Fuzzy Sets, Logic and Systems and Applications
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Lecture – 52
Mamdani Fuzzy Model

Hi welcome to the lecture number 52 of Fuzzy Sets, Logic and Systems and Applications. In this lecture, we will continue our discussion with Mamdani Fuzzy Model.

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So, in Mamdani fuzzy model, let us understand the fuzzy reasoning of Mamdani fuzzy model for the following. Mamdani fuzzy model using Max-Min and Max-Product composition for fuzzy and crisp inputs. We have already discussed the fuzzy input for Mamdani fuzzy model using Max-Min and Max-Product compositions in our previous lecture.

So, we have already discussed the Mamdani fuzzy model using Max-Min and Max-Product compositions for fuzzy input with single rule and single antecedent.

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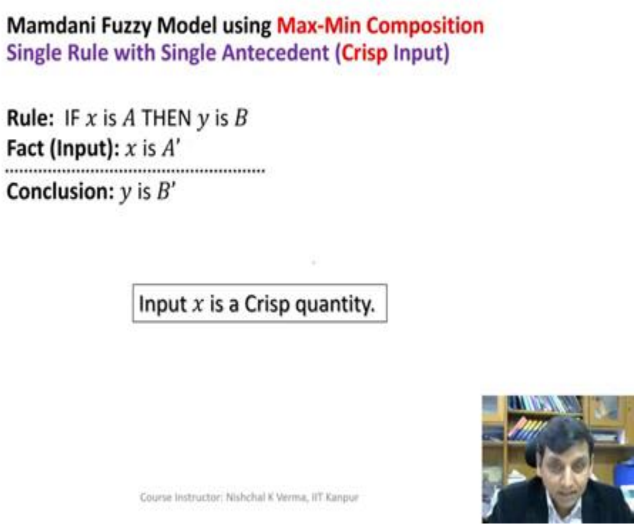

When we feed the input as the crisp value so, this means that when we apply x as crisp means the input means the crisp input we apply.

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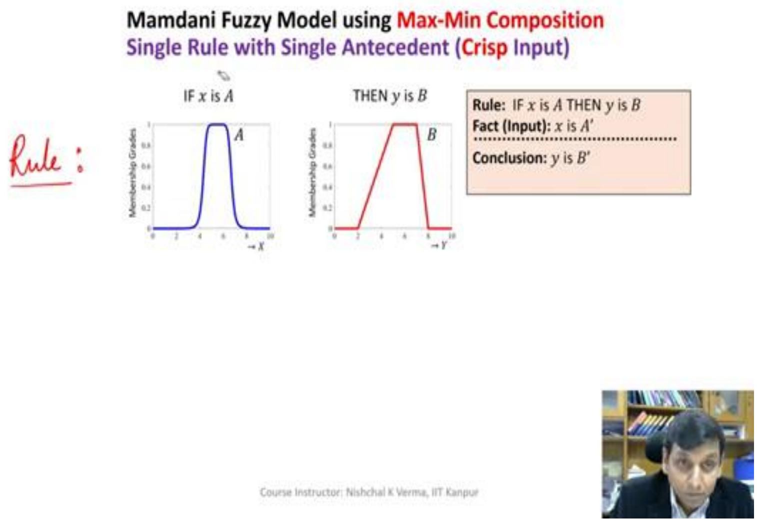

So, we see here that the rule we have a single rule in Mamdani fuzzy model. So, and in this rule we have a single antecedent here, only one antecedent. When we say only one antecedent, it means we have x is A, but when we say multiple antecedent or two antecedent and then here we have we would have x_1 is A_1 or and x_2 is may be A_2 or something like that.

So, multiple antecedent scenario would have been like that, but since here we have $x\ is\ A$, it means single input, we are we have in this fuzzy model. So, let us now apply the crisp input here like this.

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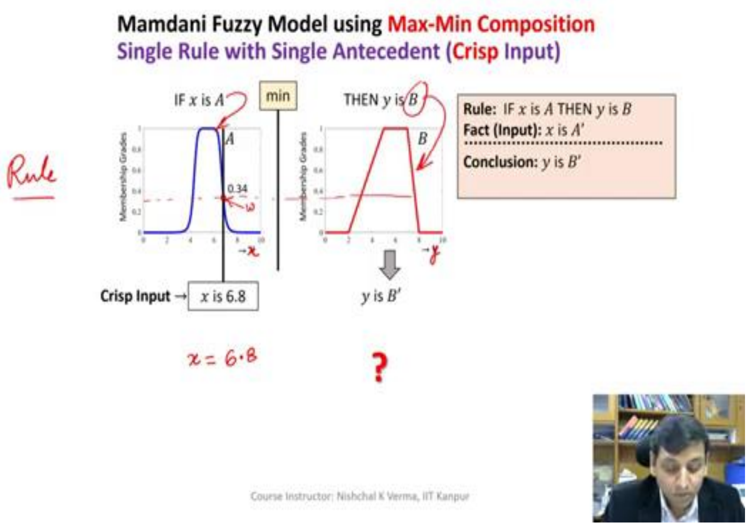

So, the value of x here is the value of x input the value of input x is a crisp value which is 6.8. So, let us now apply this. So, what we do here is we simply take this fuzzy set which is here as the input.

So, we have A, let us have A fuzzy set like this, then we find the intersection point. We draw a line at you see here in between 6 and 8, we have point 6.8. So, this line is this line gives us x is equal to 6.8. So, this value of the generic variable corresponding to this, we have the membership value 0.34. So, 0.34 we may get here.

Please note that when we have a crisp input so, the point of intersection is going to be only the single value. So, here there will not be any conflict as we have seen in the case of the fuzzy input. So, in fuzzy input, we can have multiple pointer points of intersection and then if you would have gotten the multiple points of intersections, you would have taken the maximum of these two. But here we have a single point of intersection.

So, this is our w which is the weight. So, we have a single w. So, even if we take min that is going to remain the same or even if we have we are going to take product, the w is going to remain the same because we have a single value. So, with this we will move forward and we use this value to truncate the output fuzzy set; output fuzzy set here is B. This is the B fuzzy set.

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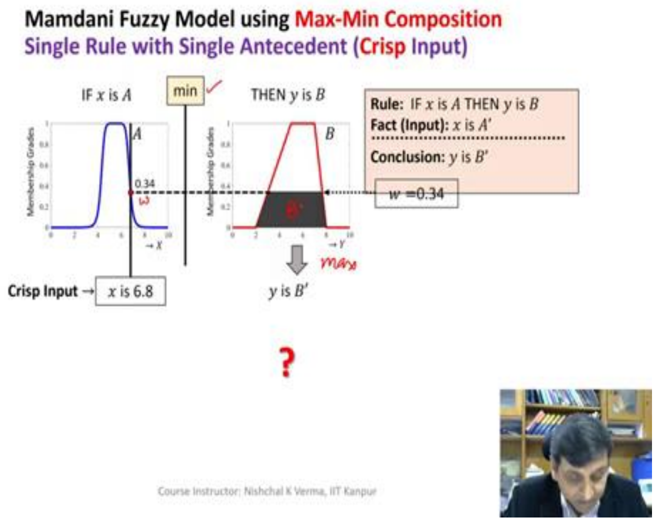

So, when we do that we see that point corresponding to the input x is equal to 6.8, we have this as the w and we use this w after either min product. This remains the same 0.34 because we have the single w.

So, we proceed further and we use this value to truncate the output fuzzy set B and again as I mentioned since we are using Max-Min composition so whatever we are taking first min and then we take the max here we, whatever we get we take max. So, max would have been applicable when we would have multiple rules applicable. And so, the outputs of all these rules would have been aggregated or output of all these rules would have been taken as a union of all these.

So, the max is not max is going to give the same value as we have here. So, this is the B' and this B' will remain the same even if we take max, same here.

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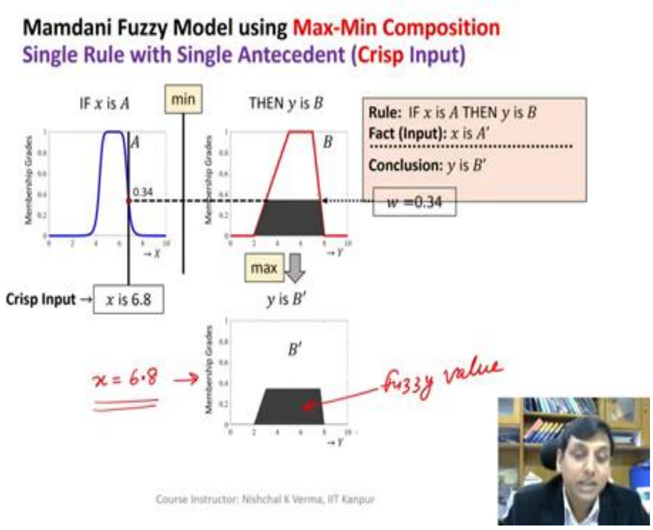

So, that is how the corresponding to x is equal to 6.8 is giving us the output as B' and this is nothing, but what? This is the fuzzy value fuzzy value, but this fuzzy value has no meaning unless we understand it like what is the corresponding crisp value out of it. We have fed the crisp value to the model, but we are getting the fuzzy value.

So, this fuzzy value has to be defuzzified to be further used. So, will discuss in the last as to how we convert this fuzzy value into the crisp value by various kinds of defuzzification models. So, here this fuzzy model which is basically having the single rule with single antecedent based on the crisp input that is x is equal to 6.8 is giving us the fuzzy output, the fuzzy value as the output which is nothing, but B'.

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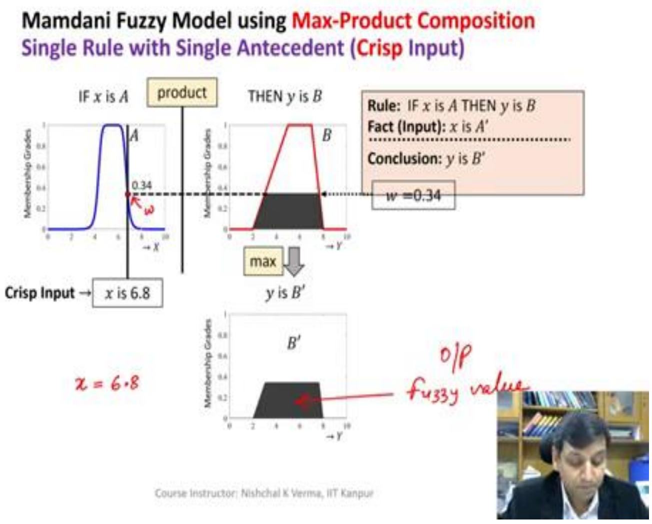

Similarly, the max-product composition for the same input is going to give us the same output. Because the product is also the product of the w here, the weight we have this single weight here.

So, the product is also going to remain the same and with this value with this weight value, we move further and we truncate and we are going to get these same output. So, x is equal to 6.8 is going to give us the same output. This is the fuzzy value as the output. So, whether we have here in this case the composition Max-Product or Max-Min, both are giving the same output.

And we know the reason because in both the cases here in this case only single rule with single antecedent is applied. So, single antecedent will produce only the single weight here, single weight w and the whether we use the max, whether we use the min or product this is going to give us the same. So, that is why the Max-Product or Max-Min both the composition are going to give us the same result.

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Now, let us move to this second scenario where we have the single rule with multiple antecedents. So, we have a Mamdani model; we have a Mamdani model here which has only one rule only one rule, but this rule has multiple antecedent means the premise part has multiple antecedents.

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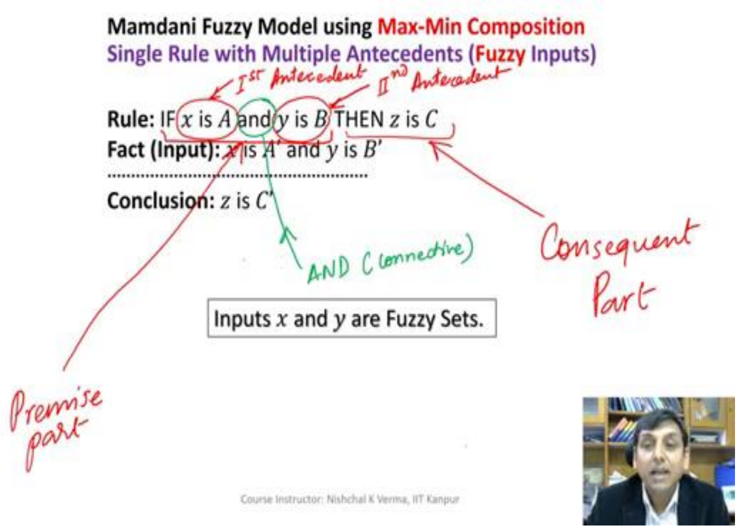

So, let us see how does it look like. So, you can see here we have a rule, this rule here and in this rule, we have the premise part which is this and then the consequent part. So, this is the premise part; premise part and this is the consequent part. So, in premise part, we see that we have two antecedents. What are those two antecedents here? First antecedent is $x\ is\ A$ and the second antecedent is so, second antecedent is $y\ is\ B$.

So, this is the I^{st} antecedent, I^{st} antecedent and $y\ is\ B$ is II, the II^{nd} one the II^{nd} antecedent. So, these two antecedents are connected by AND, so AND is nothing but a connective this could be by OR as well. So, we have a fuzzy model Mamdani fuzzy model which has a single rule, but two antecedents under the multiple antecedent case. And here also we have the inputs, but the inputs are two inputs here because we have two antecedents.

So, x and y both are the inputs to the model. Now this x and y can be either crisp or fuzzy, we will discuss both the cases. Now coming to the consequent part in consequent part, we have z as the output and C is the fuzzy value. So, consequent part is fuzzy. So, so when we see this rule, this rule has the premise part fuzzy because A and B both are fuzzy and the consequent part is also fuzzy because C is fuzzy. So, the rule type here is that the premise part is fuzzy and then the consequent part is also fuzzy.

Now, coming into the premise part, let us first understand that we have two antecedents. The first antecedent here is $x\ is\ A$, second antecedent is $y\ is\ B$ and both these antecedents are connected by connective AND, here this AND in place of AND there could be other connectives like or, but so, but in this case we have AND as the connective. So, we have in this premise part which is fuzzy we have two antecedents and these two antecedents have basically x, y as the generic variable values as the input.

So, x, y are the inputs. So, I hope this is very clear now. So, with this the fuzzy Mamdani fuzzy model is available. Now what happens when we apply the inputs x and y to this model, what is the kind of output that it generates? So, there can be two scenarios. The first scenario here is in this case that x, y can be the fuzzy input as we have seen in the previous case; x and y can be the fuzzy inputs, x can be x and y can be the crisp inputs and then here also we can have Max-Min composition and Max-Product composition.

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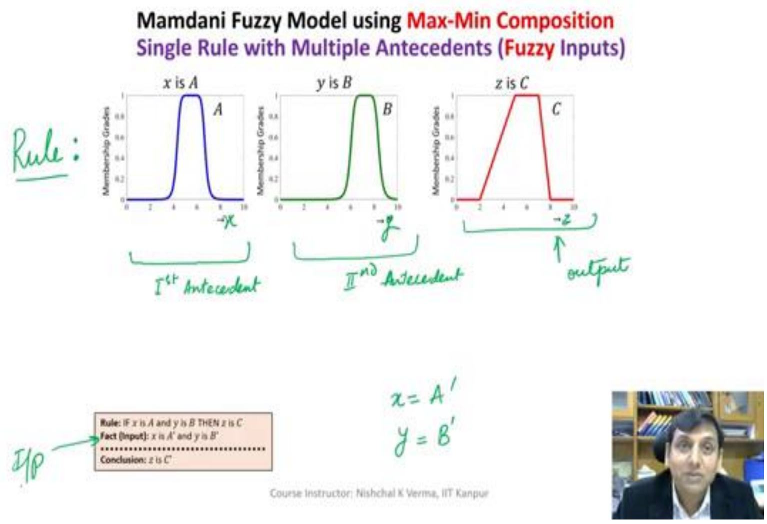

So, let us now go ahead and see the various cases. So, here we see that we have the model available and we have the rule. In this rule, we have the antecedent two antecedents. So, here we have the I^{st} antecedent. So, this is our I^{st} antecedent where we have $x\ is\ A$. So, here this is the I^{st} antecedent, the II^{nd} antecedent is here. So, I can write here the I^{st} antecedent, the II^{nd} antecedent then we have the output; this is the output.

So, this since this model is known it means A, B, C. All these three fuzzy sets are known. Now let us apply the input x, y. So, we call this as the fact here. So, let us apply the input x and y and let us apply fuzzy inputs. When we say fuzzy inputs, it means we give the fuzzy value and fuzzy value we all know that fuzzy value means a fuzzy set. So, let us give x is equal to x is equal to A', y is equal to B', x, y, z. So, now, let us do that.

So, since here x, y both are fuzzy values so, they are represented by the fuzzy sets, you see here.

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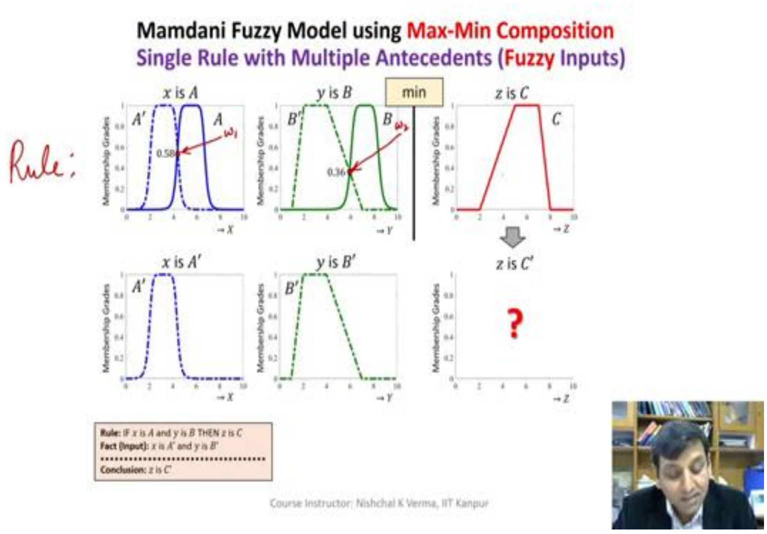

A fuzzy set A' fuzzy set, B' fuzzy set; both the fuzzy sets are here. Now let us apply this was the rule which is already known and this model Mamdani fuzzy model. When we apply this input means x is equal to A', y is equal to B', let us see what it is going to produce.

So, again like in the previous scenario, we superimpose these two fuzzy sets to their corresponding inputs input fuzzy sets. So, with A' here is superimposed to A because x is A' and then y is B'. So, this also is superimposed to the fuzzy set which is already known.

So, we see that when we superimpose these two, when we superimpose these two we see the point of intersection here and we call this as the w_1. So, let us represent this by the w_1. So, this is w_1, the intersection point. So, there is the first weight. Similarly here when B and B' intersects let us say this is w_2. So, these two are two different values of weights and these weights are nothing but the corresponding, you see here the intersection corresponding membership values.

So, you can easily get the corresponding membership value and you call this as the w_1. Here also you get the corresponding membership value and you call this as the w_2. So, these two weights are available. Now, if we apply Max-Min composition. So, we take min of these two. So, min of these two values min of 0.58, min of 0.58 and 0.36,obviously, the min of these two is going to be 0.36. So, we will take 0.36 and we proceed truncation with this value.

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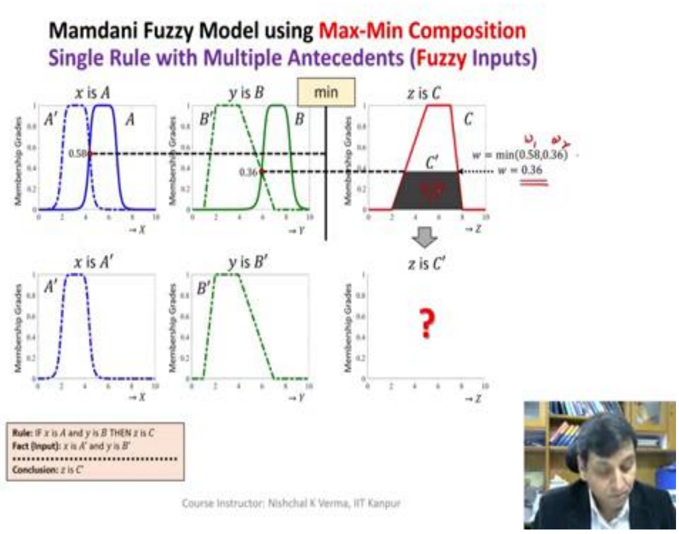

So 0.36, we use for truncation and this is what we are getting as the min value min of the two; w_1 and w_2. So, this is the output of the first rule. I mean the we have only one rule so, the output of this rule. If we would have multiple rules where we will see that in coming sides, then you would have taken max of all this, we would have aggregated the outputs of all the rules.

So, here since we have only the single rule so, this will become the output corresponding to the inputs x is equal to A', y is equal to B' connected with A connected with AND.

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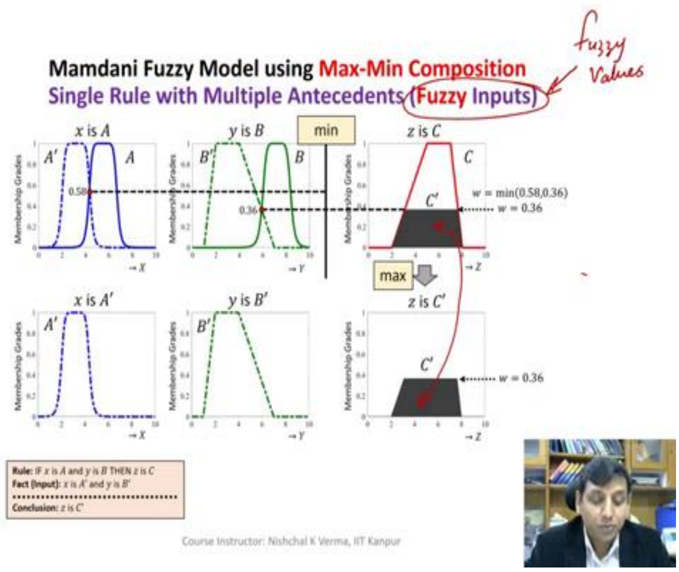

So, you see this is what is the output that we are getting. The same output here because the max is here we come since we are using Max-Min composition, max is not really applicable because we have the single value, single rule, the output is single. So, the max is giving us the same thing whatever is here is coming as the output. So, this way in the second scenario, we get the output. So, here in this case the input is fuzzy. So, you see here the input is fuzzy values fuzzy values or the fuzzy sets.

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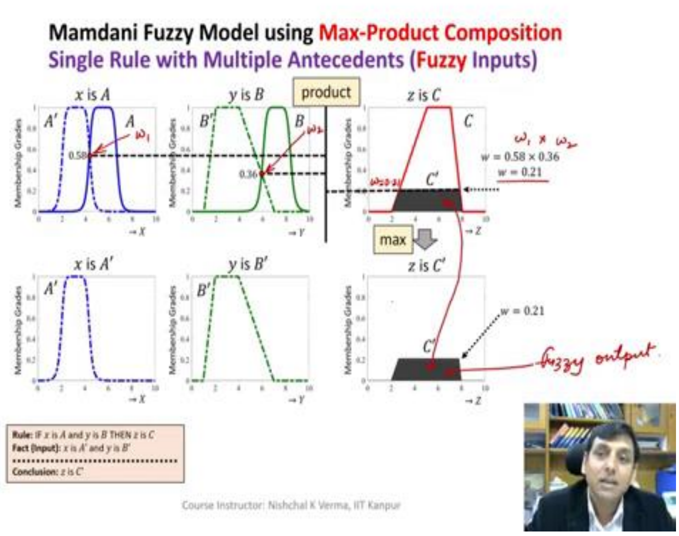

Now, in the second scenario what happens when we use when we use the Max-Product composition? So, say with the same input same set of fuzzy inputs. So, everything remains the same as we have already seen that the intersection point is designated as the w_1 and here w_2, the weights and weights we have already seen that 0.58 and 0.36.

So, here since we are using Max-Product composition instead of Max-Min where we have taken a min of w_1, w_2. Here we are taking the product of w_1 and w_2 because we are using Max-Product composition. So, we will take the product of w_1 and w_2 and the product of 0.58 and 0.36, you see there is w_1 and there is w_2. When we take the product, we are going to get the final value that is the product of this w_1 and w_2, we are getting 0.21.

So, since we are getting 0.21, we truncate with this membership value with this weight value which is 0.21. So, we see as compared to the previous case, the fuzzy value is lesser than that of the Max-Min case. So since we have now, we cannot take the max because max is really not applied, we have only single rule. So, the same output is transferred over here.

So, we see that we have the C_1 because this as the C_1 as the output. So, max Max-Product is giving us again the fuzzy output. Here is a again the fuzzy output and this we can defuzzify to get the crisp output; this fuzzy output. Fuzzy output is there in the form of a fuzzy set. So, in the second scenario, we have seen that how Max-Product, Max-Min produces the output for fuzzy inputs to two antecedents.

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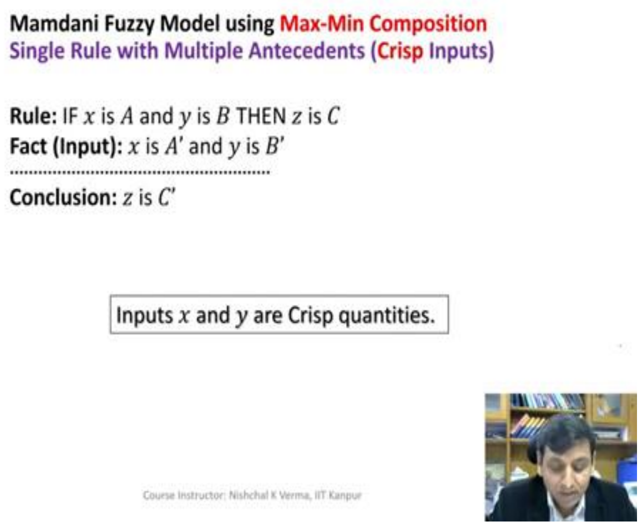

Now next is in the second scenario itself instead of fuzzy inputs, let us take the crisp inputs.

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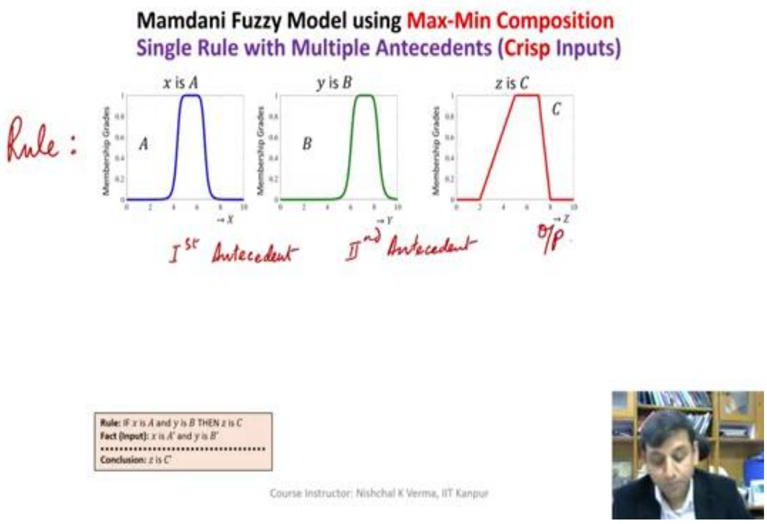

So, again on the same lines, we see that we have the rule here which is the single rule, but multiple antecedents. So, here we have two antecedents instead of A, under multiple antecedents we have two antecedents. This I^{st} antecedent I^{st} and the II^{nd}; I^{st} antecedent, II^{nd} antecedent and here we have the output. So, when we apply the inputs as crisp values, it is very simple instead of fuzzy where we applied the fuzzy value in form of fuzzy set.

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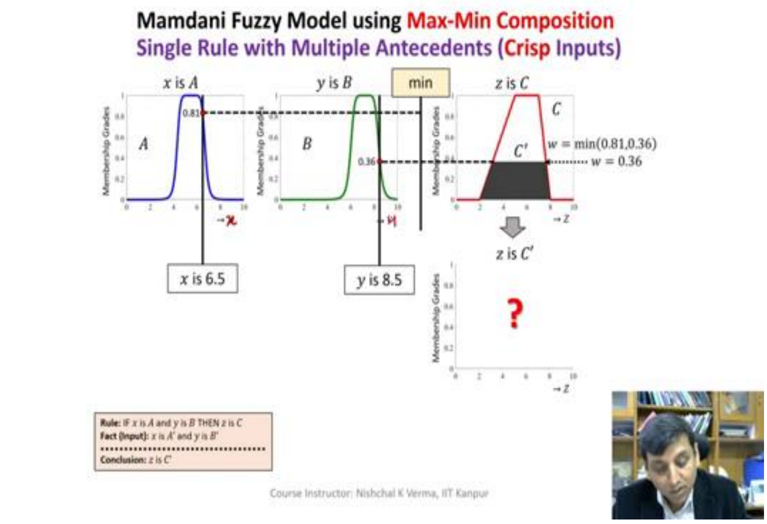

So, there we could have multiple points of intersection and we could have avoided this by taking the max within that antecedent. Here that case will never exist means the conflict will never exist rather than we are going to get always the single value single intersection value, you see here for x is equal to point for x is equal to 6.5 for here x, y. There is a small x y; here also x ,y, z.

So, when we take x is equal to 6.5, y is equal to 8.5; we get the point of intersection as w_1, point of intersection as w_2 and this way we have two points of intersection. Now for Max-Min composition, we have to take min of these two, min of w_1, w_2.

So, when we take the min of w_1 and w_2, we get again the we get 0.36 here and since we have only one rule so, maximum of the Max-Min composition does not apply here and the truncated value here, the fuzzy value will become the final output. So, this is the final output of the model corresponding to the fuzzy input x is equal to 6.5, y is equal to 8.5.

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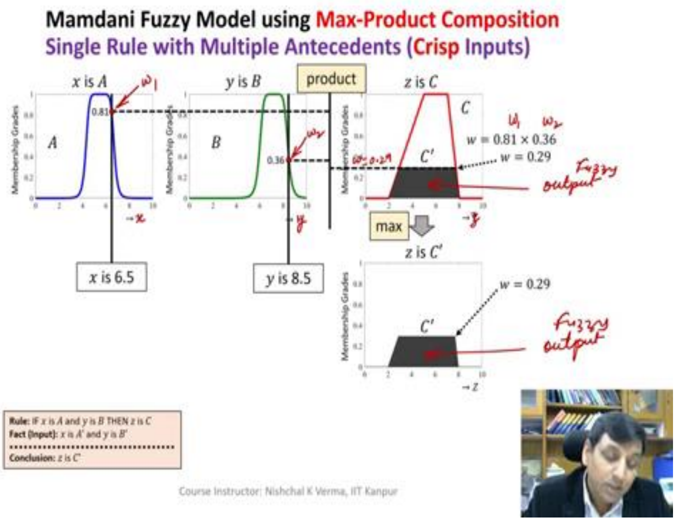

Now, what happens when we take the Max-Product composition? Since we already have this w_1, the weight w_2 the other weight corresponding to the x is equal to 6.5 and y is equal to 8.5. So, when we multiply this when we take the product of these two weights, we see here we get w is equal to 0.29. So, with this value that is w is equal to 0.29. So, with this value, we truncate the fuzzy set C.

So, with this value we get the output this is the output out of the first rule. This is the fuzzy output the truncated shaded area is the output that is fuzzy output basically fuzzy output and then again the max is not applied here because we have only one rule, single rule. So, we do not have to do anything further and the same output is the becomes the final output. So, the same is the final output that is the fuzzy output which is in the form of fuzzy set the truncated fuzzy set, I can write here if fuzzy output.

Now, we can use suitable defuzzifier to convert this fuzzy output into the crisp value.

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So, in today's lecture we have discussed the following. Mamdani fuzzy model using Max-Min and Max-Product compositions for single rule with single antecedent for crisp input. And also we have discussed the Mamdani fuzzy model using Max-Min and Max-Product compositions for single rule with multiple antecedents for fuzzy and crisp inputs. And in the next lecture, we will continue our discussion with remaining part of the Mamdani fuzzy model.

Thank you.