule is going to give us some output, but these rules should be applicable.

So, this way when we take the max, so here we take the union of this means we take the maximum which union right, we aggregate. So, when we do this we are getting this structure, this is fuzzy set fuzzy value. So, this as we can see there is a fuzzy value as the output corresponding to the fuzzy inputs x is A', y is B'. So, we can call this as the fuzzy output.

And this can be suitably converted into crisp value by using some defuzzifiers. Now, here we have discussed when x, y both were fuzzy values. Now, let us see what happens when we use x, y both are crisp values means the inputs are inputs that are fed or crisp values.

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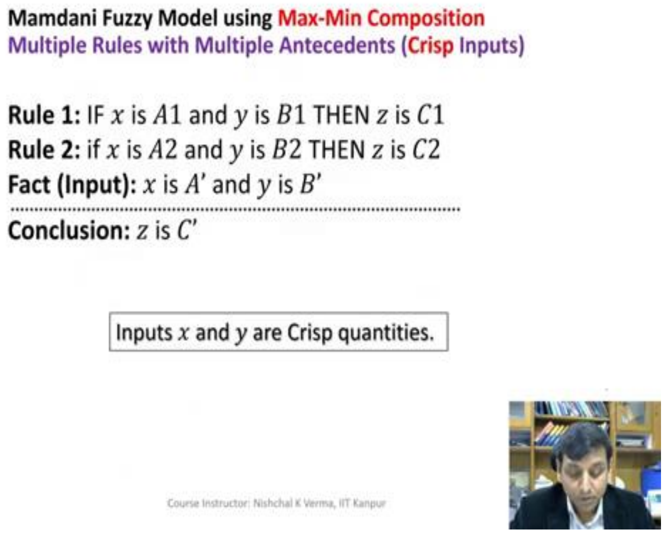

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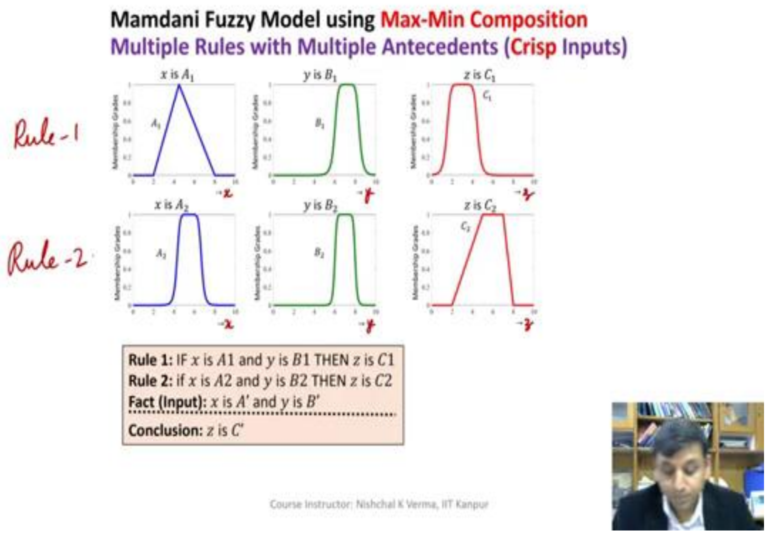

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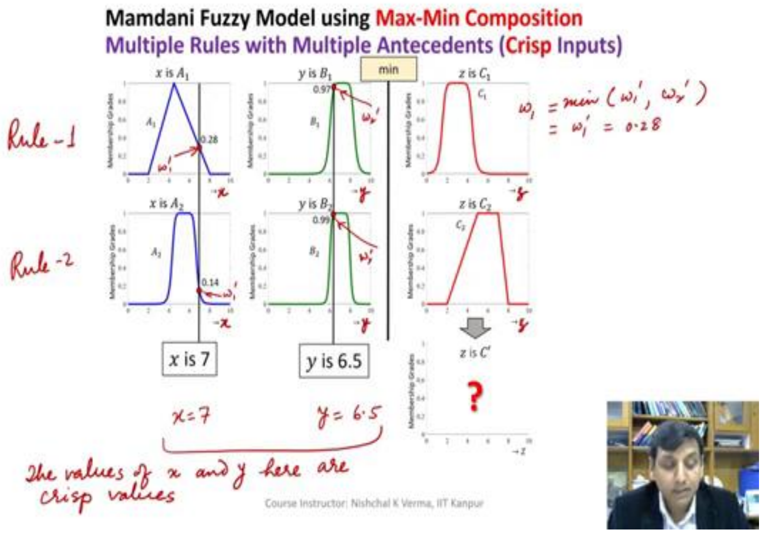

So, let us go ahead and see. So, here we have Rule number 1 here we have Rule number 2.

And we see that when we take x is equal to 7 means x the value of x is 7. The value of y is 6.5 and here please note that, all these x that we have written here is x, y, z, y, z. So, this is Rule number 1, Rule number 2 and we see that we have to antecedents here both the antecedents are connected by AND. Here also we have x, y, z, x, y, z. Now, here we see that we have these inputs the values of the inputs x, y are crisp values.

So, I can write here the values of x and y here are crisp values ok. So, when we have crisp values now we apply these crisp values and when we apply this we see if there is any intersection here on A_1 for x is equal to 7. So, we see that we have one intersection. So, we call this as w_1' similarly for y value we see whether there is any intersection here yes for y is equal to 6.5, we have on B_1 we have w_2' and similarly for Rule 2 also we see. So, here also we see that we have the intersection w 1 dash and then we have the w_2'. Now for the first Rule first, so we are taking the max min composition.

So, we will take the min of w_1' and w_2', $min(w_1', w_2')$ and this is going to give us w_1. So, let us see what we are getting here when we take the min we are going to get obviously the min w_1 is going to give us w_2 w_1' which is nothing, but zero 0.28 w_1' which is nothing but 0.28 here.

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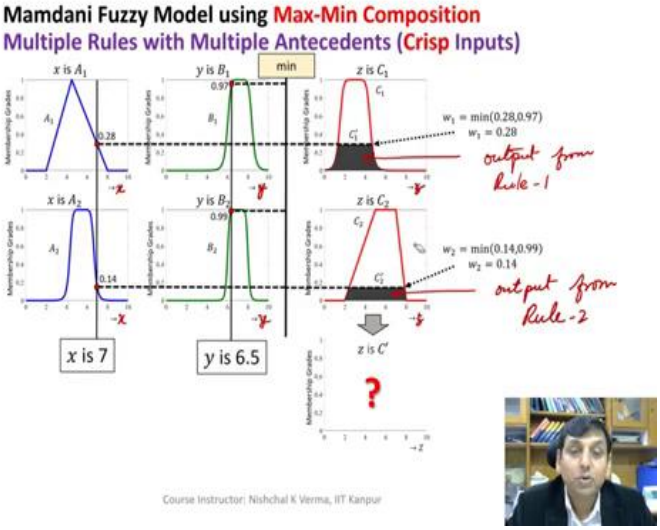

So, here we see that w_1 which is the strength of this Rule is the 0.28 and with this value 0.28 we truncate C_1, C_1 is the consequent part. So, we truncate here also we write small alright. So, this is the output of the first Rule corresponding to the crisp inputs x and y.

So, the output from Rule 1. Similarly, when we for the first for the second Rule we are getting the this as the output the fuzzy output after truncation here. So, this is the output from Rule 2. Now, since here we are using max min composition. So, we have to take the maximum of all the outputs.

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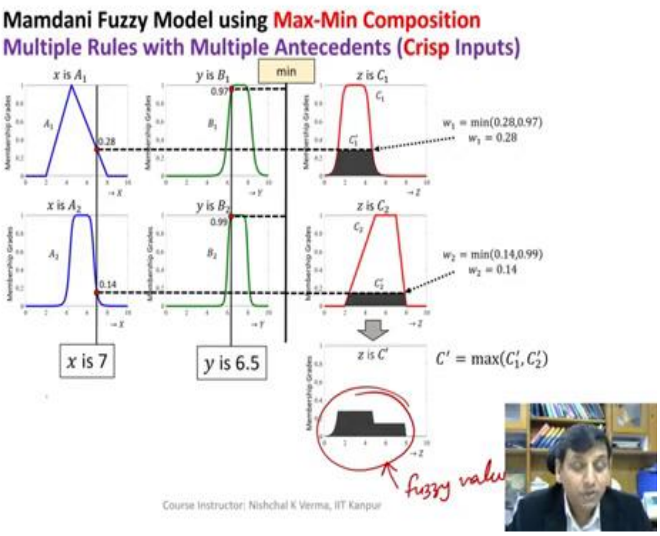

So, we take the aggregation we take the union of these two outputs coming from the Rule number 1 and Rule number 2 and this way we are getting this structure this fuzzy value as the output corresponding to the inputs as crisp values x is equal to 7 and y is equal to 6.5. So, this is what is the fuzzy value as the output ok.

So, we see that the output in all the cases here we are getting fuzzy. So, Mamdani model Mamdani fuzzy model always produces the fuzzy output and we use the defuzzifier suitable defuzzifier to convert this fuzzy output into the crisp value. Now, what happens when for the same input x is equal to 7, y is equal to 6.5 as a crisp values when we use max-product composition.

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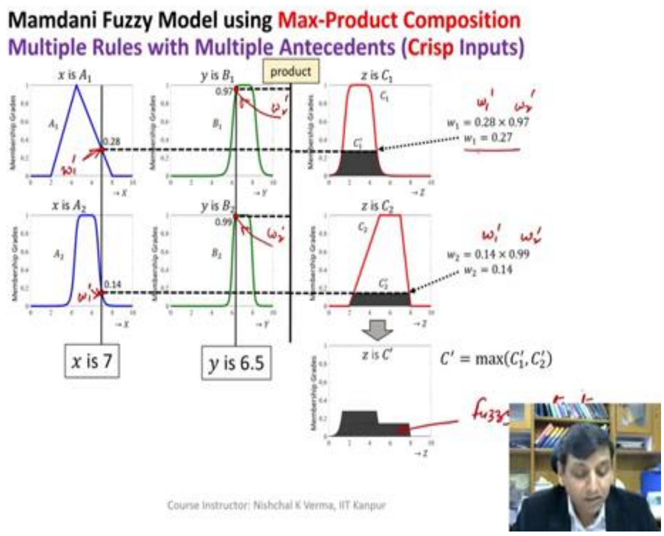

So, when we use max product composition we see that the w_1 is 0.27 we multiply w_1', w_2' here also w_1', w_2' and this is our w_1', this is our w_2', this is w_1', this is w_2'. So, these two outputs when we aggregate we are getting this as the output.

So, in this case here this is the fuzzy outcome this is the output this is the fuzzy output and this we can get we can convert as the convert into the crisp value again by the suitable defuzzifier.

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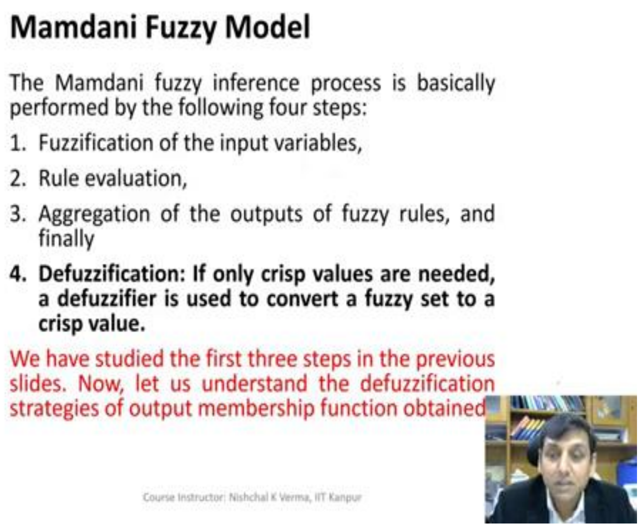

So, this way we have seen that we have discussed all the three scenarios where we have seen multiple cases with the fuzzy model Mamdani fuzzy model with single Rule single antecedent.

And again we saw all these cases with max-product, max-min compositions. Similarly, multiple antecedents with single Rule; then we discussed the multiple rules with multiple antecedents with both the compositions the max-min composition, max-product compositions and all of these cases with the fuzzy inputs and crisp inputs.

And we saw that we all the cases in all the cases we found always found the output in the fuzzy value, as the fuzzy value. And this fuzzy value can further be converted into the crisp value by using suitable defuzzification methods. Alright so, this way we have understood as to how with certain inputs to the Mamdani fuzzy model we can get the suitable output or the corresponding output.

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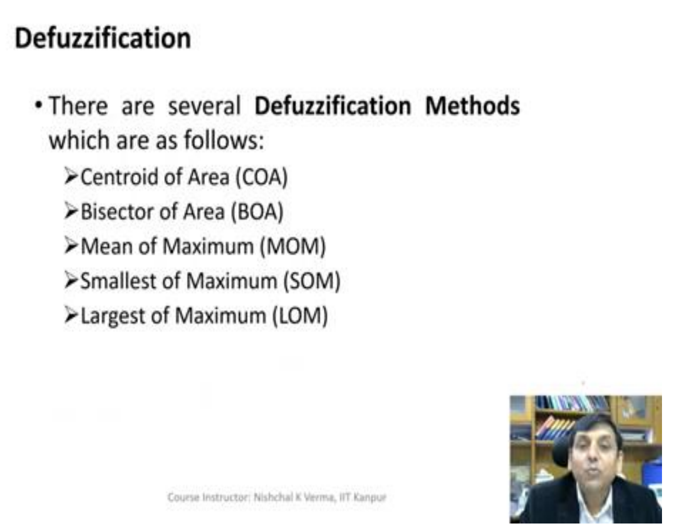

Now, when it comes to the defuzzification. So, there are several defuzzification methods, but here we will be discussing very commonly used defuzzification methods. So, the first one is the centroid of area method through which we can get the crisp value of the fuzzy value. And then we have the bisector of area, mean of maximum, smallest of maximum and largest of maximum.

So, these are some of the common commonly used defuzzification methods that we use for converting the fuzzy value into the crisp value.

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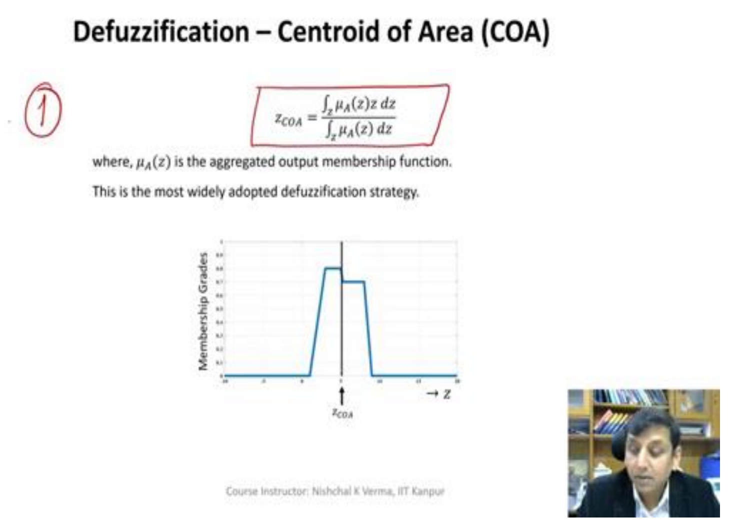

Let us discuss this one by one quickly. So, center of area centroid of area is COA and here is the formula through which we get the crisp value when we have the fuzzy value and we use this formula the center of area is very simple.

So,

$$z_{COA} = \frac{\int_z \mu_A(z)z \quad dz}{\int_z \mu_A(z) \quad dz}$$

Where, $\mu_A(z)$ is the aggregated output membership function means this is the output that the fuzzy output that fuzzy model generates.

So, by applying this, we can get crisp value corresponding to the fuzzy value. So, this is the this is the center of area which is used for converting the fuzzy value into the crisp value.

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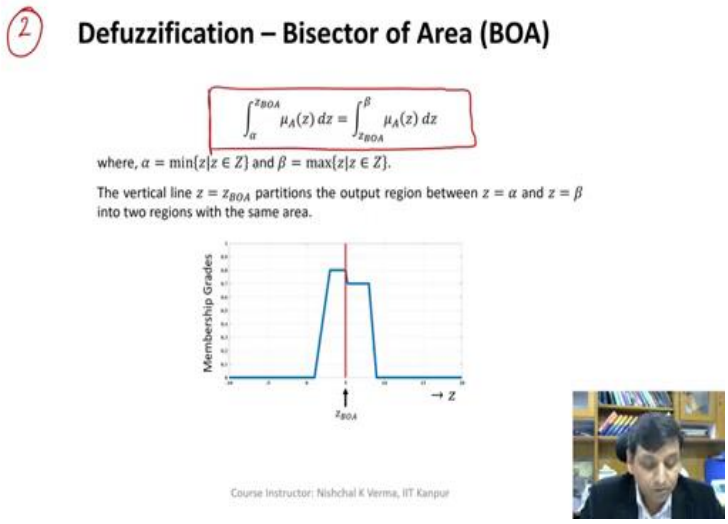

Now, the second one is bisector of area. So, bisector of area expression is here this basically does nothing, but when we have any fuzzy set as the output fuzzy value as the output of the model. So, what does what it does is basically it bisects it divides into 2.

And each of these sectors each of the parts of this equal area. So, this area is same as this area here in this case.

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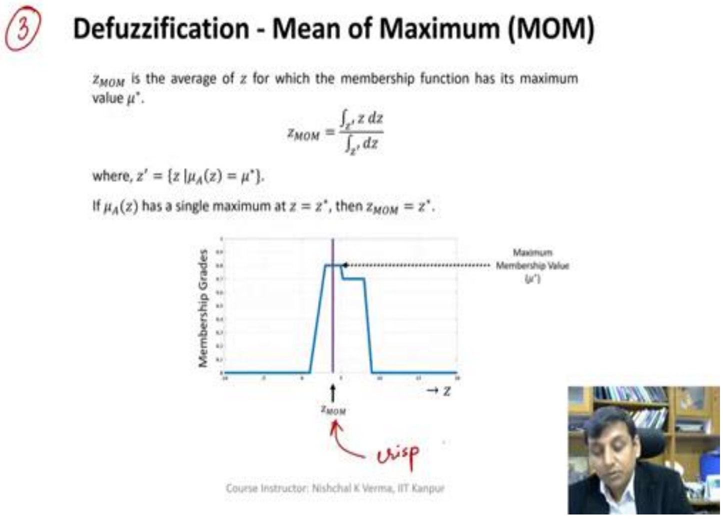

Similarly, we have the mean of maximum this is very simple, there is no calculation normally needed. So, mean of maximum here is that we have the maximum. So, we see that the maximum is here in this case.

So, we first take the maximum here all the maximum all the here the maximum power portion and then we take the mean of that. So, we basically divide this into 2 here in this case because we are taking the maximum in the mean of maximum. So, max we take the maximum and then we take the mid value.

And this value the corresponding to this value we take as the crisp equivalent to the fuzzy value that we are taking for the conversion. So, there is the crisp value as the output.

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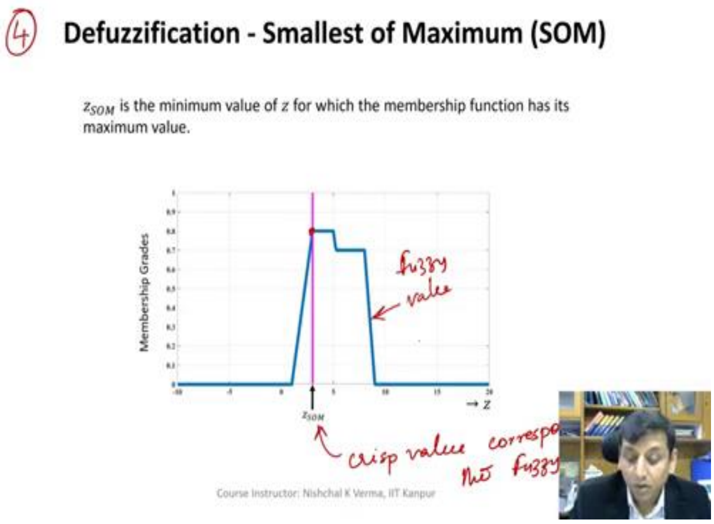

Then comes the fourth one here. The fourth here is the fourth one here is the smallest of maximum. So, in this case if we have the this as the fuzzy value mean fuzzy value.

So, we first take the maximum in this fuzzy value. So, maximum is here you see the maximum is here this portion is maximum. So, here we choose the smallest of this maximum section. So, smallest of the maximum section is this and then corresponding to this we take the output. So, this output is nothing, but the crisp value.

So, this is the crisp value this is the crisp value corresponding to three fuzzy value fuzzy value and this is fuzzy value, ok.

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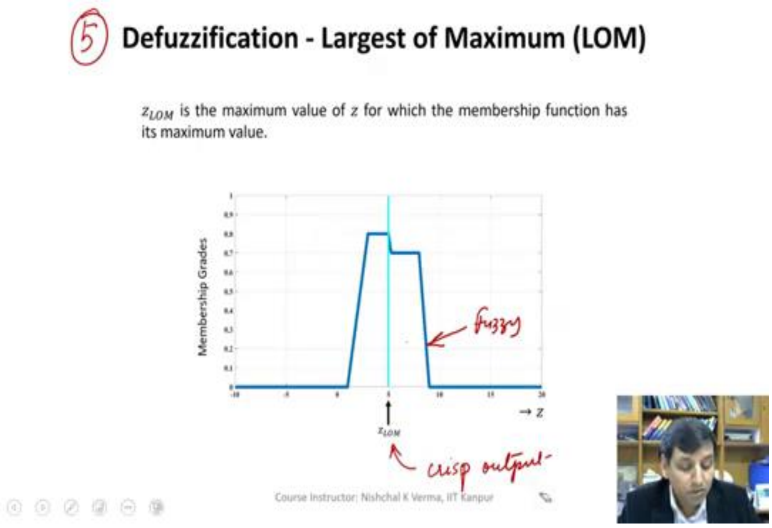

So, then comes fifth one and fifth one is the largest of maximum. So, if this is the fuzzy value here. So, this section again is the maximum. So, in this section we see which one is the largest. So, largest of the maximum will lie here this this end point corresponding to this output here the output value that is z is the crisp value.

So, this is the crisp value if we follow this criteria. So, here when we apply largest of maximum, if we have been given this as the fuzzy output and corresponding to this we will get the crisp output converted.

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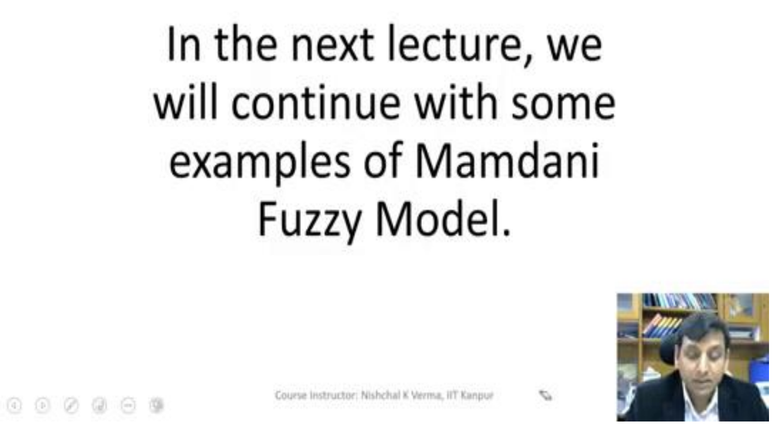

So, this way we have seen as to how the Mamdani fuzzy model operates for different scenarios, like we have the single Rule single antecedent, single Rule multiple antecedents, single multiple rules with multiple antecedents. And then with different compositions like max-min, max-product and again all of these with fuzzy inputs and the crisp inputs.

And that after that we discussed the some commonly used defuzzification methods. And with this I stop here and in the next lecture we will continue with some examples of Mamdani fuzzy model.

Thank you.